Reflections and the Focusing Effect from an Ideal Three-Dimensional Rough Surface*

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INTRODUCTION

We begin with an ideal surface in three dimensions and a simple definition of a roughness metric for it. Reflections from this surface, given the reflectance properties of a shallow-water ocean bottom, are considered with the aim of expressing the resulting reflectance analytically. Focusing effects of up to second-order reflections are discussed as well as the behavior of shadowing and obscuration for this simplified surface.

THE EGG-CARTON SURFACE

In order to simplify the description of a rough shallow-water homogeneous ocean bottom, we model it as an egg-carton surface represented by

$$Z = A^2 \sin\left(2\pi \frac{X}{L}\right)\cos\left(2\pi \frac{Y}{L}\right) + A$$

where A is the amplitude of the basic sinusoidal function in the x-direction with its corresponding length, L. X and Y represent coordinates in the xy-plane and Z represents the surface points offset from the reference plane, which also represents the average depth given a water surface. The bottom surface is oriented parallel to the water surface to emulate a bottom that is not sloping. Roughness of this ideal surface is expressed as the amplitude-to-length ratio, A/L. We suggest that any real surface of interest can be approximated by an egg-carton function and will effectively have a comparable roughness metric. Just to note, the light source is assumed to be infinitely distance so that the irradiance is uniform over the water surface.

The reflections considered are only those that find their way to the detector just below the water surface so that air-water transmission effects are eliminated while near-field observation effects are highlighted. While we limit ourselves to in-water bottom reflections, attenuation through the water is an integral part to our later understanding of the overall wavelength dependence.

Before we move on, it is important to clarify our definition of a single waveform and how it is used in the three-dimensional sense. In the two-dimensional case this is defined as a sinusoidal wave from one peak to the next, so that all

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interreflections considered are only due to other parts of the waveform. Its extension in three dimensions is the area that is formed by a valley (or depression) on the surface, that is, bounded by four peaks and the saddle ridges that connect them (this forms a diamond on the projected surface, see Figure 1). The reflections that produce the focusing though are not only limited to the single valley but to a pre-determined area of observation (more later).

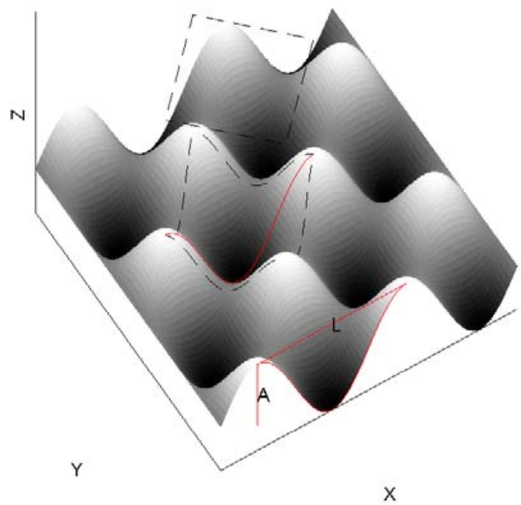

Figure 1. The egg-carton surface and what is defined here as a single waveform, whose projection is a triangular area onto a reference surface (offset here for convenience). The basic sinusoidal wave is indicated with a given length, L, and amplitude, A, that both determine the roughness of the general surface.

The detector field-of-view (FOV) is adjusted so that the same projected surface area is observed as the depth is varied or as the roughness is increased. Note that the number of waveforms remains the same if we keep the waveform length constant.

FOCUSING

The behavior of the first-order reflectance is as expected, which is similar to the two-dimensional case: decreasing for the whole surface as roughness increases (Clavano, W. R. and Philpot, W. D., 2003 , *Proceedings of the International Conference on Current Problems in Optics of Natural Waters II*, 08-12 September 2003, St. Petersburg, Russia). Figure 2a shows the radiance from each point on the surface as the roughness increases when illuminated by light incident at 30º to the vertical; totals are given in Figure 2b, which shows that the first-order reflectance viewed by a nadir-looking sensor directly above the trough.

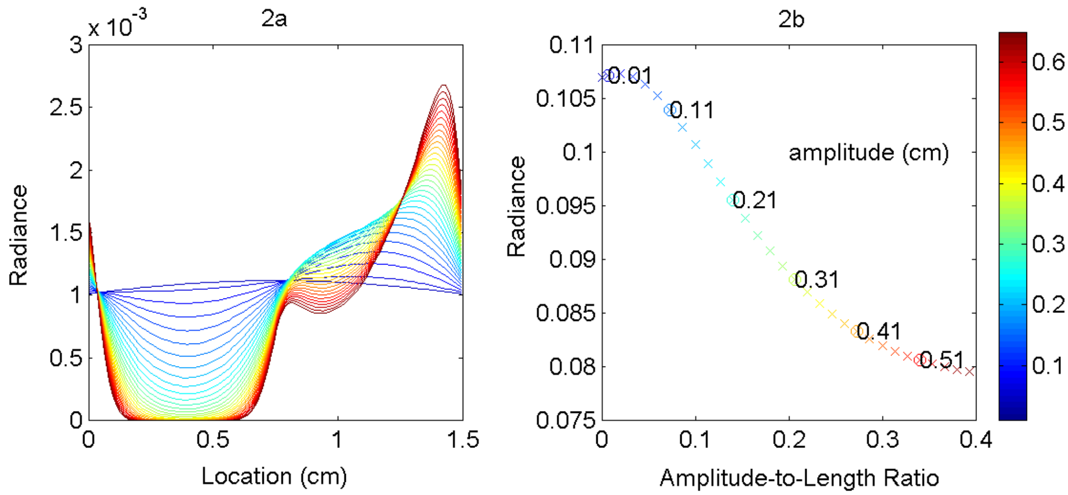

Figure 2. Radiance from each point Fig. 2a and the total first-order radiances Fig. 2b. The incidence angle (relative to the vertical) of the initial illumination is 30º.

Focusing effects occur as the incidence and reflection angles maximize the radiances received at the detector. This phenomenon is complicated by the introduction of second-order reflections that can either enhance or reduce the focusing effect depending on the viewing and incidence angles.

Along a plane about the vertical axis, a roughness ratio for a fixed-length waveform that returns the highest reflectance can be found. This in effect maximizes the bi-directional reflectance distribution function (BRDF), which when inverted along with the associated brightness patterns observed from a surface will produce an indication of its roughness. As expected though, the optimization considering all hemispherical directions is not quite so simple even when everything else is held constant.

SHADOWING AND OBSCURATION

Shadowing from direct illumination is determined simply by the angle of incident light and the slopes of the surface along the solar plane. In contrast, shadowed areas for interreflected light are those simply not within the "line-of-sight" of other parts of the illuminated surface. Figure (3) shows an example of a point on the surface marked "*" whose interreflections illuminate the areas within its line-of-sight, shown in brighter shades on the figure.

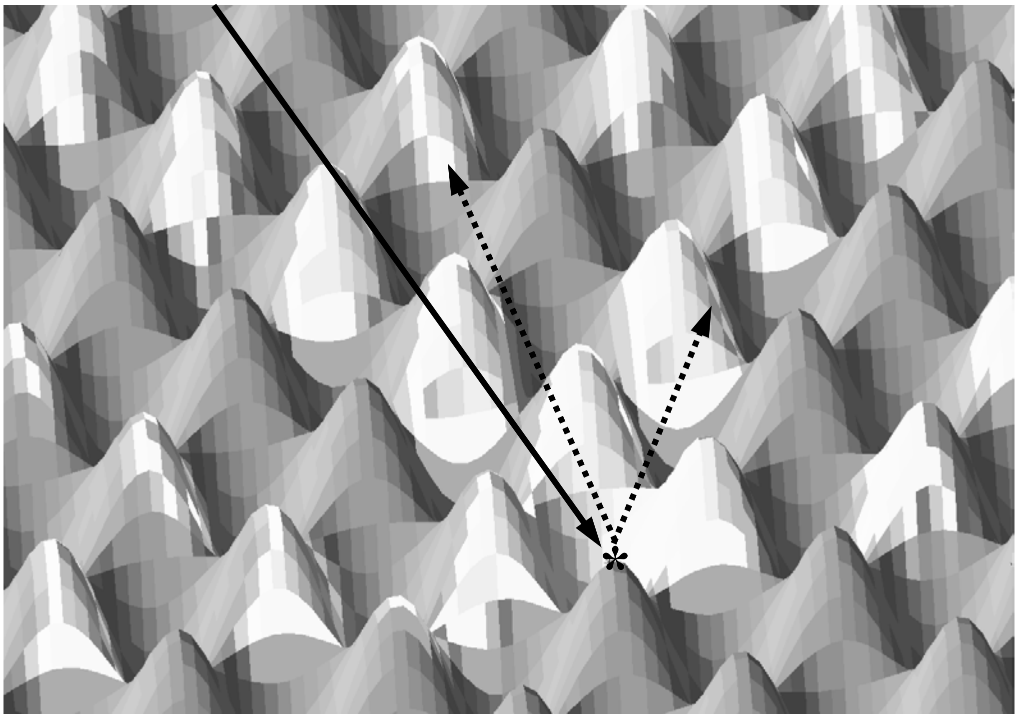

Figure 3. Areas within the line-of-sight of a source point (*) are indicated with brighter shades. The continuous line represents the incidence direction and the dashed lines are examples of interreflections.

If we were to look at a transect of the surface from a source point, we would see areas that are shadowed and those that are not (Figure 4). In the first-order, shadowing will contribute to the drop in the final total reflectance. There is no abrupt change though as the surface gets rougher because only increasingly large areas are shadowed (refer back to Figure 2a and notice the zeroing out of radiances as the bottom gets rougher).

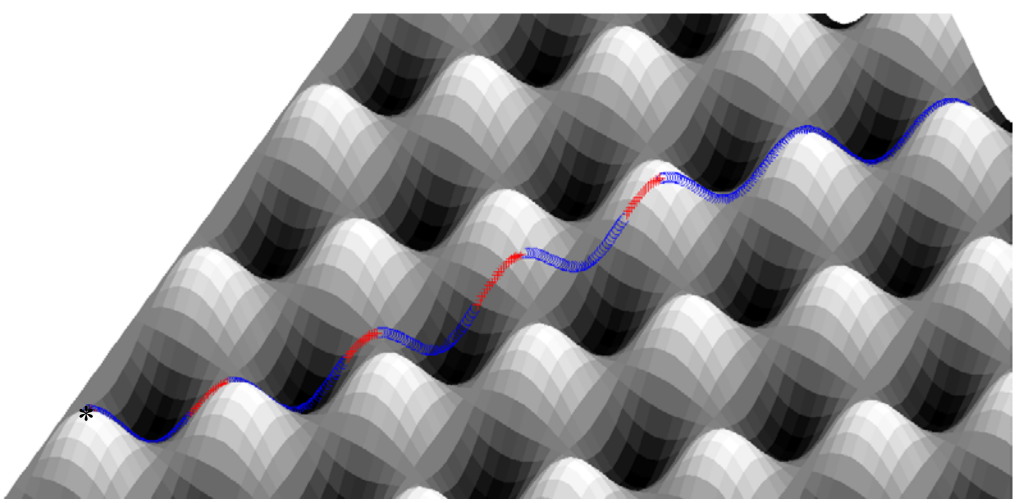

Figure (4). Shadowed areas (○) and illuminated areas (+) as seen from a source point (*) looking out along one azimuthal direction.

In higher-order reflections, these shadowed areas, while they may not be directly illuminated, will have the potential to interreflect light. For a flat surface, just as in the two-dimensional case, there will be no second-order reflections. As roughness increases, the total second-order surface radiances will increase until shadowing occurs. Results of the increase in second-order reflectance radiance are shown in Figure 5b for the two-dimensional case; similar are expected for the ideal three-dimensional surface presented.

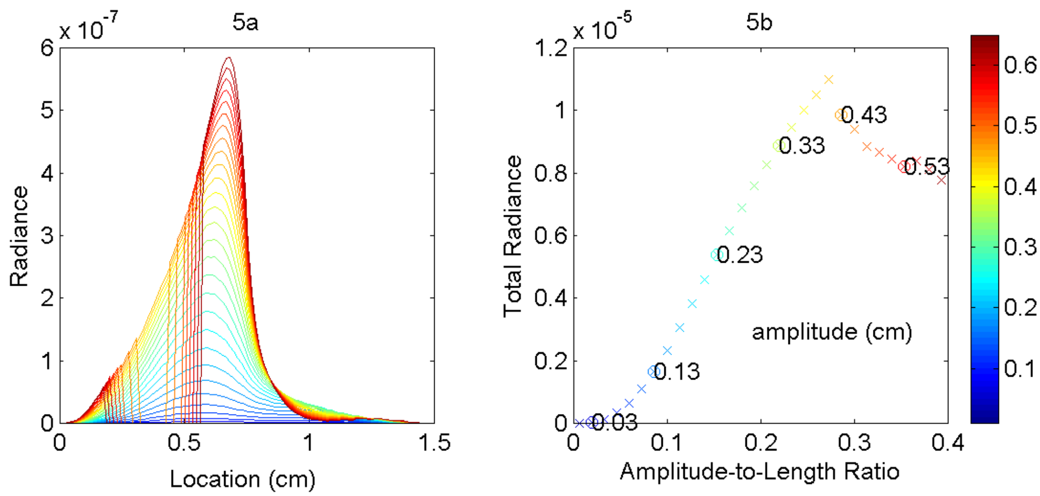

Figure (5). Total radiances from each point along a two-dimensional waveform (Fig. 5a), and total radiances at the detector (Fig. 5b).

Obscuration occurs when parts of the surface are not in the line-of-sight of the detector. It does not occur for a detector viewing at nadir until it comes close enough to the surface, assuming that its FOV is adjusted to the bounds of the surface of interest. Obscuration between points along the surface is the same thing as one point or the other being shadowed. It will have a similar effect to shadowing in reducing the radiances of higher-order reflections as roughness increases. The contribution to the total radiance will be much less that that of shadowing.

LIMITATIONS AND FUTURE CONSIDERATIONS

The potential of higher-order reflections will increase as the roughness increases; whether or not the magnitude of higher order reflectances will significantly affect the total final reflectance will depend on the accuracy required for a specific application. The distribution of higher order reflections for an ideal sinusoidal surface is expected to resemble a Gamma-like function, to be affected by shadowing or obscuration. While the interreflecting areas of a point on the surface of interest can be fully determined (but limited to a certain radius), only the areas that are within the detector footprint are considered in the total reflectance, i.e., interreflections from adjacent areas are included.