



Hydrologic Discovery Through Physical Analysis - Honoring the Scientific Legacies of **Wilfried H. Brutsaert and Jean-Yves Parlange**



Infiltration into soils



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Definition of Infiltration

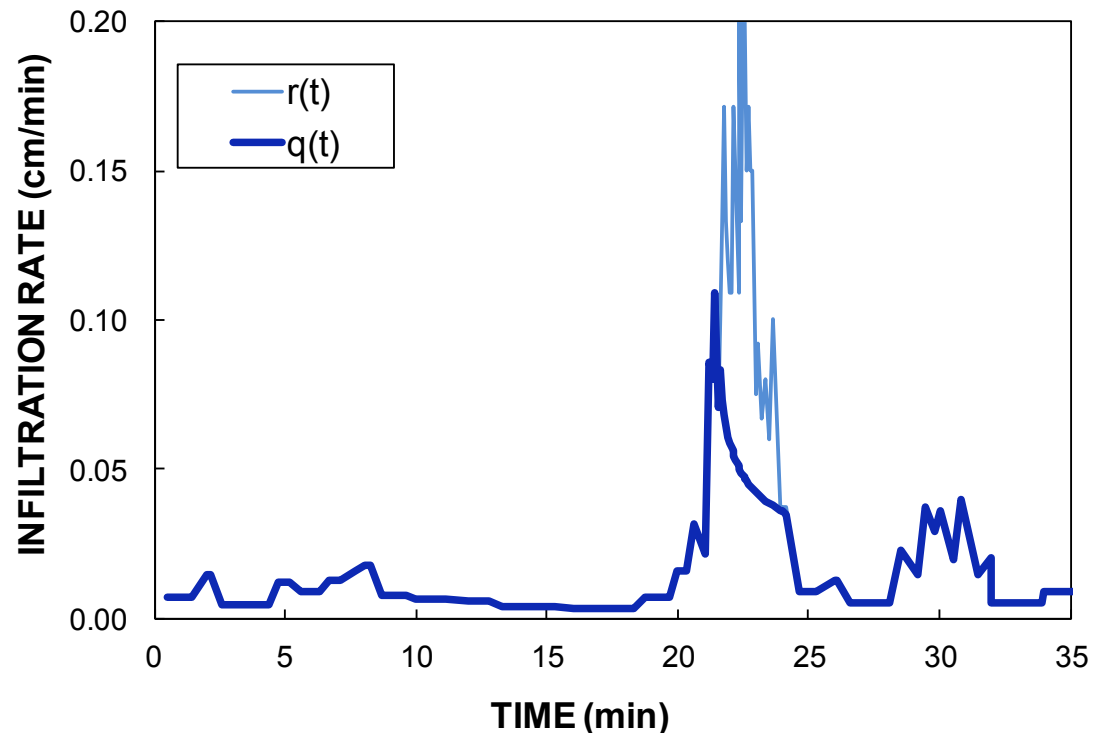
- Infiltration: Entry of water into the soil surface and *its subsequent vertical motion through the soil profile* (Brutsaert, 2005).

- Infiltration is a complex process, which depends upon several factors: water supply rate; distribution of the hydraulic properties within the soil profile; initial and boundary conditions; topography; water quality; soil salinity and sodicity; temperature gradients within the soil profile....



Infiltration regimes

- Two infiltration regimes could be identified:
- When all the water supplied to the soil surface infiltrates (flux-type or Neumann boundary condition);
- When only part of the water supplied to the soil surface infiltrates while the remaining part remains ponded or runs off (concentration-type or Dirichlet boundary condition). This case is termed the “**infiltration capacity**” of a soil.



The first “falling-head ponded infiltration with evaporation” experiment at the global scale



“and the ark floated on the face of the waters.....the waters prevailed above the mountains, covering them **fifteen cubits deep**.

And the waters continued to abate until the tenth month; **in the tenth month, on the first day of the month**, the tops of the mountains were seen.”

(Genesis, 7-8)

Information on *ponding depth; time; cumulative infiltration and rate.*

Infiltration experiment at the laboratory scale

Freshwater irrigated soil

Wastewater irrigated soil



Water movement in soils: a complex issue

It is likely that, from the dawn of humanity, men (and women) were puzzled by the fact that water was disappearing slowly from ponds. Until now, the movement of water into soils is perceived as a very complex problem, if one judges from these two citations made at an interval of 5 centuries:



“I can foretell the way of celestial bodies, but can say little about the movement of a small drop of water”

Galileo Galilei (1564-1642)
(from Hillel, 1998)

“Mis en face de la realite d'un milieu poreux, l'esprit peut s'effrayer d'abord de la tache a entreprendre”

Georges Matheron (1930-2000)
(from Philip, 1970)

Evolution of the conceptual perception of flow through porous media

H. Darcy (1856)

$$q = \frac{V}{At} = \frac{Q}{A} = -K_s \frac{\Delta H}{L}$$

E. Buckingham (1907)

$$q = -K(\psi) \frac{\partial H}{\partial z} = -K(\psi) \frac{\partial(\psi + z)}{\partial z} = -K(\psi) \left(\frac{\partial \psi}{\partial z} + 1 \right)$$

L. A. Richards (1931)

$$\frac{\partial \theta}{\partial t} = \nabla \cdot [K(\psi) \nabla H]$$

1-D vertical and isothermal infiltration
into homogeneous non-swelling soil

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[K(\psi) \left(\frac{\partial \psi}{\partial z} + 1 \right) \right]$$

Diffusion equation form (non-linear
Fokker-Planck diffusion convection
equation)

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left(D(\theta) \frac{\partial \theta}{\partial z} + K(\theta) \right)$$

Solution of the unsaturated flow equation – time expansion approach

J.R. Philip (1957, 1969)

- Fails to converge to $q=K_s$ for large values of t
- $$I(t) = S t^{\frac{1}{2}} + A_1 t^{\frac{2}{3}} + A_2 t^{\frac{3}{4}} + K$$

- Truncated expression for vertical infiltration:
- $$I = S \sqrt{t} + A_1 t$$

- The theoretical limit $q=K_s$ for large values of t requires:

$$A_1 = K_s$$

- Experimental data suggest
(Philip, 1969; Talsma and Parlange, 1972)

$$\frac{1}{3} K_s \leq A_1 \leq \frac{2}{3} K_s$$

- **Brutsaert (1977)** has shown that for some cases (mainly narrow pore size distributions), the time expansion method can behave properly for both small and large t values:

$$I(t) = K_s t + \frac{S^2}{\beta K_s} \left\{ 1 - \left[1 + \beta \left(\frac{K_s \sqrt{t}}{S} \right) \right]^{-1} \right\}$$

Solution of the unsaturated flow equation – the integral approach

J. Y. Parlange (1971, 1972)

- Different form of the flow equation

$$\frac{\partial z}{\partial t} + \frac{\partial}{\partial \theta} \left[\frac{D}{(\partial z / \partial \theta)} \right] = \frac{dK}{d\theta}$$

$$t = 0, \quad z > 0, \quad \theta = \theta_0$$

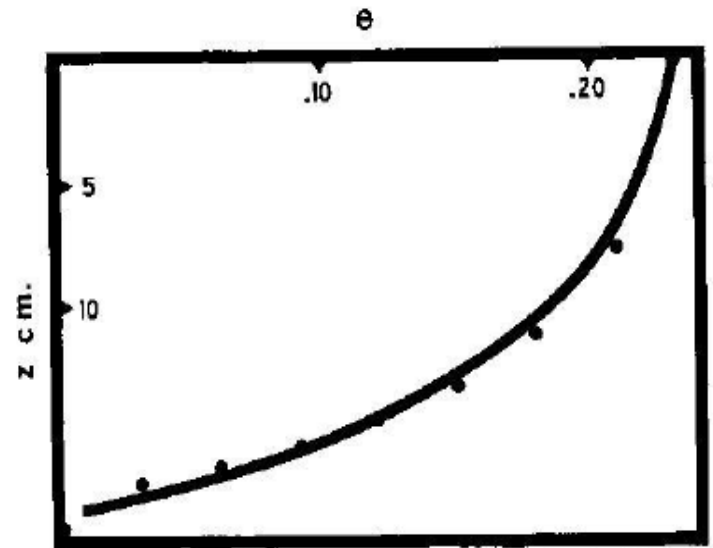
$$t \geq 0, \quad z = 0, \quad \theta = \theta_s$$

- Integral representation solved iteratively (double integration technique)

$$\int_{\theta}^{\theta_s} (\partial z / \partial t) d\theta + D_s (\partial z / \partial \theta)_s - D (\partial z / \partial \theta) = K_s$$

- For initially dry soils ($\theta = \theta_0 = 0$ and $K = K_0 = 0$)

$$\int_{\theta_0}^{\theta_s} (\partial z / \partial t) d\theta + D_s (\partial z / \partial \theta)_s = K_s$$



Parlange, 1972

Solution of the unsaturated flow equation – the integral approach

- The integral representation and the concept of iterative solution of the unsaturated flow equation introduced by Parlange (1971) laid the ground for the **flux-concentration method** proposed by Philip few years latter :

“We present a new quasi-analytical technique for solving the flow equation. It has affinities with Parlange’s method,.....” (Philip and Knight, 1974)

- It was also applied by Brutsaert (1976) to derive a general form for horizontal infiltration :

$$\phi = x t^{-1/2} = \left(2 / \int_0^1 D_w S_n^a dS_n \right)^{1/2} \int_{S_n}^1 D_w(y) y^b dy$$

where a and b are constants that depend on the approximation used in the solution.

Universal model of the unsaturated flow equation

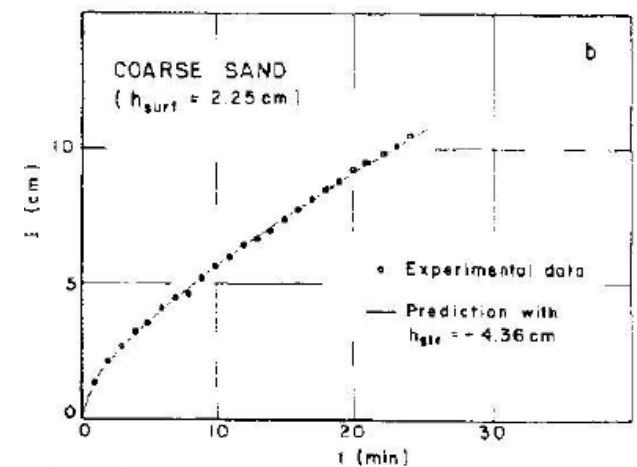
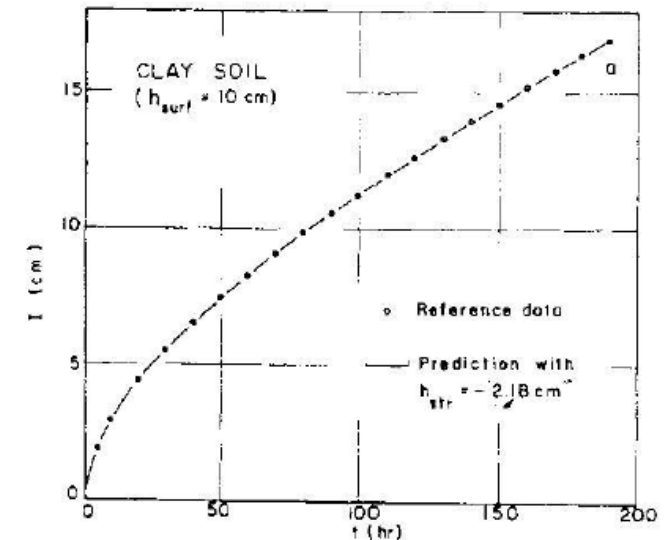
Parlange et al., 1982

$$t = \frac{S^2}{2K_s^2(1-\delta)} \left[\frac{2K_s}{S^2} I - \ln \frac{\exp\left(\frac{2\delta K_s I}{S^2}\right) + \delta - 1}{\delta} \right]$$

(Assuming K_0 is very small compared to K_s)

$$\delta = \frac{1}{\theta_s - \theta_0} \int_{\theta_0}^{\theta_s} \frac{K_s - K(\theta)}{K_s} d\theta$$

- For $\delta=0$, the model reduces to the Green-Ampt solution (**Parlange et al., 1985**)
- Generalized solution of **Haverkamp et al. (1990)**
- Explicit infiltration equation of **Barry et al. (1995)**



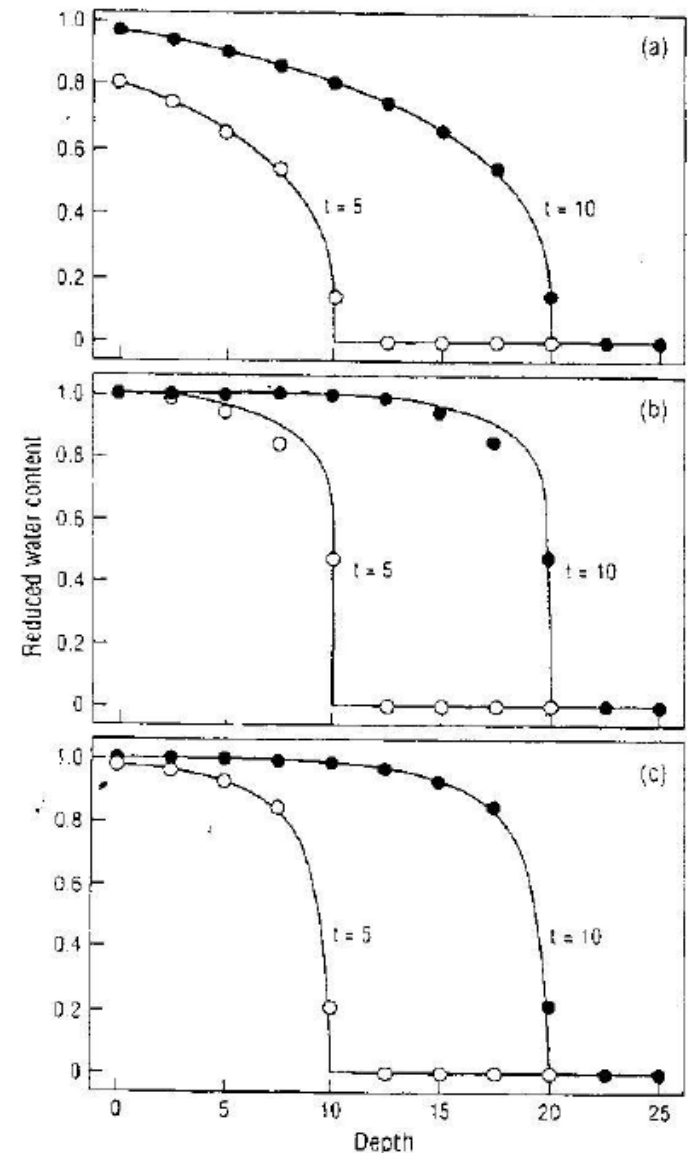
Haverkamp et al., 1990

Solution of the flow equation – the *Heaslet and Alksne* technique

$$\int_0^{\theta_s} \frac{D d\theta}{q(\theta / \theta_s) - K} = z + M(t) z^2$$

$$I(t) = \int_0^{\theta_s} \frac{D \theta d\theta}{q(\theta / \theta_s) - K} - M(t) \int_0^{\theta_s} z^2 d\theta$$

- Initially developed for a power-law diffusivity
- Extended to arbitrary soil water diffusivities (Prasad and Romkens, 1982; M. Parlange et al., 1992)
- Improves the solution of Parlange (1972)
- Provides accurate results
- Led to analytical approximations of Richards' equation (Barry et al., 1993; Parlange et al., 1997; Parlange et al., 1999)



Ross and Parlange (1994)

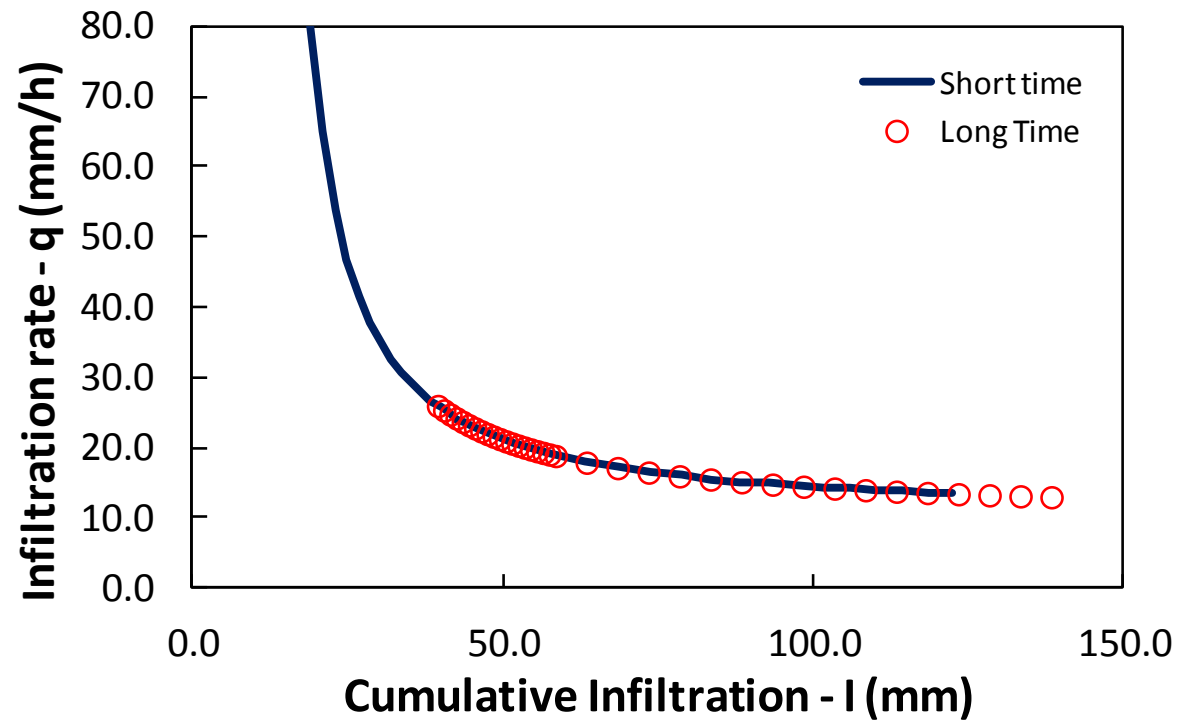
Back to the Universal Model of Parlange et al. (1982)

Cumulative infiltration for:
Short time

$$I = St^{1/2} + \frac{(2 - \delta)}{3} K_s t$$

Long time

$$I = K_s t + \frac{S^2}{2K_s(1 - \delta)} \ln\left(\frac{1}{\delta}\right)$$

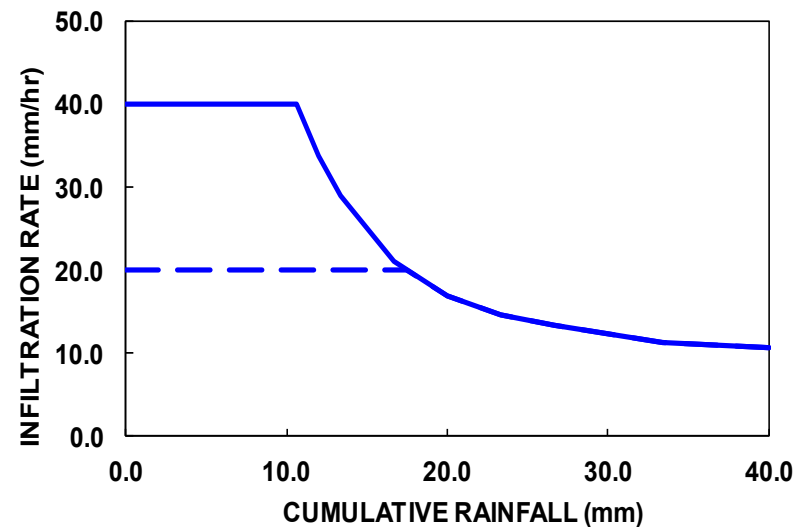
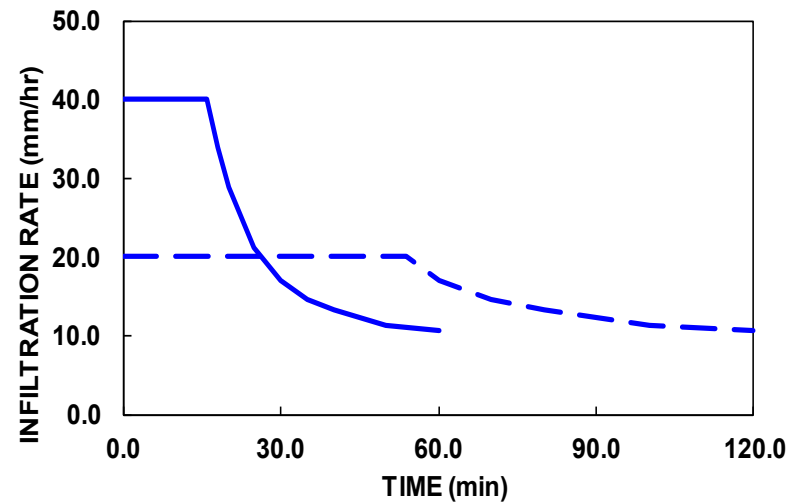


Infiltration rate , q (Espinoza, 1999)

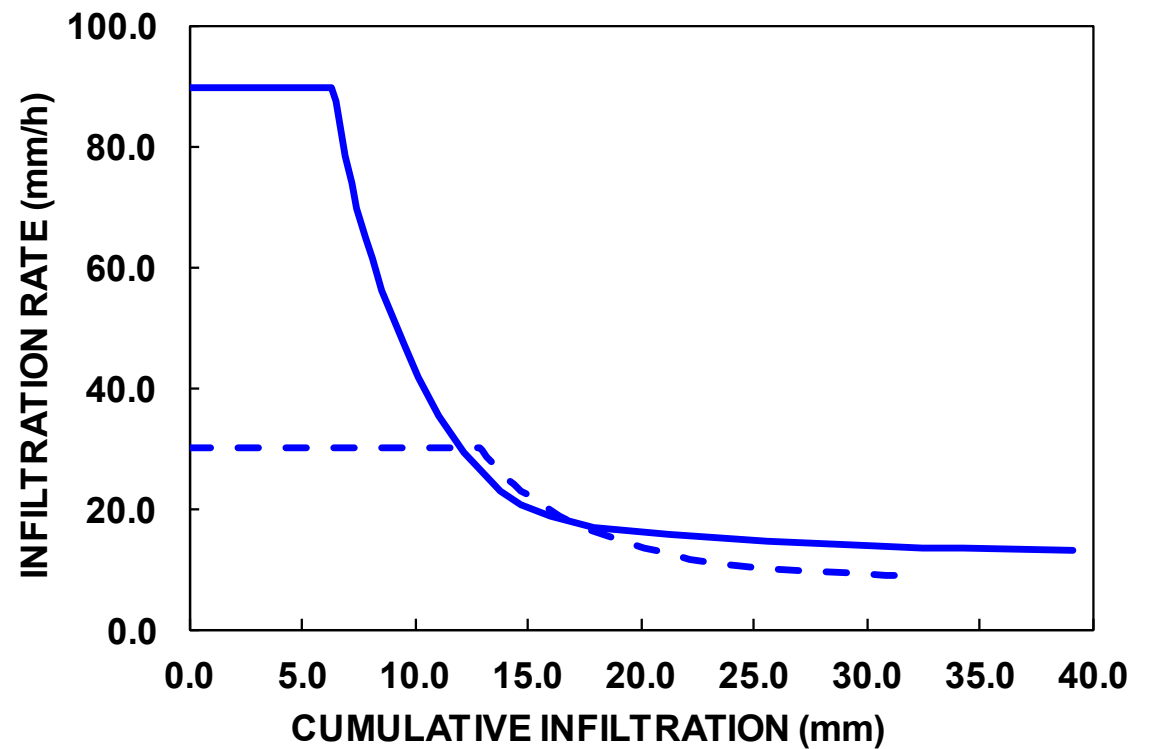
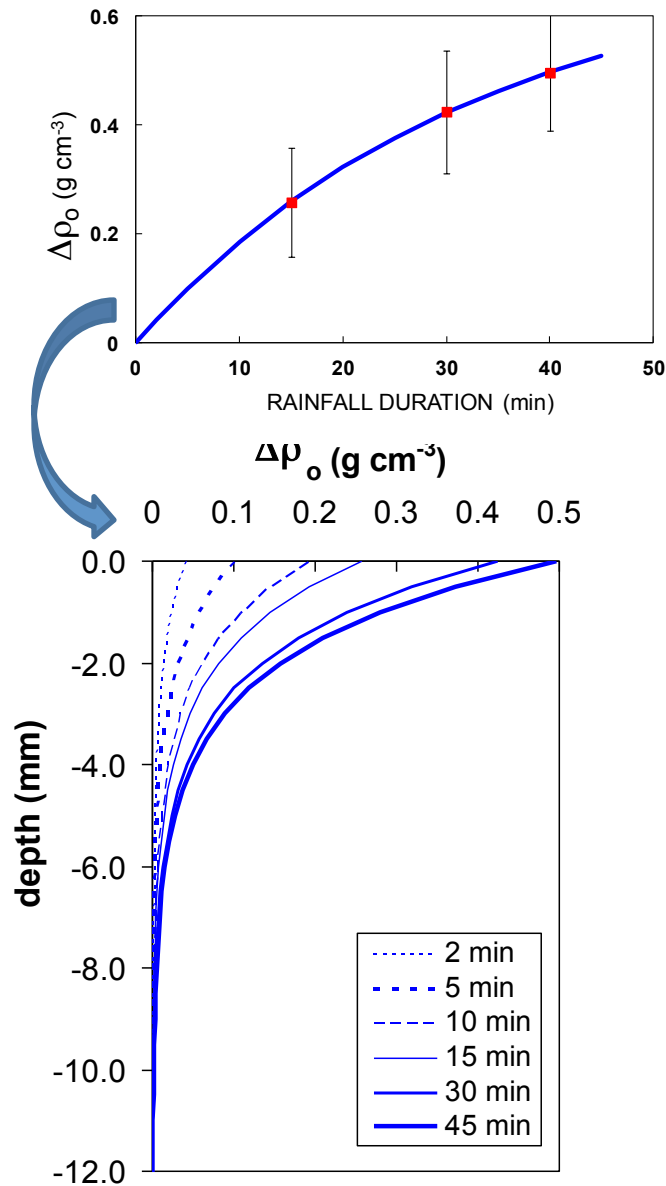
$$q = K_s - \delta K_s \left[1 - \exp\left(\frac{2I\delta K_s}{S^2}\right) \right]^{-1}$$

The robustness of the $q(I)$ relationship

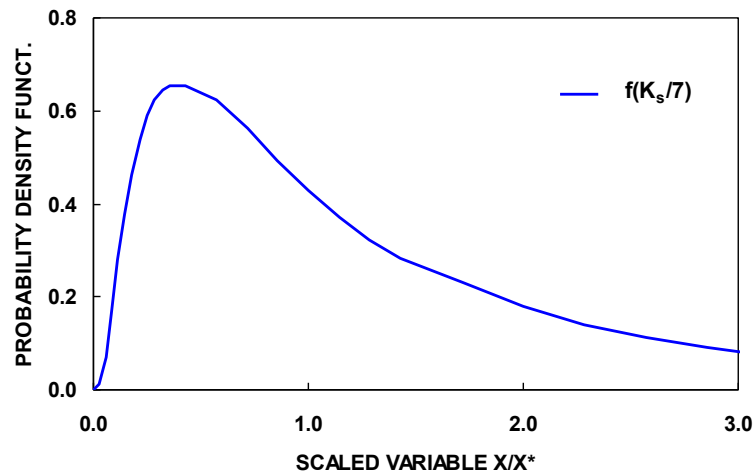
- In homogeneous soils, infiltration rate curves for postponding conditions all collapse to the same $q(I)$ function
- This property is at the basis for the Time Compression Approximation (TCA) that allows to predict the shift from pre-ponding to post-ponding infiltration, or in other words, to predict the time of ponding. **Brutsaert** and **Parlange** have contributed significantly to this important domain also but it will not be covered in this presentation
- The uniqueness of $q(I)$ holds for layered soil profiles (**Smith, 1990**)
- Parlange contributed to inject some physics in models of infiltration into sealed soils by deriving hydraulic functions to the seal layer (**Romkens et al., 1986; Baumhardt et al., 1990**)



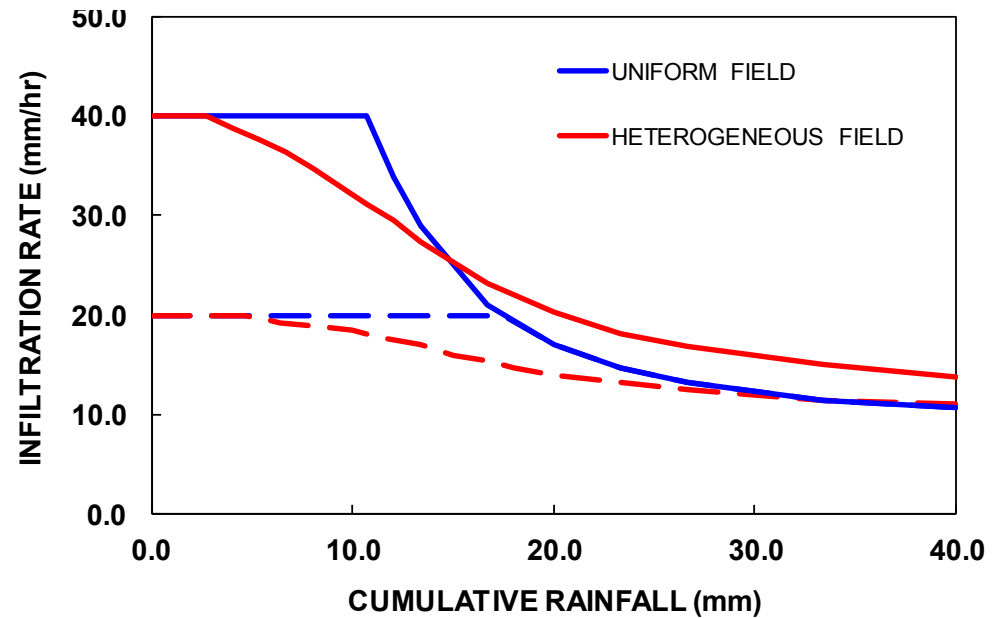
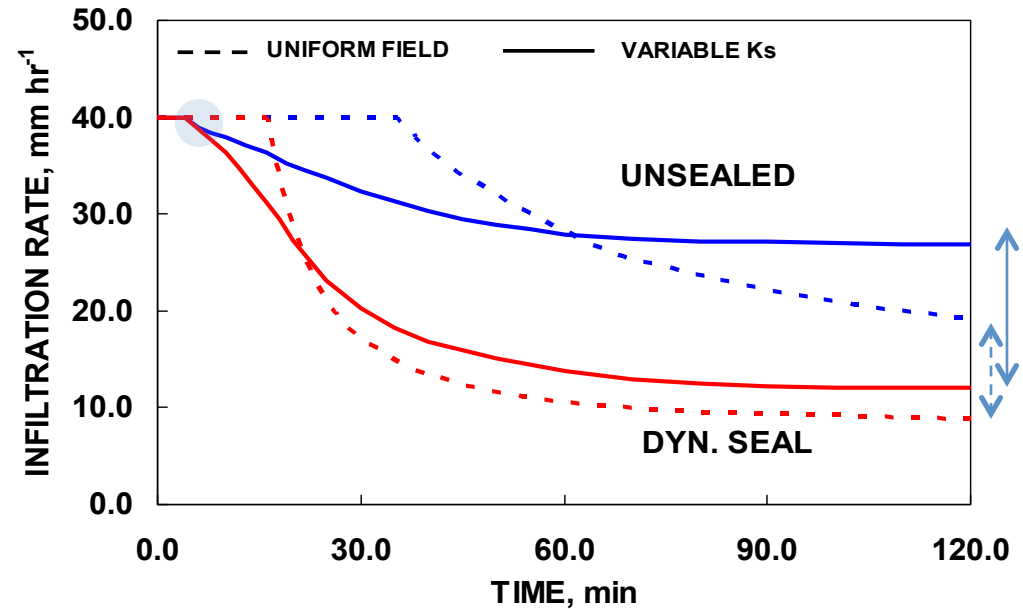
Infiltration during soil surface sealing



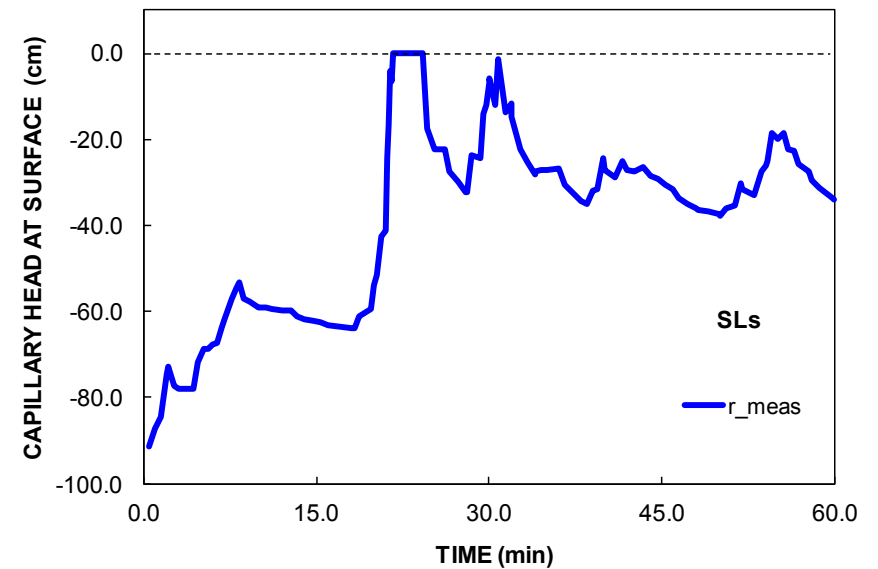
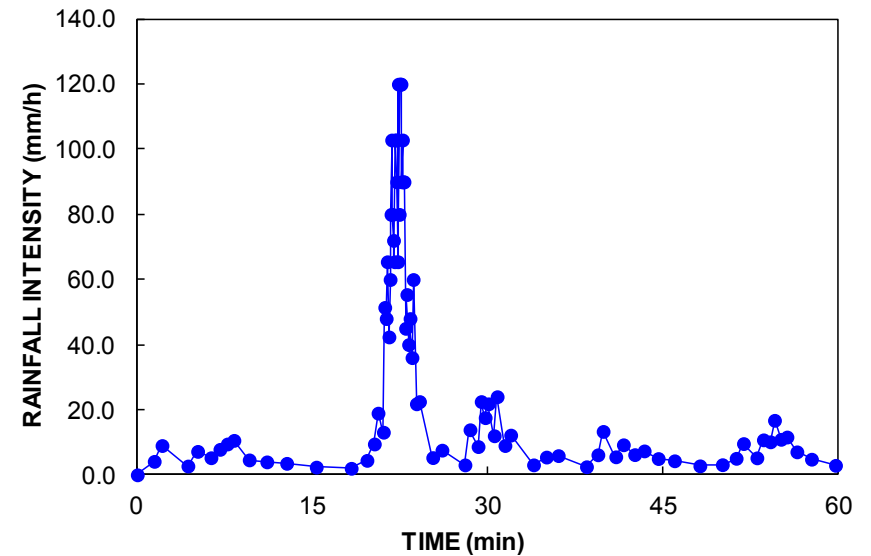
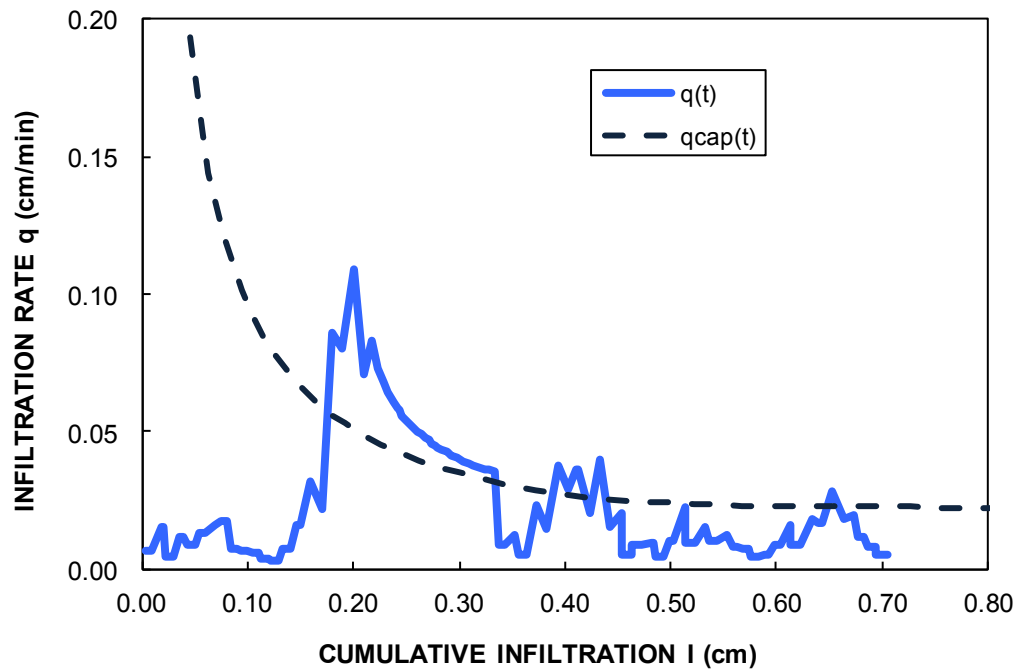
Infiltration under spatially variable soil properties:



$$\bar{q}(t) = \sum_i m_i q_i(t)$$



Infiltration during rainfall with variable intensity



Assouline, Selker and Parlange (2007)

Conclusions

- Brutsaert and Parlange have made seminal contributions to the understanding and the ability to quantify the complex process of infiltration
- We should be grateful because:
 - we had the chance to learn directly from them
 - they have left some unresolved issues so that we could get the opportunity to apply what we have learned

Thank you for your attention

