and hence we should have more accurately for the first branch pipe $\zeta_1 = 00263$, and for the other $\zeta_2 = 0{,}0270$, and hence with the best accuracy which the formula admits

$$d_1 = \sqrt[5]{\frac{0{,}0263 \,.\, 600 + 0{,}391}{1706}} = \sqrt[5]{\frac{16{,}171}{1706}} = 0{,}394 \text{ feet} = 4{,}7 \text{ inches, and}$$

$$d_2 = \sqrt[5]{\frac{0.0270 \,.\, 200 + 0{,}505}{169{,}7}} = \sqrt[5]{\frac{5{,}905}{169{,}7}} = 0{,}511 \text{ feet} = 6{,}13 \text{ inches.}$$

CHAPTER IV.

OF VERTICAL WATER WHEELS.

§ 81. *Water Power.*—Water acts as a *moving power*, or *moves machines* either by its *weight*, or by its *vis viva*, and in the latter case it may act either by *pressure* or by *impact.* In the action of water by its weight, it is supported on some surface connected with the machine, that sinks under the weight; and in the action by its *vis viva* it comes against a surface yielding to it, in a horizontal direction generally, which is, in like manner, an integral part of the machine. If Q be the quantity of water (or $Q\gamma$ the weight of water) available as power, per second, and h, the *fall*, or the perpendicular height through which the water falls in giving out its mechanical effect, then the mechanical effect produced is: $L = Q\gamma \,.\, h = Q h \gamma$. If, again, c be the velocity with which the water comes upon any machine, the mechanical effect produced by its *vis viva*, is:

$$L = Q\gamma \frac{c^2}{2g} = \frac{c^2}{2g} Q\gamma.$$

That water may pass from rest to the velocity c, a fall, or height due to the velocity $h = \frac{c^2}{2g}$ is necessary, and, therefore, in the second instance we may also put $L = h\,Q\gamma$. So that *the mechanical effect inherent in water is the product of its weight into the height from which it falls*, as in the case of other bodies.

Water sometimes acts by its weight and *vis viva* simultaneously, by combining the effects of an acquired velocity c, with the fall h through which it sinks on the machine. In this case, the mechanical effect produced is again:

$$L = Q\gamma \,.\, h + Q\gamma \frac{c^2}{2g} = \left(h + \frac{c^2}{2g}\right) Q\gamma.$$

The mechanical effect Pv yielded by a machine is of course always less than the above available mechanical effect $Q h \gamma$; because many *losses* occur. In the first place, *all the water* cannot always be brought to work; secondly, a part of the fall is generally lost; thirdly, the water retains a certain amount of *vis viva* after having quitted the machine; and, fourthly, there are the passive resistances of friction, &c., interfering. The *efficiency* of a water-power machine may be represented by $\mu = \frac{P v}{Q h \gamma}$, and the merits of different machines are proportional to the approximation of this ratio in their case, to unity.

From the general formula $L = Q h \gamma$, it is manifest that fall and quantity of water are convertible terms; so that, by doubling the height of a fall with a given quantity of water, we have the same power as by doubling the quantity of water, and retaining the original height.

Example. There is a fall of 10 feet yielding 12 cubic feet of water per second. The machine uses only 8,5 feet, however, and the water leaves it with a velocity of 9 feet per second, and the friction is ascertained to be 750 feet lbs.; required the efficiency of this machine.

The available mechanical effect $L = 12 \times 10 \times 62{,}5 = 7500$ feet lbs. (Pruss.), and the effect of the fall used $= 12 \times 8{,}5 \times 62{,}5 = 6375$ feet lbs. The mechanical effect lost from the *vis viva* retained in the water leaving the machine is $0{,}0165 \times 9^2 \times 12 \times 62{,}5 = 941{,}2$ feet lbs.; and the mechanical effect consumed by friction $= 750$ feet lbs.; and, therefore, the useful effect of this machine $Pv = 6375 - (941{,}2 + 750) = 4683{,}8$ feet lbs., and the efficiency $= \frac{4683{,}8}{7500} = .624$.

§ 82. *Water Wheels.*—The machines used as recipients of water-power, are either wheels, (water wheels, Fr. *roues hydrauliques;* Ger. *Wasserräder;*) or engines with pistons, *water-pressure engines,* (Fr. *machines à colonnes d'eau;* Ger. *Wassersäulen-maschinen.*) Water wheels are essentially "the wheel and axle," with water as power. Pressure engines consist of a column of water, pressing on a movable piston.

Water wheels are either *vertical,* the axle of the wheel being horizontal, or they are *horizontal,* the axle of the wheel being vertical.

Vertical water wheels, concerning which we shall first treat, are either overshot, (Fr. *roues en dessus;* Ger. *Oberschlägige,*) or breast wheels, (Fr. *roues de côté;* Ger. *Mittelschlägige,*) or undershot, (Fr. *roues en dessus;* Ger. *Unterschlägige.*) The water comes on to the wheel near the top or summit, in overshot wheels; near the middle or level of the axle, in breast; and near the bottom in undershot wheels. In the first, the water's weight is chiefly the source of mechanical effect, whilst in undershot wheels it is the inertia of the water, and in breast wheels, the *weight and inertia* both that are usually effective. Undershot wheels sometimes hang freely between boats in a wide stream, and sometimes in a confined course, which is either straight or curved. Breast wheels are generally hung in a curved channel or course. It is, perhaps, necessary to distinguish from the above-named vertical wheels, Poncelet's wheel, in which the water acts by pressure in its ascent and descent on curved buckets.

§ 83. *Bucket Wheels.*—All vertical water wheels consist of an axle of wood or iron, with two journals or gudgeons—of two or more annular *crowns* or *shroudings*—of a set of arms connecting the shrouding with the axle, and of a series of *cells* or *buckets* between the shrouding—and, lastly, of a *flooring,* which reaching from crown to crown on their under side, forms a close cylinder. The buckets divide the annular space bounded by the shroudings on the flooring into a series of compartments, which, when the buckets are placed more tangentially than radially, form *water troughs* or *cells*. This latter is the general construction of the buckets of overshot and breast wheels, which are thus distinct from the simple *floats* of undershot wheels. For overshot wheels, the water is led on to the wheel by a trough or channel having a regulating sluice, and falls thence into the second or third cell from the summit of the wheel. If, then, the wheel be once in motion, each cell gets partially filled with water as it passes the discharge of the water trough or lead, and retains the water till near to the bottom of the wheel, when it falls out, so that there is always a certain number of cells filled with water on *one side* of the wheel, and this keeps the wheel continuously revolving. Overshot wheels have been constructed for falls varying from 8 to 50 feet, and sometimes even up to 64 feet in height, and for quantities of water varying in every degree up to 50 cubic feet of water per second. It is often more advantageous to put up two or three smaller wheels, than one very large one; for the weight of the parts becomes inconvenient.

The *fall* of a water wheel should be measured as *between the surface of the water at the pentrough*, or regulating sluice, and the surface of water in the *tail race*, the depth of which latter will depend of course on the quantity of water, and on the breadth, and the inclination of the race. In order to lose as little of the effect as possible, the bottom of the wheel should be as near as possible to the surface of the race, so that the height from the surface of water in the pentrough to the bottom of the wheel may also serve as a true measure of the *height of fall*. If there be any risk of *backwater* in the race, the wheel must be hung at an extra elevation accordingly.

§ 84. *Construction of Water Wheels.*—Water wheels are made of wood or of iron, or of both these materials combined. The manner of uniting the axle and arms together is various. In the case of wooden wheels, they are either strapped or bolted on to the side of a square axle, as shown in Fig. 171, or they are let into the axle by morticing, or passed through it. The latter construction is bad, and only applicable to light wheels. The arms of the framed wheel, Fig. 171, may be strengthened by braces or auxiliary arms. Such wheels of 20 to 50 feet diameter are erected for pumping water, for driving ore mills, &c., in the Freiberg mining district. *A* is the axle, *B* and *C* are the journals or gudgeons, *DE*, *FG*, &c., are the main arms, *HM*, *HL* are the auxiliary arms; *DFG*, and $D_1F_1G_1$ are the shroudings of the wheel; *K* is the pentrough end. The crowns are two rings of wood composed of 8 to 16 pieces of 3 to 5 inch-thick segments. The whole is put together with screw bolts. There are cross tie-bolts for uniting the two crowns. The interior of the crowns are *grooved* out to receive the buckets. The open wheel *N* is a part of the mechanism for transmitting the motion.

Fig. 171.

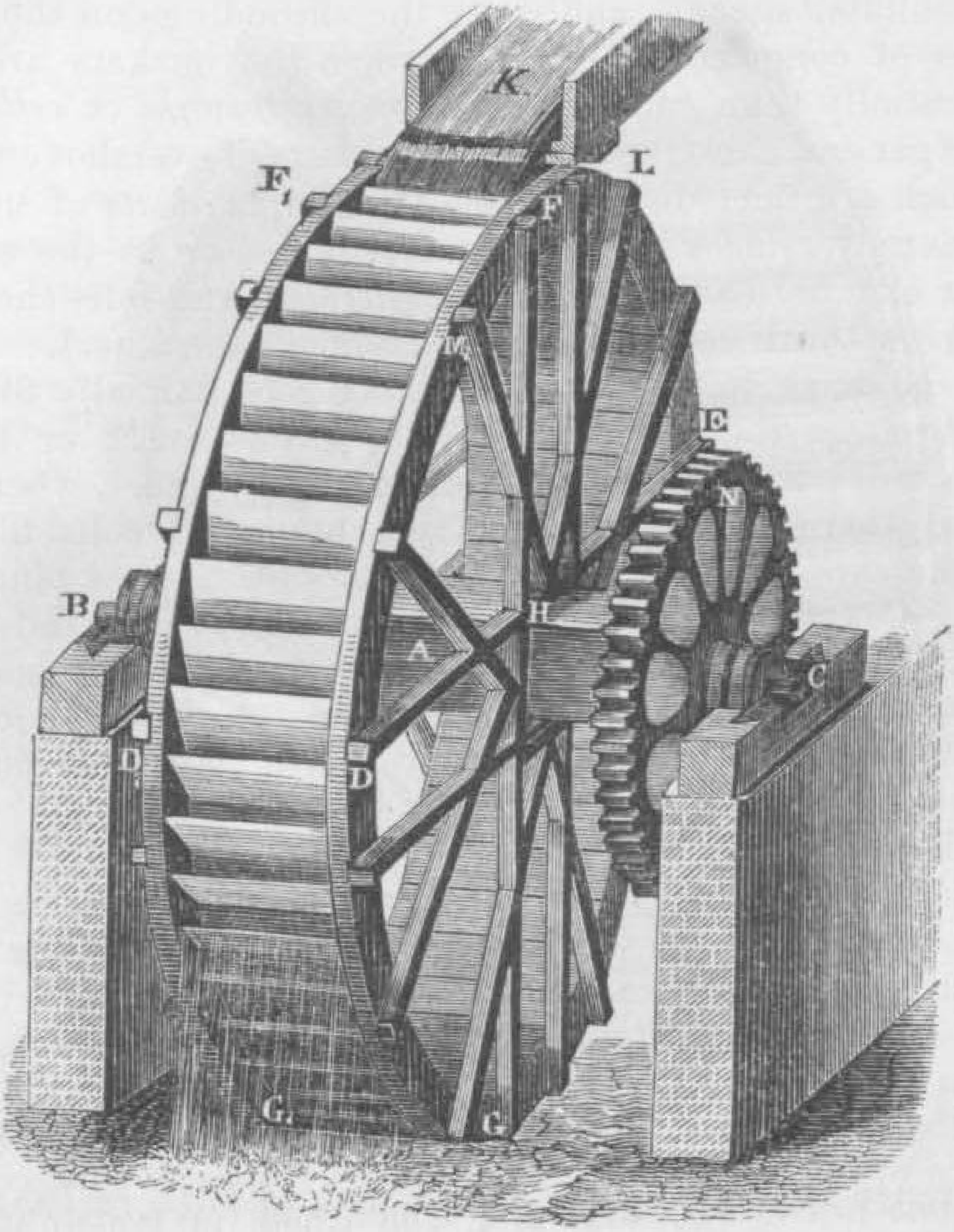
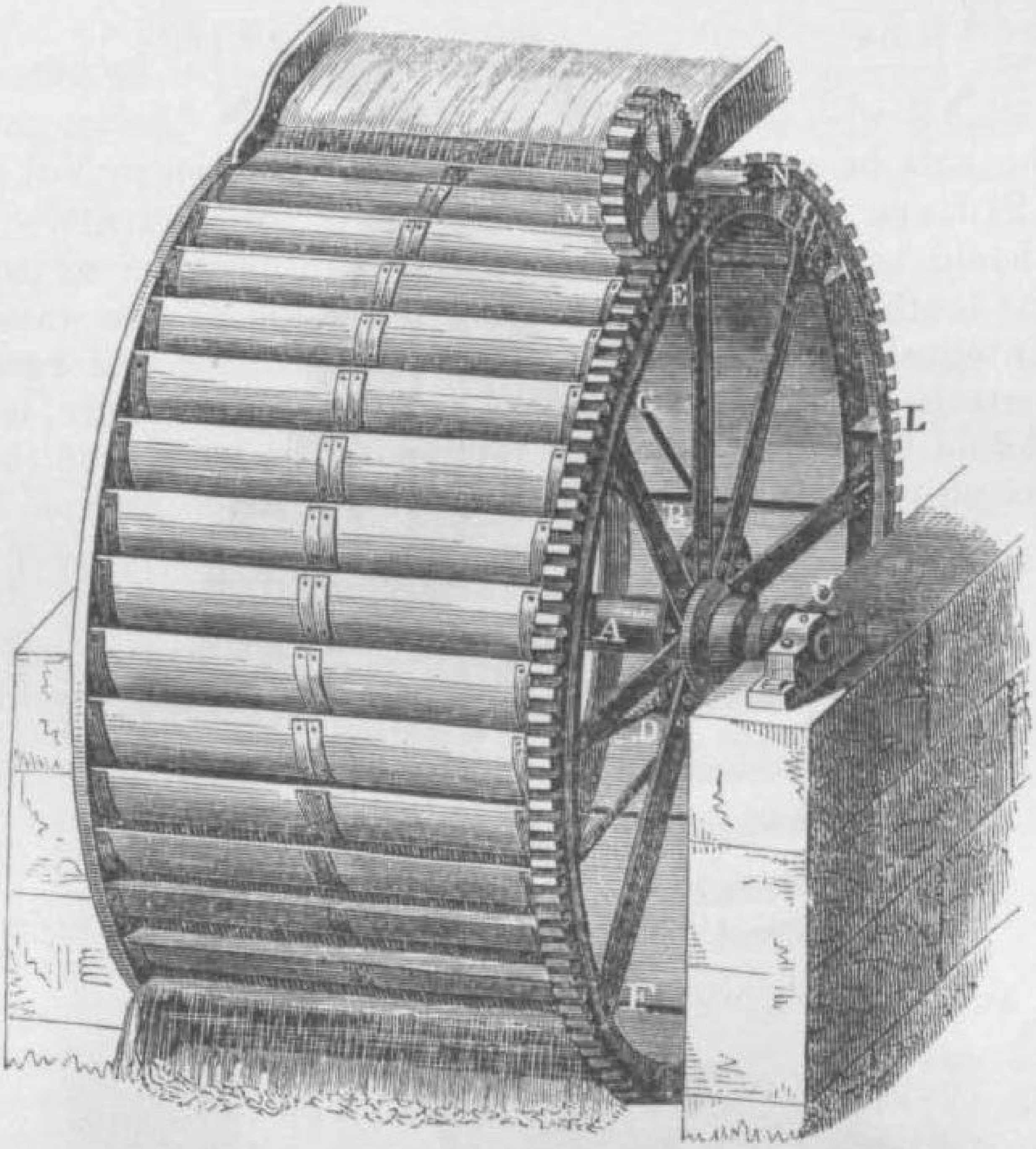


Fig. 172 is an iron water wheel. Cast iron discs, or *naves BD*, are set on the axle *AC*, and to these the arms are attached by bolts. An intermediate ring or crown is introduced when the wheel becomes more than 7 or 8 feet wide, and this has either a separate set of arms or diagonal arms, as shown by *BG*, &c., brought from this to the nave of the outer crowns. Through-bolts are introduced to bind the whole firmly together. The prime mover in the train of mechanism is often, as shown in Fig. 172, a toothed wheel, forming the periphery of an outside crown *ELF*, and this works into a pinion on a lying shaft *MN*. In practice, this pinion should be rather below than above the level of the axle, and on the side on which the water is. The buckets are of *sheet iron*, and bolted to ribs of *angle iron*, *cast on* the inner surface of the crowns, or *fastened to* them.

§ 85. *Dimensions of Parts.*—The axle, the gudgeons, and the arms of the wheel, must have dimensions proportioned to the weight and power of the wheel. To find these, the principles and rules of the third section of the first volume are to be applied. The dimensions of the axle may be determined either in reference to the moment of inertia of the wheel and the resistance to torsion of the axle,

Fig. 172.



or in reference to the weight of the wheel, and the resistance of the axle to transverse strain. In Vol. I. § 211, it has been shown, that in the case of a solid round cast iron axle of radius $= r$, acted upon by the statical moment of an effort P equal to Pa, that $Pa = 12600\, r^3$, where r and a are expressed in inches. Hence $r = \sqrt[3]{\frac{Pa}{12600}}$ inches $=$ the radius of axle; and if a be expressed in feet, then the diameter of the axle

$$d = \sqrt[3]{\frac{8 \,.\, 12\, Paw}{12600}} = \sqrt[3]{\frac{4\, Pa}{525}} = 0{,}197\, \sqrt[3]{Pa} \text{ inches.}$$

But the mechanical effect corresponding to the moment Pa, u, being the number of revolutions of the wheel per minute, is

$L = Pv = P\,\frac{\pi u a}{30}$ feet lbs., or, in horses' power, $L = \frac{P \,.\, \pi u a}{30 \,.\, 550}$, hence $Pa = \frac{16500\, L}{\pi u}$, and

$$d = 0{,}197\, \sqrt[3]{\frac{16500}{\pi}} \,.\, \sqrt[3]{\frac{L}{u}} = 3{,}34\, \sqrt[3]{\frac{L}{u}} \text{ inches.}$$

But for greater security, we generally make $d = 6{,}12\, \sqrt[3]{\frac{L}{u}}$ inches.

If the axle be square, the side of the square

$$s = \sqrt[3]{\frac{3\pi}{8\sqrt{2}}} \cdot d = 0{,}94\, d, \text{ i. e., } s = 5{,}75 \sqrt[3]{\frac{L}{u}} \text{ inches.}$$

If the axle be made hollow, the formulas given in Vol. I. § 209 and § 210 are to be used with the above co-efficients. Wooden axles should be from 3 to 4 times as large in diameter as iron axles.

If the toothed wheel, transmitting the power of the water wheel, be an integral part of it, as in Fig. 172, the axle undergoes a less torsion-strain by the moment of the power, and, therefore, its dimensions should be determined in reference to the weight of the wheel. For this we may make use of the formulas given in Vol. I. § 202, $Q\left(\frac{l_1 l_2}{l} - \frac{c}{8}\right) = \frac{K}{6} bh^2$, in which we substitute for Q, G the weight of the water wheel, c the breadth of the wheel, l the length of the axle, and l_1 and l_2 the distance of the centre of the wheel from the two gudgeons. Hence for a square axle:

$$h = b = s = \sqrt[3]{\frac{6\,G}{K}\left(\frac{l_1 l_2}{l} - \frac{c}{8}\right)}.$$

And if for $\frac{K}{6}$ we put 1000 lbs. as a minimum, and expressing l, l_1, and l_2, and c in feet, we get for square cast iron axles:

$$s = 0{,}229 \sqrt[3]{G\left(\frac{l_1 l_2}{l} - \frac{c}{8}\right)} \text{ inches,}$$

and, on the other hand, for round cast iron axles:

$$d = s\sqrt[3]{\frac{16}{3\pi}} = 1{,}193 \cdot 5 = 0{,}272 \sqrt[3]{G\left(\frac{l_1 l_2}{l} - \frac{c}{8}\right)}.$$

Wooden axles must be made at least as large again.

The diameter of the gudgeon d_1 is deduced from the well-known formula given in Vol. I. § 196, $Pl = \frac{\pi}{4} r^3 K$, substituting in it for $r = \frac{d_1}{2}$, and l the length of the gudgeon, which is generally about equal to d_1, its diameter. Hence we should have for the diameter $d_1 = \sqrt{\frac{32}{\pi K}} \cdot P$, for which we may put in practice $d_1 = 0.48 \sqrt{P}$, P being the pressure on the gudgeon. Buchanan's rule is $d_1 = 0{,}241 \sqrt[3]{P}$ inches.

The arms of the wheel must evidently be of strength sufficient to resist the moment of rotation. If this moment be again taken $= Pa$, and the number of the arms in each set of arms of the wheel $= n$, so that for a double set of arms the total number of arms $= 2\,n$, then the moment which a single arm has to resist $= \frac{Pa}{2n}$. If, now, $b =$ the breadth, and h the thickness of an arm, and if the length of the arm be equal to the radius of the wheel $= a$, then, from Vol.

I. § 196, we have $\frac{Pa}{2n} = bh^2 \frac{K}{6}$, or, as b is made $= mh$, or, in iron generally, $\frac{1}{5} h$, and in wood, $\frac{5}{7} h$, *i. e.*, $\frac{Pa}{2n} = mh^3 \frac{K}{6}$, and hence the thickness of the arms sought, measured in the direction of the plane of revolution, is: $h = \sqrt[3]{\frac{3\,P\,a}{m\,n\,K}}$. If we introduce the effect, and number of revolutions of the wheel, then, for cast iron arms, $h = 10{,}4 \sqrt[3]{\frac{L}{nu}}$ inches. And, as the diameter of the axle was found $d = 6{,}12 w \sqrt[3]{\frac{L}{u}}$, we have also $h = \frac{1{,}7\,d}{\sqrt[3]{n}}$, or $\frac{h}{d} = \frac{1{,}7}{\sqrt[3]{n}}$, and, therefore, for 4, 6, 8, 10, 12, 16 arms, the values of $\frac{h}{d} =$ 1,08, 0,94, 0,85, 0,79, 0,75, 0,67, and from h, we deduce the breadth b, measured in the direction of the axis.

For wooden arms $h = 13{,}6 \sqrt[3]{\frac{L}{nu}}$, and hence we can deduce $b = \frac{5}{7} h$.

According to Rettenbacher, the number of arms in a set, or to one crown (of which there are always two at least), is $n = 2\left(\frac{a}{3} + 1\right)$. If a wheel be 8 feet wide, or wider, the number of sets of arms should not be less than three.

Example. A cast iron water wheel, weighing 35,000 lbs, gives an effect of 60 horse-power, making 4 revolutions per minute; required the dimensions of its principal parts. The diameter of a solid axle is $d = 6{,}12 \sqrt[3]{\frac{60}{4}} = 13{,}2$ inches, and that of its gudgeons $d_1 = 0{,}048 \sqrt{\frac{35000}{2}} = 6\frac{1}{3}$ inches, which might be made 7 inches. Buchanan's formula gives $d_1 = 0{,}241 \sqrt[3]{17500} = 6\frac{1}{4}$ inches. For the arms, supposing two sets of 12 each, the thickness $h = \frac{1{,}7 \times 13{,}2}{\sqrt[3]{12}} = 10$ inches nearly, and the breadth $b = \frac{1}{5}\,10 = 2$ inches (h being in the direction of the plane of revolution).

§ 86. *Axles and Gudgeons.*—We must make special allusion to the manner of putting the gudgeons in the axles, and to the *plummer* blocks on which they rest. For wooden axles, oak, or larch, or beech, answers exceedingly well. They are dressed into polygons, when the arms are to be framed on the axle, and they are squared when the arms are to be morticed through, or into the axle. The

Fig. 173.

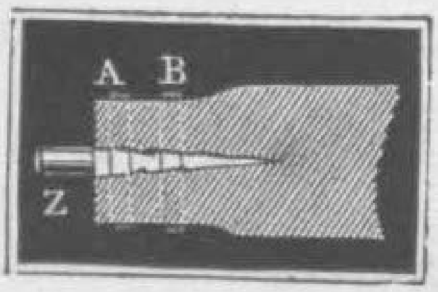
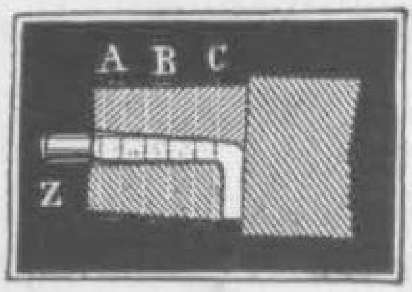


Fig. 174.



gudgeons are either spiked in, as shown at Z, Fig. 173, or they are *hooped*, as shown in Fig. 174. Also, plate or flat ends are used, as in Fig. 175 (and these are the most common), or rings, as at Fig. 176, or compound gudgeons, as at Fig. 177. To strengthen the

Fig. 175.

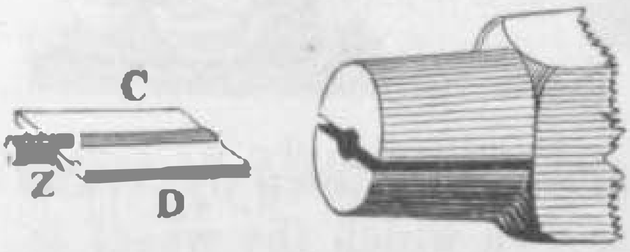


Fig. 176.

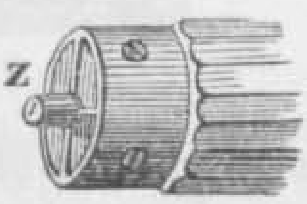
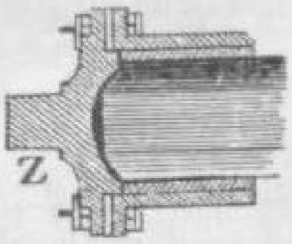


Fig. 177.



neck of the axle, to prevent its splitting, it is dressed off conically, and three iron rings, $\frac{1}{4}$ to $\frac{1}{2}$ inch thick, and $1\frac{1}{2}$ to 3 inches broad, are driven on while hot. The plates in the flat gudgeon ends are from 1 to 3 inches thick, and about an inch narrower than the diameter of the axle. The ring attachment is convenient, when a spur wheel is to be placed at the neck of the axle; the compound gudgeon is applied when much *wear* is anticipated, because the end plates are easily removed and renewed. Cast iron axles are either hollow or solid, either round or polygonal in section, and sometimes *ribbed* or feathered to increase their stiffness.

For solid axles the gudgeon is generally in one piece with the axle. Fig. 178 is a simple round axle, Fig. 179 is a feathered axle,

Fig. 178.

Fig. 179.

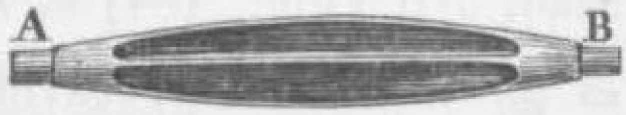
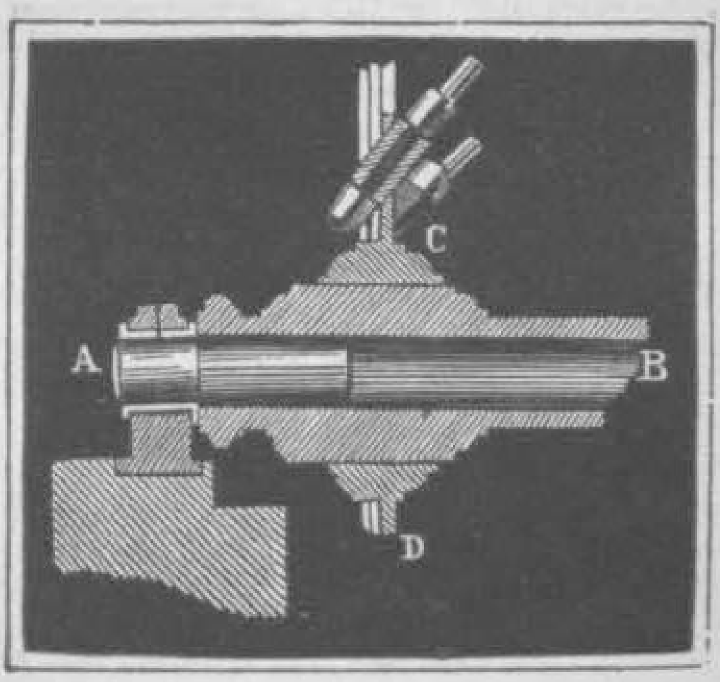


Fig. 180.



and Fig. 180 is the end of a hollow iron axle, with a gudgeon put in, and an arm plate or nave set upon it.

The gudgeons rest on supports termed *plummers* or *plumbing* blocks, which, to afford a permanent seat for the wheel, are placed on substantially founded walls. The plumbing block is lined with a brass or other movable seat for the gudgeon. These seats are either of *brass* (hence termed generally *brasses*), or of gun-metal (8 parts copper, 1 part tin), or of white metal; sometimes they are of wood, though seldom.

The gudgcons rest on a wooden block in Fig. 171. Fig. 181 is a simple, uncovered, cast iron block. Fig. 182 is an open block, with a metal seat, or lining, and Fig. 183 is a close or covered block with

Fig. 181.

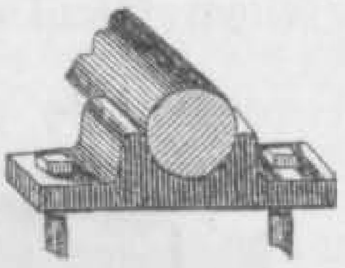


Fig. 182.

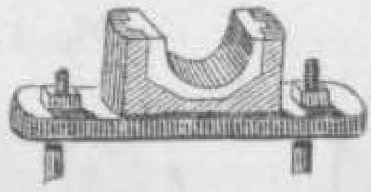
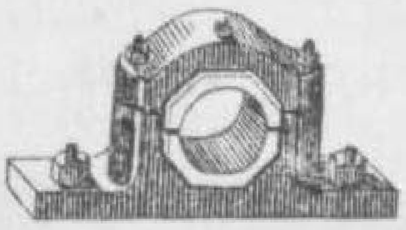


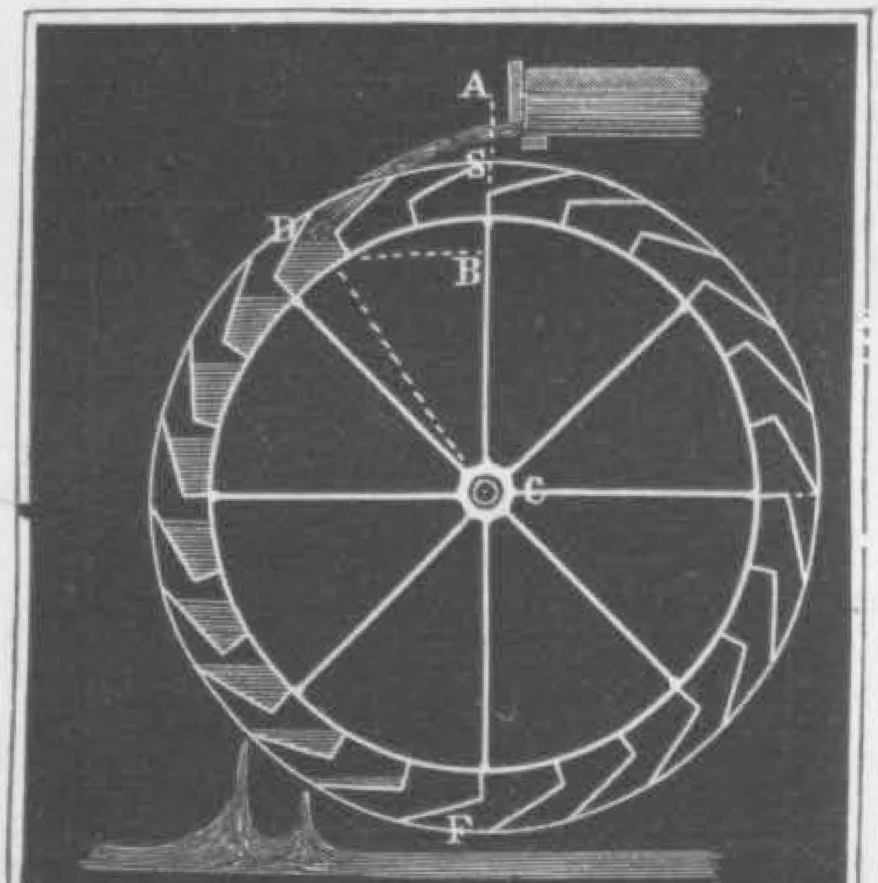
Fig. 183.



metallic lining. The plumbing blocks are bolted down by means of bolts and sole plates to the walls or beams on which the wheel is to rest. The cover of blocks is always provided with a hole, through which grease can be supplied. The inside of the cover is sometimes grooved, so that the grease diffuses more readily over the gudgeon. And, wherever it is desired to reduce the resistance from friction to a minimum, a grease cup, affording a *constant supply*, is placed in communication with a hole in the plumbing-block cover.

§ 87. *The Proportions of Water Wheels.*—The first or main element of a water wheel is the velocity of the circumference v, or the number of revolutions u. It will be seen in the sequel, that overshot wheels should have a very small velocity. Many wheels have a velocity of 10 feet per second, but 5 feet is more suitable, yet under $2\frac{1}{2}$ feet is not advisable. The velocity c of the water entering the wheel, should depend on the velocity of the wheel, and is either equal to this, or greater in a certain proportion. For creating the velocity c, a fall or height of head, AB (Fig. 184) $= h_1 = \frac{c^2}{2g}$ is necessary, leaving of the total fall $AF = h$, only the fall on the wheel $= BF = h_2 = h - h_1 = h - \frac{c^2}{2g}$.

Fig. 184.



As even in the case of the most perfect discharge, 6 per cent. of *vis viva* is lost, it is advisable to take it as 10 per cent. in this case, and, therefore, to put the effective fall required to bring the water on to the wheel with suitable velocity $h_1 = 1,1 \cdot \frac{c^2}{2g}$, and hence $h_2 = h - 1,1 \cdot \frac{c^2}{2g}$. From the fall on the wheel h^2, we deduce the semi-diameter of the wheel $CF = CS = a$, by assuming the angle

$SCD = \theta$, by which the point of entrance of the water D deviates from the summit S as given.

Then $h_2 = CF + CB = a + a \cos. \theta = (1 + \cos. \theta)\, a$, and hence, inversely, $a = \frac{h - h_1}{1 + \cos. \theta}$. From the radius of the wheel a, and the velocity v at the circumference, the number of revolutions per minute $u = \frac{30\, v}{\pi\, a}$.

When u is given, we can determine a and v. As $v = \frac{\pi u a}{30}$, and $c = x \frac{\pi u a}{30}$, in which x is a given ratio $\frac{c}{v}$, we have:

$$(1 + \cos. \theta)\, a = h - \frac{1{,}1}{2g} \times \left(\frac{x \,.\, \pi u a}{30}\right)^2,$$

and hence $a = \frac{h - 0{,}000193\, (x\, u\, a)^2}{1 + \cos. \theta}$, and the solution of this quadratic equation gives:

$$1.\ a = \frac{\sqrt{0{,}000772\, (x\, u)^2\, h + (1 + \cos. \theta)^2} - (1 + \cos. \theta)}{0{,}000386\, (x\, u)^2}, \text{ and}$$

hence:

$$2.\ v = \frac{\pi u a}{30} = 0{,}1047 \,.\, u\, a.$$

Example 1. For a fall of 30 feet, a wheel is to be constructed to have 8 feet velocity at circumference, and taking on the water, at 12° from the summit with twice the above velocity. What is the radius of wheel required, and what the number of revolutions? $c = 2 \times 8 = 16$ feet, and hence $h_1 = 1{,}1 \times 0{,}0155 \times 16^2 = 4{,}36$ feet, and

$$a = \frac{30 - 4{,}36}{1 + \cos.\ 12^\circ} = \frac{25{,}64}{1{,}978} = 12{,}9 \text{ feet; lastly, } u = \frac{30 \times 8}{\pi \times 12{,}9} = 5{,}92.$$

Example 2. If, inversely, the number of revolutions be 5, then for the above fall, and other proportions $x = 2$, and the radius of the wheel:

$$a = \frac{\sqrt{2{,}316 + 3{,}9125} - 1{,}978}{0{,}0386} = \frac{0{,}5177}{0{,}0386} = 13{,}41 \text{ feet.}$$

Again, the velocity at the circumference $v = 0{,}1047 \times 5 \times 13{,}41 = 7{,}02$ feet, the velocity at entrance $= 14{,}04$ feet, and lastly, the height of fall due to this latter velocity $= h_1 = 1{,}1 + 0{,}0155 \times 14{,}04^2 = 3{,}47$ feet.

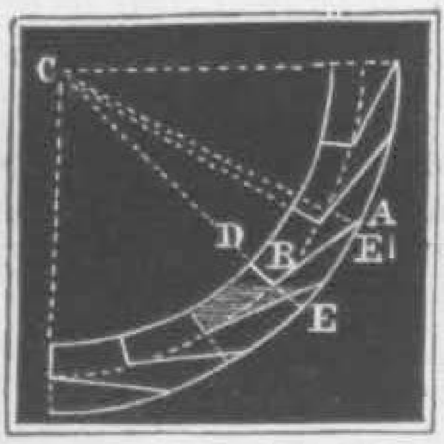
§ 88. The proportions of the wheel, in reference to *depth* of the shrouding and width of the wheel, are important. The depth of the crown (or *water space*) is made 10 to 12 inches, and sometimes even 14 to 15 inches, and this proportion is chosen, because the water in a wheel with *shallow* shrouding, acts with greater leverage than it would do on a wheel of equal radius with deeper crowns. As to the width or breadth of the wheel, it depends on the capacity to be given to the wheel. If d be the depth of crowns, and e the width of the wheel (or distance between the internal surfaces of the crowns), then the section of the annular space above the flooring of the wheel is $= d\, e$, and if v be the velocity at the middle of the crown's depth, the capacity presented to the water, per second, is $d\, e\, .\, v$. But this cannot be considered equal to the quantity of water delivered on the wheel, because a certain portion of this capacity is taken up by the substance of the buckets, and it is also inexpedient to fill up the buckets to the brim. We must, therefore, put $d\,e\,v = \varepsilon\,Q$, in which equation $\varepsilon > 1$; ε is usually $= 3$ to 5, the former when the buckets are filled rather in excess, the latter when they are deficiently filled. The width of wheel is, however, now determined:

$$e = \frac{\varepsilon\,Q}{d\,v}, \text{ or as } v = \frac{\pi\,a\,u}{30}, \text{whence } e = \frac{30\,\varepsilon\,Q}{\pi\,u\,a\,d} = 9{,}55\,\frac{\varepsilon\,Q}{u\,a\,d},$$

and taking $\varepsilon = 4$, then $e = 38{,}2\,\frac{Q}{u\,a\,d}$. That wheels of very great diameter may not be too narrow, it is advisable to assume $\varepsilon = 5$.

The number of buckets n is another important element in the construction of water wheels. The more cells there are, the longer will the water be retained on the wheel. But this number has its limits, because the buckets occupy space, taken from the capacity of the wheel, and the more the capacity is diminished for a given quantity of water delivered on the wheel, the sooner the water will leave it. As iron, that is sheet iron buckets, are much thinner than those of wood, we may adopt a greater number of iron buckets, than we should do of wooden buckets. We may follow the rule, to place the buckets at such a distance from each other, that at that point where the wheel begins to *spill*, or lose its water, the bucket next above, ABD, Fig. 185, shall not dip into the water of the one below at B, for if we put the buckets closer than this, the upper bucket diminishes the capacity of that under it, and so what we gain in one respect is lost in another. The number of buckets is generally made $n = 5\,a$ to $6\,a$, or according to Langsdorf $n = 18 + 3\,a$; in which expressions a is the radius of the wheel in feet: or the distance between any two buckets is made $= 7\left(1 + \frac{d}{10}\right)$ inches. From the given, or thus found number of buckets n, we have the angle of subdivision β, *i. e.*, the central angle between two adjacent buckets, $\beta = \frac{360^\circ}{n}$.

Fig. 185.



Example. Suppose an overshot wheel of 15 feet radius, having 1 foot depth of crown and taking 10 cubic feet of water per second, makes 5 revolutions per minute, the *width* of the wheel must be $38{,}2\,\frac{10}{5\,.\,15\,.\,1} = 5{,}1$ feet, and the distance between two buckets is to be $7\left(1 + \frac{12}{10}\right) = \frac{77}{5} = 15{,}4$ inches, and hence the number of buckets $= \frac{2\,.\,\pi\,.\,15\,.\,12}{15{,}4} = 73$, or 72 for the sake of easier division of the circle. The angle of subdivision is $\beta = \frac{360}{72} = 5^\circ$.

§ 89. *Form of Buckets.*—The form of the cells or buckets is of much consequence to the efficiency of water wheels. The buckets must have such form and position, that the water may enter freely, remain in them to as near the bottom of the wheel as possible, but no further. By the various forms adopted, these requirements are more or less perfectly fulfilled. The two requirements are in fact often, to a certain extent, incompatible; for if the cells be made very close, the entrance, as well as the exit of the water, becomes much impeded. If the buckets be merely plane-boards, as shown at AD, Fig. 186, the entrance of the water is quite free certainly, but then it leaves the cells too soon, so that there is a great loss of mechanical effect. To prevent this too early loss of water, the bucket would have to be very long, and the angle ADE, at which the bucket inclines to the radius CE, very large, *i. e.*, nearly a right angle. As this is a practical difficulty in construction, it is preferred to make the bucket in two parts, or by a second piece DB, to give the bucket a *bottom* or *flooring* of its own. The bottom DB is sometimes termed the start, or *shoulder*, and the outer piece BA, the *arm*, or *wrist*. The former is generally placed in the direction of the radius, sometimes at right angles to the outer piece, or arm. The circle passing through the *elbow* B, made by the junction of the shoulder and *arm*, is termed the *division circle*. In the older construction of wheels, this circle is generally found placed at $\frac{1}{3}$ of the depth of the shrouding from the interior, or the sole of the wheel. As, however, the capacity of a cell is greater the wider the shoulder-blade DB (Fig. 187) is, or the greater the angle ABE (which we term the *elbow angle*), we now usually find the division circle in the middle of the depth of the shrouding. The capacity of a cell will then depend only on the width or position of the arm. The simplest construction of buckets, is to make the end A of the arm AB start from the prolongation of

Fig. 186.

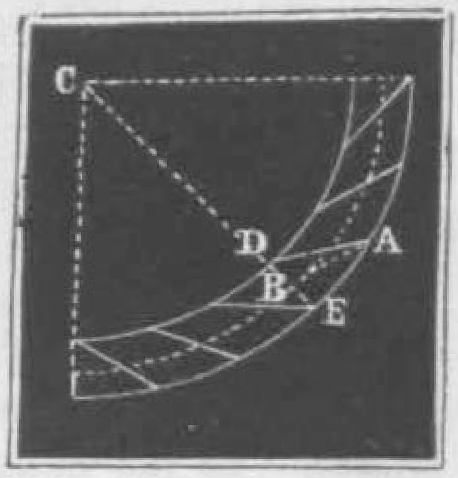


Fig. 187.

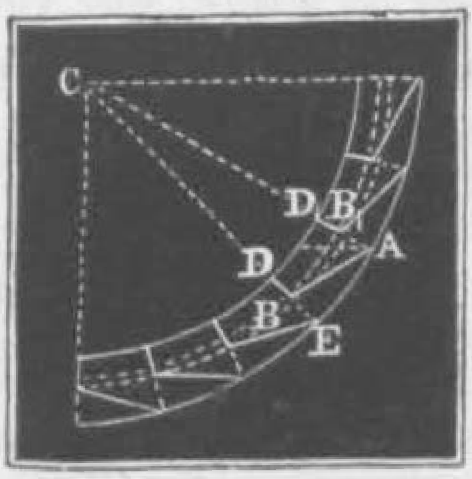
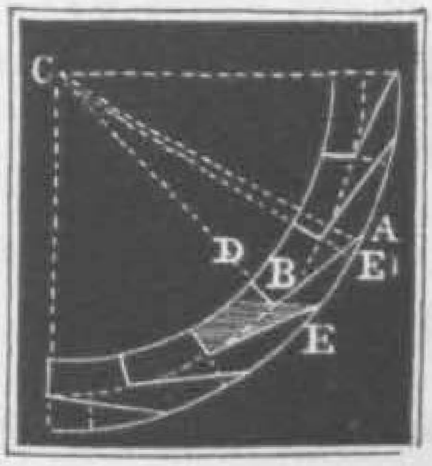


Fig. 188.



the shoulder next above it $D_1 B_1$, or, by letting the arm be included between the sides of the division angle $\beta = \frac{360^\circ}{n}$. But this construction does not close or cover the cells sufficiently, except for very shallow shrouding, and, therefore, the usual plan, for wheels up to 35 to 40 feet diameter, is to let the *arm* extend over $\frac{5}{4}$ of the dimension angle, or the arc EA is made $= \frac{5}{4}\, EE_1$, Fig. 188. From the radius

$CA = a$, and the central angle $ACB = \beta_1$, included by the *arm*, we can easily find the elbow angle $ABE = \delta$. In the triangle ACB as $CBD = CE - BED = a - \frac{b}{2}$, then

$$tang.\ \delta = \frac{a\ sin.\ \beta_1}{\frac{b}{2} - a\,(1 - cos.\ \beta_1)}.$$

Example. A 30 feet wheel, shrouding 10 inches deep, is to have (according to Langsdorf's rule) $18 + 3 \times 15 = 63$ buckets, or, say 64, and each is to extend over $\frac{5}{4}$ of the division angle. What will be the elbow angle? We have:

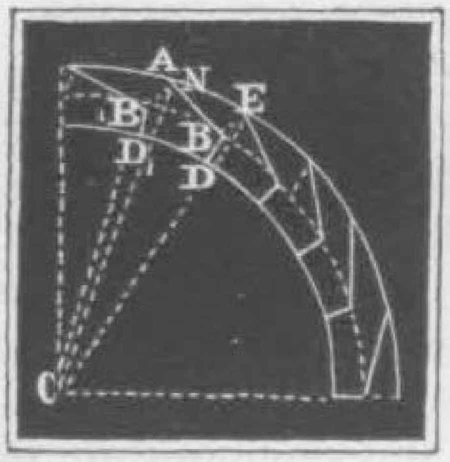
$$\beta = \frac{360}{64} = 5\tfrac{5}{8}^{\circ}, \text{ hence } \beta = \tfrac{5}{4} \times \tfrac{45}{8} = 7\tfrac{1}{32}^{\circ} = 7^{\circ}, 1', 52'', 5, \text{ and}$$

$$tang\,\delta = \frac{15\ sin.\ 7^{\circ}, 1', 52'', 5}{\frac{5}{12} - 15\,(1 - cos.\ 7^{\circ}, 1', 52'', 5)} = \frac{36\ .\ 0{,}12241}{1 - 36 \times 0{,}00752} = \frac{4{,}40678}{0{,}72928},$$

hence $\delta = 80^{\circ}, 30'$.

§ 90. The position of the *arm* of the bucket may likewise be determined, by adopting as a rule, that the least section of a cell shall be somewhat greater than the section of the water coming on to the wheel. If the cells come exactly under the water-*jet*, then this construction would permit the water to enter freely, and the air to escape unhindered. This may be done as follows. From a point of division B_1, Fig. 189, in the division circle (or *line of pitch of the buckets*) with a radius greater by one or two inches than the thickness of the water-jet, or layer, describe a circle, and from the next adjacent point of division B, draw a tangent BA to this circle. This is the position of the arm of the bucket required, for then the least width B_1N of the cell ABD_1 is equal to the radius of that circle. But in order to determine the thickness of the layer of water coming (from the pentrough) on to the wheel, we must know the fall h_1 from the surface of the water in the pentrough to the point B_1N; also the quantity of water, and the width e of the wheel. As $Q = d_1\, e\, \sqrt{2gh_1}$, therefore, the thickness in question $d_1 = \frac{Q}{e\sqrt{2gh_1}}$, and to allow of free escape of air, we make

Fig. 189.



$$d_1 = B_1N = \frac{Q}{e\sqrt{2gh_1}} + 1 \text{ or } 2 \text{ inches.}$$

If equal spaces for the exit of air and entrance of water be allowed, then $d_1 = \frac{2\,Q}{e\sqrt{2gh_1}}$ would be the equation to be satisfied.

Buckets in *three parts*, as $AEBD$, Fig. 190, give, *cæteris paribus*, more capacity than those in two parts, without any greater contraction. There is, therefore, mechanical advantage in this form, though it be more expensive to execute. Curved buckets are best of all, as shown in Fig. 191, and this is the great advantage of sheet iron, to which this form can readily be given. If the section of bucket is to be composed of two segments of circles, then it is only necessary to find the position of the *arm* of a bucket, by any of the planes above

Fig. 190.

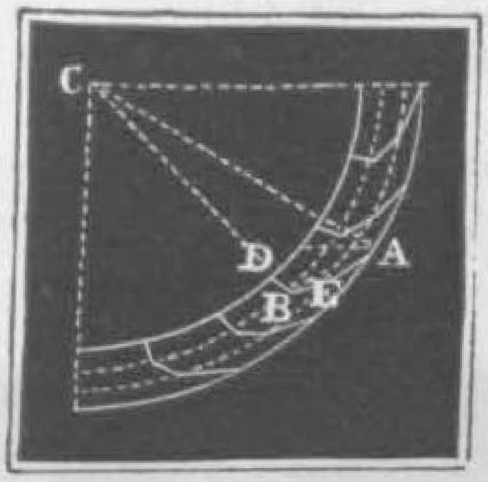
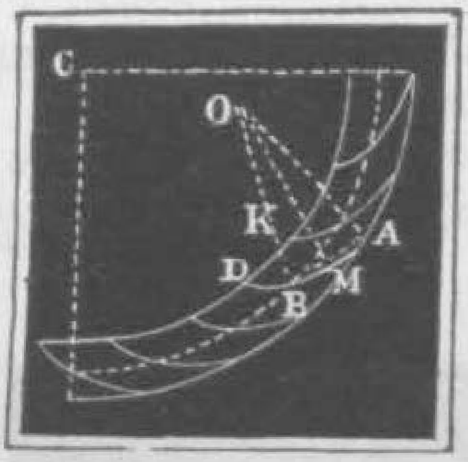


Fig. 191.



given—at its bisection M (Fig. 191), to erect a perpendicular, and from any point O at will to describe an arc with the radius OB, and from any point K in it, to describe another arc to complete the bucket ABD of a suitable form.

Example. If, in the wheel of our example to § 88, the jet or layer of water falling on the wheel has $2\frac{1}{2}$ feet fall, then as:

$$Q = 10, \text{ and } c = 5{,}1,\ d_1 = \frac{0{,}1265\,.\,10}{5{,}1\,.\,\sqrt{2{,}5}} = \frac{1{,}265}{8{,}064} = 0{,}157 \text{ feet.}$$

If now we allow an equal thickness for the exit of the air, then the least distance of two buckets becomes 0,314 feet, or $3\frac{3}{4}$ inches.

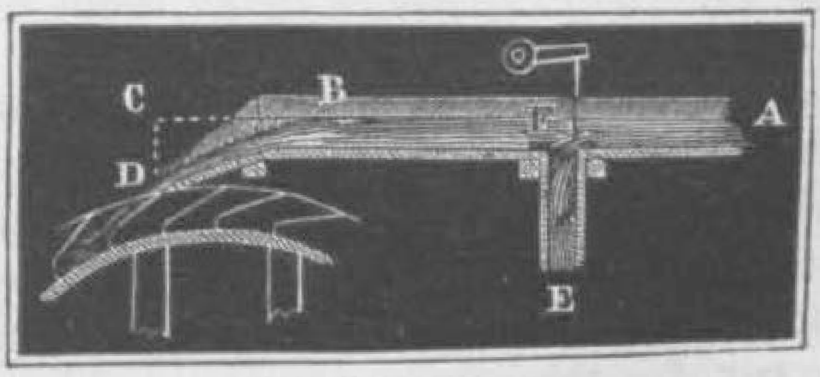
Remark. To find the elbow angle δ, in that construction of bucket which is based on the thickness of the layer of water, let us put:

$$\delta = 180^\circ - CBA = 180^\circ - CBB_1 - B_1BA = 180^\circ - \left(90 - \frac{\beta}{2}\right) - \phi = 90^\circ + \frac{\beta}{2} - \phi,$$

but $sin.\ \phi = \dfrac{B_1 N}{B\,B_1} = \dfrac{d_1}{2\,a_1 sin.\,\frac{\beta}{2}}$, when d_1 is the least distance between two buckets, and a_1 the radius of the *division circle.* For the last example, $\beta = 5^\circ$, $d_1 = 0{,}314$, $a_1 = 14{,}5$ feet, hence $sin.\ \phi = \dfrac{0{,}314}{29\ sin\ 2\frac{1}{2}^\circ} = \dfrac{0{,}314}{1{,}265} = 0{,}2482$, hence $\phi = 14^\circ,\ 22'$, and $\delta = 90^\circ + 2^\circ, 30' - 14^\circ\ 22' = 78^\circ, 8'$.

§ 91. *Sluices, Pentroughs,* or *Penstocks.*—The method of bringing the water on the wheel is of no small importance. Either the water falls freely out of the lead or trough, or it is pent up by a sluice, or pentrough, or penstock, before entering the wheel. In the former case, the velocity of entrance depends on the inclination of the trough or the height of fall. In the second case, it may be regulated by adjusting the height of head created, and, therefore, this latter method should be preferred. Fig. 192 shows a trough without a regulating sluice; but there is a *waste board* at F by which the *quantity* of water can be regulated. If the water flows along the

Fig. 192.



trough with a velocity c_1, and if the fall from the end of it to the centre of the cell $= h_1$, the velocity

$$c = \sqrt{2\,g\,h_1 + c_1^2} = \sqrt{2\,g\,h_1 + \left(\frac{Q}{F}\right)^2},$$

if Q be the quantity of water, and F the sectional area of the water coming on the wheel.

The penstock (Fr. *vannes;* Ger. *Spannschutze*) is either vertical, horizontal, or inclined. Fig. 193 shows the arrangement of a hori-

Fig. 193.

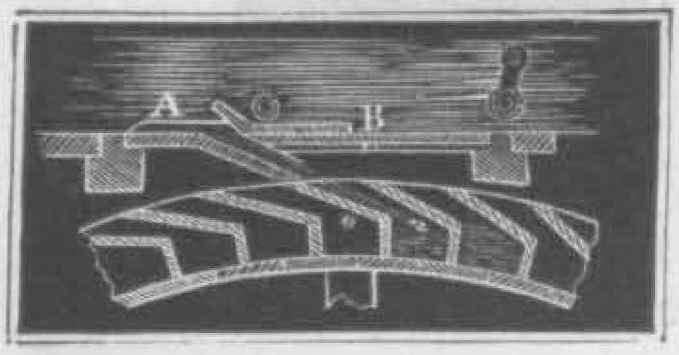
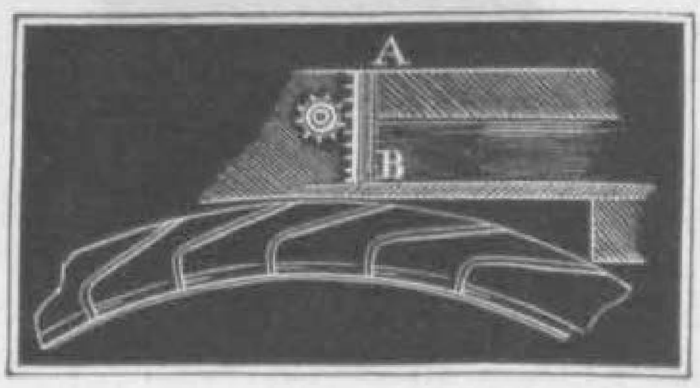


Fig. 194.



zontal sluice, and Fig. 194, that of a vertical sluice. The construction of inclined sluices as shown in Figs. 195 and 196. The one,

Fig. 195.

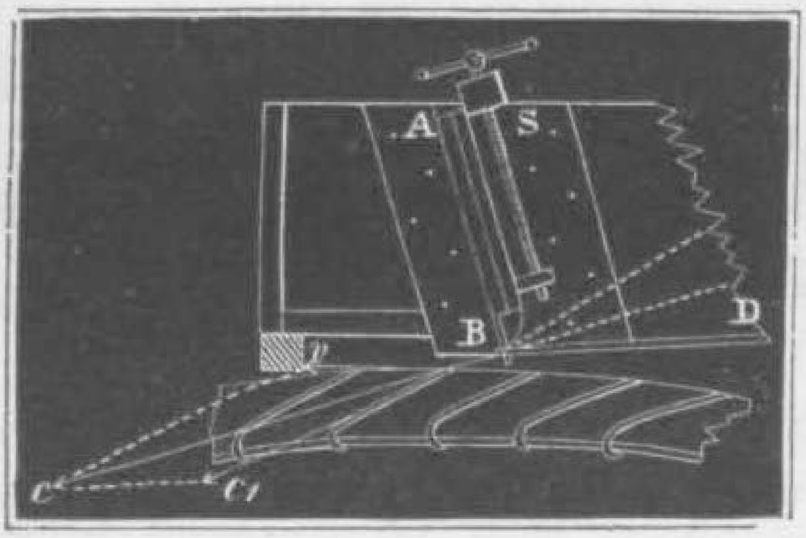


Fig. 196.

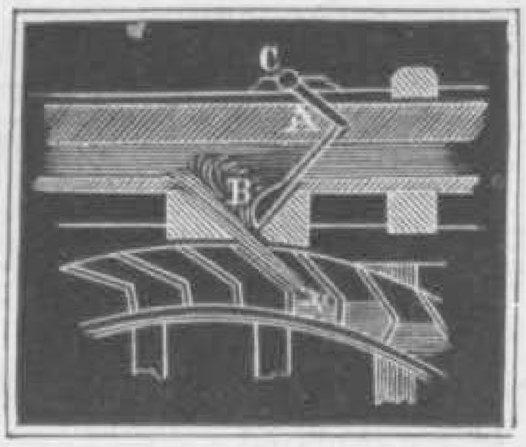


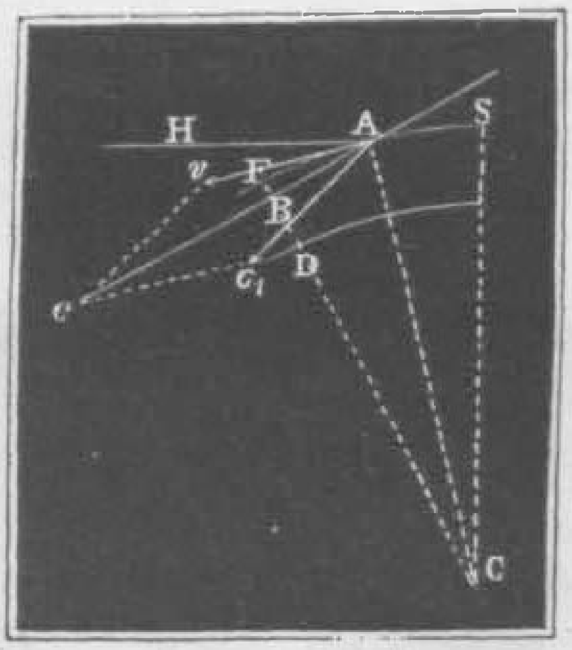
Fig. 195, is the arrangement general in the Freiberg district, the sluice being raised and depressed by means of a screw S. In Fig. 196, a simple lever is used for these purposes. It is a general rule for these penstocks, to make them as smooth as possible inside, and to round off the edges of the orifice, so as to adapt it to the form of the *contracted vein*, that the resistance may be the least possible. If the water, after passing the sluicc, fall quite freely, and if we can place the plane of the orifice at right angles to the jet of water, it becomes then advisable to make the orifice as in a *thin plate*, but in that case, care must be taken that partial contraction does not occur, for this gives rise to an obliquity of the jet (Vol. I. § 319).

In the discharge from penstocks, the velocity of discharge is deduced from the height h_1 by the formula $c_1 = \phi\,\sqrt{2\,g\,h_1}$, and if h_2 be the height of fall after passing the orifice, to the centre of the cell, then the velocity of entrance $c = \sqrt{c_1^2 + 2\,g\,h^2} = \sqrt{2g\,(\phi^2\,h_1 + h_2)}$. If we take the velocityco-efficient $\phi = 0{,}95$, then $c = \sqrt{2g\,(0{,}95\,h_1 + h_2)}$.

We see from this, that for equal falls the velocity of entrance must be very nearly equal, whether it flow on freely, or be discharged from a sluice, on to the wheel.

§ 92. That the water may enter unimpeded into the wheel cells, it must not come in contact with the bucket at the outer circumference, but nearer to the inner circumference or bottom of the cells. Hence, not only must the outer edge of the buckets be sharpened off, but the layer of water AC, Fig. 197, must be so directed that its velocity may be decomposed into two others, one of which is in the direction of the velocity of the wheel $Av = v$, and the other in the direction AB of the *arm* or *wrist* of the bucket. As we may assume the direction of the outer element of the bucket—the velocity at the outer circumference of the wheel v, at right angles to the radius AC of the wheel—and the velocity c of the water coming on to the wheel, to be given, we shall have the required direction of the water layers if we draw through v a parallel to AB, and with c as radius, describe an arc from A as centre, and draw from A to the intersection of the arc with the parallel, the straight line Ac, or by calculation as follows:

Fig 197.



The angle which the velocity v of the circumference makes with the outer element of the bucket $AB = v\,AB = \phi$, may be deduced from the elbow angle $ABE = \delta$, and the division angle $ACBD = \beta_1$, by the equation $\delta = ACB + BAC = \beta_1 + 90^\circ - \phi$, and hence $\phi = 90^\circ - (\delta - \beta_1)$.

From ϕ, v and c we have the angle $c\,AB = \psi$, by which the direction of the layer of water must deviate from that of the arm of the bucket, in order that the water may enter the cells unimpededw for $\frac{sin.\,\psi}{sin.\,\phi} = \frac{v}{c}$, and, therefore,

$$sin.\,\psi = \frac{v\,sin.\,\phi}{c} = \frac{v\,cos.\,(\delta - \beta_1)}{c}.$$ (See Vol. I. § 32.)

Again; the angle $c\,AH$ of the direction of the water layer to the horizon, is $\nu_1 = \phi - \psi + \theta$, θ being as above the angle ACS, by which the point of entrance of the A water on the wheel, deviates from the summit S.

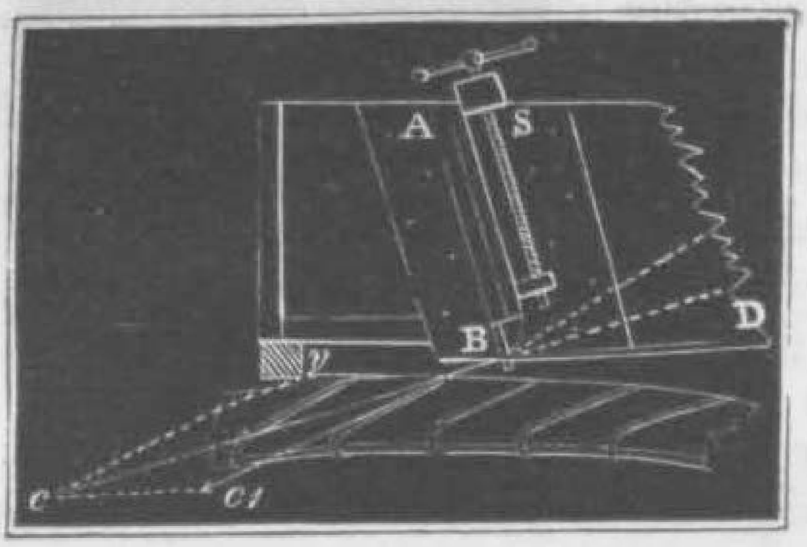
The relative velocity $Ac_1 = c_1$ with which the water enters the cells is $c_1 = \frac{c\,sin.\,(\phi - \psi)}{sin.\,\phi}$.

Example. Suppose a water wheel, the velocity at the circumference of which $v = 10$ feet, the velocity of the water $c = 15$ feet, the elbow angle $= 70\frac{1}{2}^\circ$, the division angle $\beta_1 = 4\frac{1}{2}^\circ$, and the point of entrance of the water deviates 12° from the summit: then $\phi = 90^\circ - (70\frac{1}{2}^\circ - 4\frac{1}{2}^\circ) = 24^\circ$, and, therefore, $sin.\,\psi = \frac{10}{15}\,sin.\,24^\circ = 0{,}27116$, hence $\psi = 15^\circ, 44'$. Thus, that the water may enter unimpeded, the ~~deviation of the~~ layer must be $15\frac{3}{4}°$ from that of the arm or outer element of the bucket. The angle of inclination to the horizon, or $\nu_1 = 24° - 15\frac{3}{4}° + 12° = 20\frac{1}{4}°$, and the relative velocity is $c_1 = \frac{15 \sin. 8°. 16'}{\sin. 24°} = 5{,}303$ feet.

Remark. It is generally considered, in older works on this subject, that the water layer should *enter the wheel in the direction of the arm of the bucket*, but this rule is only true when $v = 0$, or $\delta - \beta_1 = 90°$, *and these cases never occur.* The deviation ψ is of course very small for a wheel revolving slowly, but never so small as to allow of our assuming it as 0. When the water enters in the direction of the outer element of the bucket, the bucket strikes against the water, and throws it before it with a velocity $v \sin. \phi$, by which *vis viva* is lost, and water spilt.

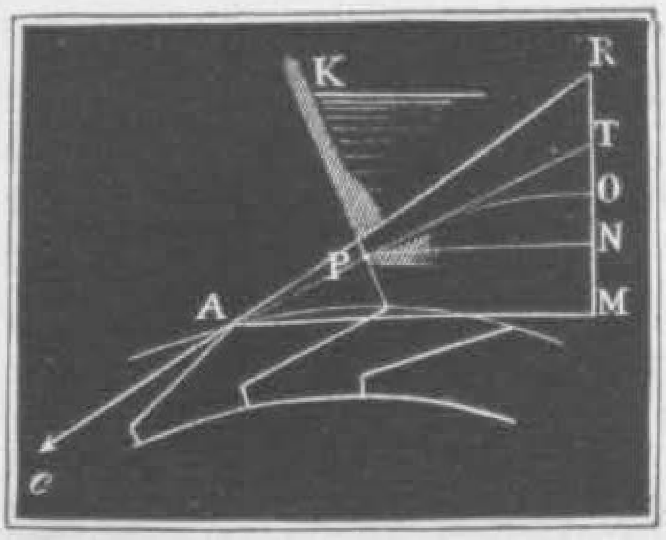
§ 93. That the water may reach the wheel with the direction required, either the sluice-opening is laid close up to the point at which the water is to enter the wheel, and the sluice is set at right angles to the direction of the layers of water, or an additional trough is laid in the required direction of the layer, or the sluice is so placed, that the direction of the parabolic curve, formed by the water in its free descent, may be that required. The Fig. 198 shows the pen-trough used in the Freiberg district, in which the bottom piece BD, and the lower part of the sluice-board, are set obliquely to the direction of the water layer, so that each makes an angle of about $14\frac{1}{2}°$ with the direction of the axis of it.

Fig. 198.



In order to find the direction of the sluice-board, when part of the water falls freely into the wheel, we have to recur to the theory of projectiles, given in Vol. I. § 38, &c. From the velocity $Ac = c$, Fig. 199, and the angle of inclination $RAM = \nu_1$ of the required direction of the layer to the horizon, the vertical co-ordinate MO of the apex of the parabola is:

Fig. 199.



$$x_1 = \frac{c^2 \sin. \nu_1{}^2}{2g},$$ and, on the other hand, the horizontal co-ordinate:

$$AM = y_1 = \frac{c^2 \sin. 2\nu_1}{2g}.$$

If, now, we wish to place the sluice-aperture at any point P of this parabolic curve, and if we know the height $MN = a$, of this point above the point of entrance A, then for the co-ordinates of this point $ON = a$, and $NP = y$, we have the formulas: $x = x_1 - a$, and

$$y = y_1 \sqrt{\frac{x}{x_1}} = y_1 \sqrt{\frac{a}{x_1}},$$

and for the angle of inclination $TPN = \nu$, which the parabola makes with the horizon on this point,

$$tang.\ \nu = \frac{TN}{PN} = \frac{2\ ON}{PN} = \frac{2x}{y}.$$

The plane PK of the sluice-board must be set at right angles to the tangent PT; and thus we find the required position of the sluice board, if we set off the abscissa ON in the opposite direction OT, draw PT, and erect a perpendicular to it PK.

If the sluice-aperture be set at the apex of the parabola, then the sluice-board will have to be vertical.

The velocity of discharge at P is $c_0 = \sqrt{c^2 - 2\,g\,a}$, and the corresponding theoretical pressure height $h_0 = \frac{c^2}{2g} - a$, or the effective height $= 1,1\left(\frac{c^2}{2g} - a\right)$, when the orifice is nearly rounded. The breadth of the sluice-orifice is made a little less than the breadth of the wheel.

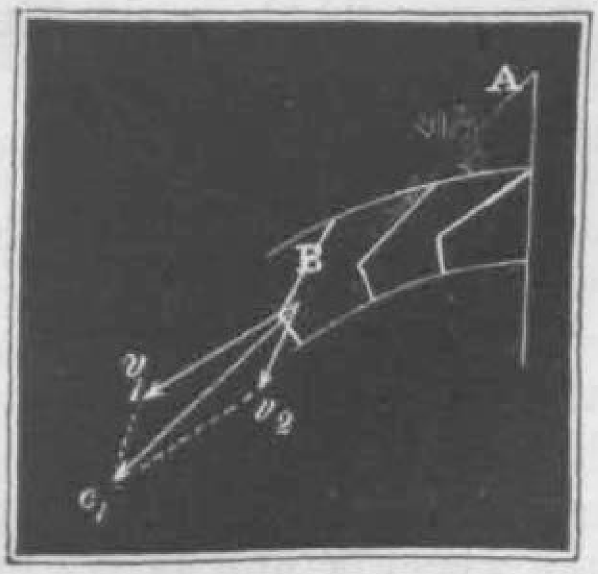
Example. For velocity $c = 15$ feet, and angle $\nu_1 = 20\frac{1}{2}^\circ$ (see example to last paragraph), the co-ordinates of the parabola's apex are $x_1 = 0.0155 \cdot 15^4\ (sin.\ 20\frac{1}{2}^\circ)^2 = 0,43$ feet, and $y_1 = 0,0155\ .\ 15^2\ .\ sin.\ 40\frac{1}{2}^\circ = 2,33$ feet. If, now, the centre of the sluice-aperture is to be 4 inches $= 0,333$ feet above the point of entrance, then the co-ordinates from the centre of the opening:

$$x = 0,43 - 0,33 = 0,1,\ y = 2,33\ \sqrt{\frac{0.1}{0,43}} = 1,11 \text{ feet, and}$$

$tang\ \nu = \frac{0.2}{1,11} = 9^\circ,\ 58'$, and hence the inclination of the sluice-board to the horizon is $= 90^\circ - \nu = 90^\circ - 9^\circ,\ 58' = 80^\circ,\ 2'$.

§ 94. *Effect of Impact.*—In the overshot wheel, the water acts in some degree by impact, but chiefly by its weight. We determine the effect of the shock, by deducting from the whole effect corresponding to the *vis viva* which the water entering the wheel possesses, the mechanical effect retained by the water when it leaves the wheel, and that lost by the oscillatory and eddying motion of the water in the cells. The velocity of the water leaving the wheel may be assumed as equal to the velocity v_1 of the wheel in the division circle, and hence the mechanical effect retained in this water is $\frac{v_1^2}{2g} Q \gamma$. The mechanical effect lost by the oscillation and eddying motion of the water may be put equal to $\frac{v_2^2}{2g} Q \gamma$, where v_2 is the velocity suddenly lost by the water entering the wheel. If, therefore, c_1 be the velocity Bc_1, Fig. 200, of the water entering the wheel, the mechanical effect still inherent in its *vis viva* is:

Fig. 200.



$$L_1 = \left(\frac{c_1^2 - v_1^2 - v_2^2}{2\,g}\right) Q\,\gamma.$$

But the velocity c_1 may be decomposed into two others $Bv_1 = v_1$, and $Bv_2 = v_2$, of which v_1 is exactly the velocity retained by the water as it moves on with the wheel, and, therefore, v_2 is the velocity *lost*. If we put the angle $c_1 B v_1$, which the direction of the entrance velocity c_1 of the water makes with a tangent $B v_1$ (the direction of the velocity of the circumference) $= \mu$, when we have $v_2^2 = c_1^2 + v_1^2 - 2\, c_1\, v_1 \cos. \mu$, and, therefore, the mechanical effect in question

$$L_1 = \left(\frac{c_1^2 - v_1^2 - c_1^2 - v_1^2 + 2\, c_1\, v_1 \cos. \mu}{2\, g}\right) Q\, \gamma =$$

$$\frac{(c_1 \cos. \mu - v_1)\, v_1}{g}\, Q\, \gamma, \text{ or as } \frac{1}{g} = 0{,}031,$$

and $\gamma = 62{,}5$, $L = 2{,}008\, (c_1 \cos. \mu - v_1)\, v_1\, Q$ feet lbs.

It is evident that the mechanical effect of impact is so much the greater, the greater c_1 is, and the less μ; and by comparing with Vol. I. § 386, it follows that this effect is a maximum when $v_1 = \frac{1}{2}\, c_1 \cos. \mu$. The maximum effect corresponding to this latter ratio is $\frac{1}{2}\, \frac{c_1^2 \cos. \mu^2}{2g}\, Q\, \gamma$; or when $\mu = 0$, or $\cos. \mu = 1$, then $L = \frac{1}{2} \cdot \frac{c_1^2}{2g}\, Q\, \gamma$. As $\frac{c_1^2}{2g}$ is the fall due to the velocity c_1, it follows, *that, in the most favorable case, the effect of impact is only half the available effect.* Hence the least possible part of the fall should be spent to produce impact, as much as possible being employed as weight. Suppose, for instance, we make $c_1 \cos. \mu = v_1$, therefore, $c_1 = \frac{v_1}{\cos. \mu}$, we sacrifice a height of fall $\frac{v_1^2}{2g \cos. \mu^2}$, without having any mechanical effect in return, but if we make $c_1 = \frac{2\, v_1}{\cos. \mu}$, we expend four times that fall, viz: $4 \cdot \frac{v_1^2}{2g \cos. \mu^2}$, and yet we have only

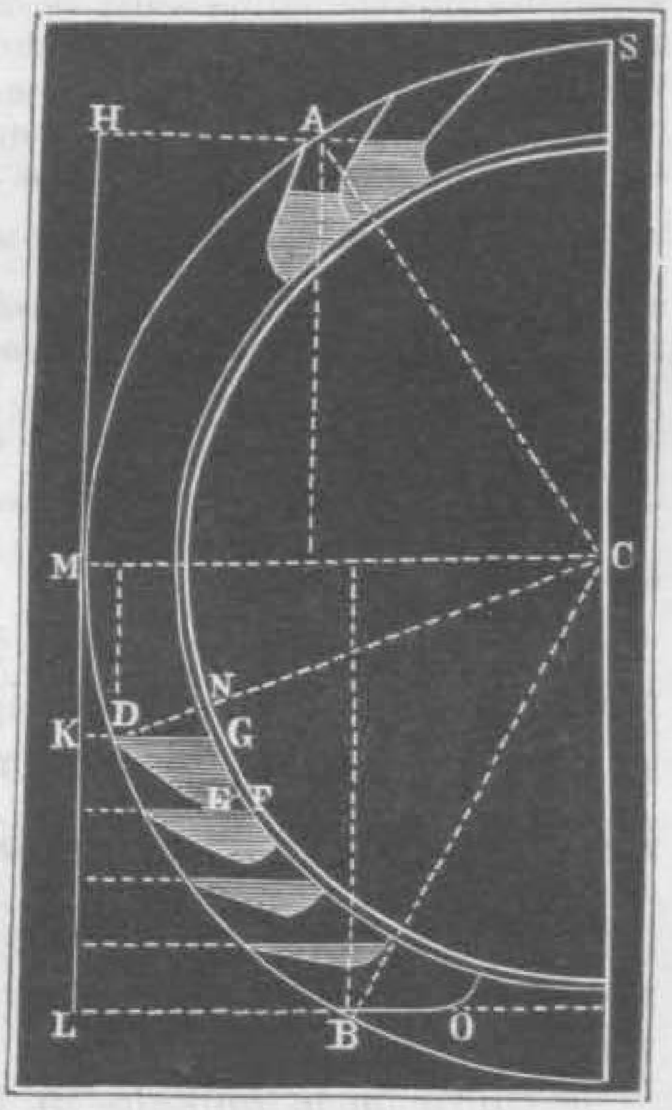
$$\tfrac{1}{2} \cdot \frac{4\, v_1^2}{2g}\, Q\, \gamma = 2 \cdot \frac{v_1^2}{2g}\, Q\, \gamma,$$

and lose thereby the amount of fall represented by:

$\left(\frac{4}{\cos. \mu^2} - 2\right) \frac{v_1^2}{2g}$, and even if we assume $\mu = 0$, or $\cos. \mu = 1$, the loss of fall is $2 \cdot \frac{v_1^2}{2g}$, or double as much as when we avoid all shock, or bring the water on to the wheel with the velocity with which the wheel revolves. Again, we perceive *that the efficiency of the wheel will be greater the less v_1 is, or the slower the wheel revolves.* It is true that the capacity of the wheel, its width e, and, therefore, its height, must be greater as the velocity of revolution v is less; and as the journals of a wheel must be of greater diameter the heavier the wheel is, and as the moment of friction increases as the radius of the journal, the mechanical effect consumed by the journal friction in the case of the wheel revolving slowly, may be greater than in one moving more rapidly; and hence we perceive that it by no means follows as a matter of course, that the slower a wheel revolves, the greater its efficiency will be.

§ 95. *Effect of the Water's Weight.*—The cells of a water wheel, when filled, form an annular water space AB, Fig. 201, which is termed the water arc, as the water enters at the upper part of this arc, and leaves it at the lower end, its height h is the effective fall, and, therefore, the mechanical effect given off by the weight of water $= h \cdot Q\gamma$. The height of the water arc may be subdivided into three parts. The first part HM lies above the centre of the wheel, and depends on the angle $SCA = \theta$, by which the point of entrance deviates from the vertical passing through the summit of the wheel. If, again, we put the radius of the wheel $CA = a$, the height of the upper part of the water arc $MH = a \cos. \theta$. The second part MK lies below the centre of the wheel, and depends upon the point D, at which the wheel begins to lose water, or to *spill*. If we put the angle MCD by which this point lies below the centre of the wheel $= \lambda$, then this second height $MK = a \sin. \lambda$. The third part includes the arc DB, in the course of which the wheel empties each bucket in turn. If we put MCB, the angle by which the point B, at which the buckets are emptied, deviates from M the centre of the wheel $= \lambda_1$, then the height $KL = a\,(\sin. \lambda_1 - \sin. \lambda)$.

Fig. 201.



Whilst now in the two upper parts of the arc, the water has its entire effect, it communicates only a part of its mechanical effect to the wheel in this third part, because here it gradually quits the wheel, and, therefore, the total effect of the water's weight must be represented by $a\,(\cos. \theta + \sin. \lambda)\, Q\gamma + a\,(\sin. \lambda_1 - \sin. \lambda)\, Q_1 \gamma$, when Q_1 is the mean quantity of water effective in the lower division of the water arc.

If we combine with this, the effect of the impact of the water, we have the total mechanical effect of an overshot water wheel:

$$L = Pv = \left(\frac{(c_1 \cos. \mu - v_1)\, v_1}{g} + a\,(\cos. \theta + \sin. \lambda)\right) Q\gamma$$
$$+ a\,(\sin. \lambda_1 - \sin. \lambda)\, Q_1 \gamma;$$

or, if we put the height $a\,(\cos. \theta + \sin. \lambda)$ of the part of water arc taking up the entire effect of the water $= h_1$, and the remaining part $a\,\sin. \lambda_1 - \sin. \lambda) = h_2$, and the ratio $\frac{Q_1}{Q} = \varkappa$, then:

$$L = Pv = \left(\frac{(c_1 \cos. \mu - v_1)\, v_1}{g} + h_1 + x\, h_2\right) Q \gamma,$$

and the force at circumference of the wheel:

$$P = \left(\frac{(c_1 \cos. \mu - v_1)\, v_1}{g} + h_1 + x\, h_2\right) \frac{Q}{v} \gamma.$$

Example. The velocity of entrance of the water on an overshot wheel of 30 feet diameter is $c_1 = 15$ feet, the velocity v_1 of the division circle $= 9\frac{1}{4}$ feet. The angle by which the direction of the water layer deviates from the direction of motion of the wheel at the point of entrance, is $8\frac{1}{2}^\circ$, and the deviation of this point from the summit of the wheel is 12°. The deviation of the point where the wheel begins to lose water from the centre of the wheel $\lambda = 58\frac{1}{2}^\circ$, and the deviation of the lowest point in the water arc from the centre of the wheel, or $\lambda_1 = 70\frac{1}{2}^\circ$. Lastly, the quantity of water going on to the wheel $Q = 5$ cubic feet per second, and $x = \frac{Q_1}{Q}$ is assumed as $\frac{1}{2}$. Required the effect of the wheel. First, the effective *impact fall* $= 0{,}031\ (15 \cos. 8\frac{1}{2}^\circ - 9\frac{1}{4})\, 9\frac{1}{4} = 1{,}60$ feet; and the effective *weight fall* is:
$15\ (\cos. 12^\circ + \sin. 58\frac{1}{2}^\circ) + \frac{1}{2}\ (\sin. 70\frac{1}{2}^\circ - \sin. 58\frac{1}{2}^\circ) = 15\ (1{,}8307 + 0{,}0450 = 28{,}14$ feet, and hence the total effect of the wheel is $(160 + 28{,}14)\ .\ 5\ .\ 62{,}5 = 9256$ feet lbs. $=$ 17 horse power, and the force at the division circle is $\frac{9256}{9\frac{1}{4}} =$ 1000 pounds, nearly.

§ 96. We easily perceive from this, that for the exact determination of the effect of the weight of the water on an overshot wheel, it is essential to know the two limits of the arc in which the wheel loses its water, and the ratio $x = \frac{Q_1}{Q}$, of the mean quantity of water contained in the cells in this part of the water arc, to that originally received by them. On this subject we must now endeavor to ascertain the necessary rules.

If there be n buckets in the wheel, and if it make u revolutions per minute, there are presented $\frac{nu}{60}$ cells per minute to receive the quantity of water Q, and, therefore, into each cell there goes the quantity $V = Q \div \frac{nu}{60} = \frac{60\ Q}{nu}$. If e be, as hitherto, the width of the wheel, then the section of the prism of water in any cell $= F_0 = \frac{V}{e} = \frac{60\ Q}{n\, u\, e}$.

If now $DEFG$, Fig. 201, be the cell at which the water begins to spill, then the section: $F_0 =$ segment $DEF +$ triangle DFG, or as the triangle $DFG =$ triangle $DFN -$ triangle GN, then $F_0 =$ segment $DEF +$ triangle $DFN -$ triangle DGN. If we put the area of the segment $DEF = S$, and that of the triangle $DFN = D$, then the triangle $DGN = S + D - F_0$; but as the triangle DGN is also equal to $\frac{DN\,.\,NG}{2} = \frac{1}{2}\, d^2 \tang. \lambda$ nearly, we have approximately (and the more accurately the greater the number of buckets), $\tang. \lambda = \frac{S + D - F_0}{\frac{1}{2}\, d^2}$. Thus the angle $MCD = \lambda$, corresponding to the point at which the wheel begins to empty itself, is determined.

Each cell will have emptied itself when the outer element of the bucket becomes horizontal; if, therefore, the angle CBO, which this outer element, or the wrist of the bucket, makes with the radius $= \lambda_1$, then λ_1 gives us the angle MCB, which fixes the point where the cells have emptied themselves. In order, therefore, to find the effect of the water on the discharging arc, let us divide the height $KL = a\,(sin.\ \lambda_1 - sin.\ \lambda)$ into an even number of equal parts, indicate the position of the bucket for each of these points of division, draw horizontal lines through the sections of the water in the cells for each of these positions, and reckon the areas $F_1, F_2, F_3 \ldots F_n$ of these sections. The mean value F of these may be determined by the Simpsonian rule, putting

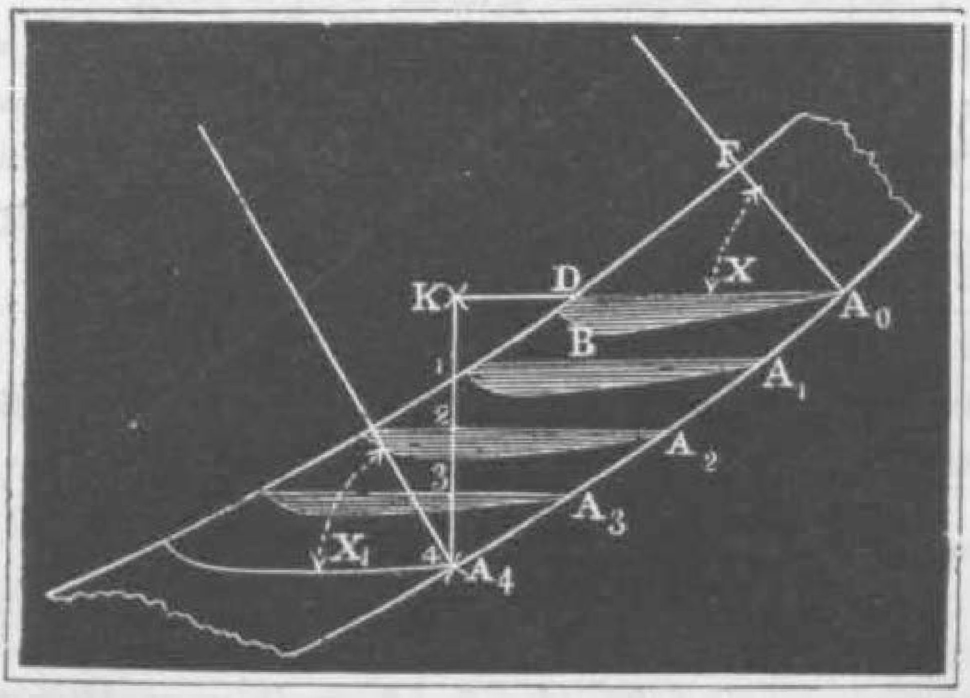
$$F = \frac{F_0 + F_n + 4\,(F_1 + F_3 + \ldots + F_{n-1}) + 2\,(F_2 + F_4 + \ldots F_{n-2})}{3\,n},$$

and from this we get the ratio of the mean quantity of water in a cell in the discharging arc to the quantity in a cell before it begins to empty itself:

$$x = \frac{Q_1}{Q} = \frac{F}{F_0} = \frac{F_0 + F_n + 4(F_1 + F_3 + \ldots + F_{n-1}) + 2(F_2 + F_4 + \ldots F_{n-2})}{3\,n\,F_0}.$$

Example. There are 300 cubic feet of water per minute supplied to a water wheel 40 feet in diameter, making 4 revolutions per minute. What is the effect of such a wheel? If we suppose the depth of the shrouding to be 1 foot, then the width of the wheel $= \frac{4\,.\,300}{\pi\,.\,40\,.\,1\,.\,4} = \frac{30}{4\pi} = 2{,}4$ feet. If there be 136 buckets on the wheel, the quantity of water in each cell $V = \frac{300}{4\,.\,136} = \frac{75}{136} = 0{,}5515$ cubic feet, and, hence, the section: $F = \frac{0{,}5515}{2{,}4}$ square feet $= \frac{144\,.\,0{,}5515}{2{,}4} = 33{,}09$ square inches. By accurate measurement on the buckets themselves, as they are represented in Fig. 202, the

Fig. 202.



area of the segment A_0BD is 24,50 square inches, and that of the triangle $A_0FD = 102$ square inches, hence for the commencement of discharge

$$tang.\ \lambda = \frac{24.50 + 102 - 30.09}{\frac{1}{2}\,.\,144} = \frac{93{,}41}{72} = 1{,}2973,$$ and, therefore, $\lambda = 52°,\ 22\frac{1}{2}'$. The angle at which the wrist of the bucket meets the radius is $\lambda_1 = 62°,\ 30'$, and, therefore, the height KA_4 of the part of the discharging arc retaining water is $= a\,(sin.\ \lambda_1 - sin.\ \lambda) = 20\,(0{,}8870 - 0{,}7920) = 1.79$ feet. If within this height we delineate three relative positions of a bucket, we find by measurement and calculations the section of the water

space in the bucket for these positions: $F_1 = 24{,}50$; $F_2 = 14{,}48$, and $F_3 = 6{,}60$ square inches. As, now, the section at the commencement is $F_0 = 33{,}09$, and at the end it is $F_4 = 0$, we shall find the ratio:

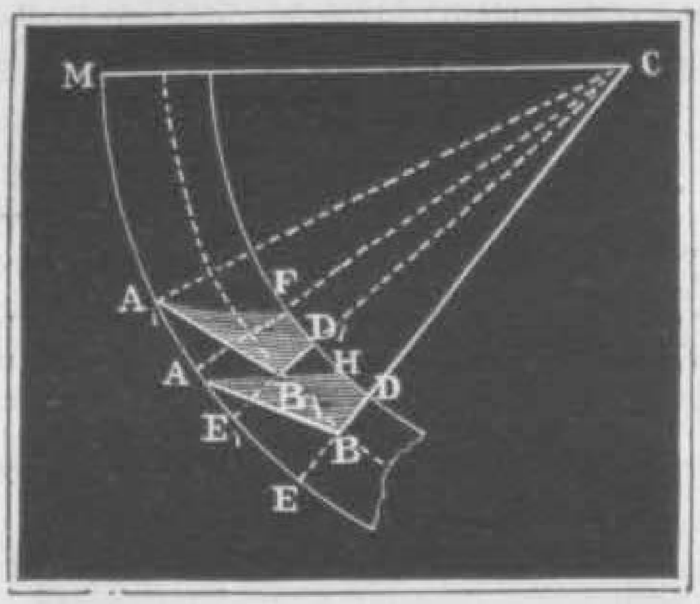
$$\varkappa = \frac{F}{F_0} = \frac{33{,}09 + 4\,(24{,}50 + 6{,}60) + 2\,.\,14{,}48}{12\,.\,33{,}09} = \frac{15{,}5375}{33{,}09} = 0{,}469.$$

If, again, the water enter the wheel at 10° below the summit, and with a velocity $c_1 = \frac{v_1}{cos.\,\mu}$, so that the water acts without shock, then the whole mechanical effect given off by the wheel, neglecting the friction of the axle, is:

$$L = a\,[cos.\,\theta + sin.\,\lambda + 0{,}469\,(sin.\,\lambda_1 - sin.\,\lambda)] \times 5 \times 62{,}25$$
$$= 20\,(0{,}9848 + 0{,}7920 + 0{,}469\,.\,0{,}085)\,6600n = 1{,}8167n\,6225 = 11308 \text{ feet lbs.} = 23{,}5$$
horse power.

§ 97. *Number of Buckets.*—As we have above indicated, the capacity of the wheel to hold water should be made as great as possible, or the buckets should retain the water as long as possible, so that, *cæteris paribus*, the maximum effect of the fall of water is obtained when the buckets are placed so close, that the water surface AH, Fig. 203, in the bucket beginning to empty itself, is in contact with the bucket $A_1B_1D_1$ next above it. If we take this condition as basis, we can deduce a formula for determining the number of buckets. From the angle of discharge $MCF = FAH = \lambda$, and the depth of the shrouding $AF = d$, we have, approximately, $FH = d\,.\,tang.\,\lambda$. If, now, we assume the *division circle* to be at half the depth of the shrouding, we may then put

Fig. 203.



$$D_1H = D_1F = \tfrac{1}{2}\,FH = \tfrac{1}{2}\,d\,tang.\,\lambda.$$

If, again, we introduce the angle of division $ECE_1 = \beta$, and the bucket angle $ACE = \beta_1$, we get the angle $ACE_1 = \beta_1 - \beta$, and, approximately, the arc $D_1F = a_1\,(\beta_1 - \beta)$. By equating the two values of D_1F we have $a_1\,(\beta_1 - \beta) = \frac{1}{2}\,d\,tang.\,\lambda$, and, therefore,

$$\beta = \beta_1 - \tfrac{1}{2}\,\frac{d}{a_1}\,tang.\,\lambda.$$

If the thickness of the buckets $s = 0$, then of course the number of buckets would be

$$n = \frac{360^\circ}{\beta^0} = \frac{2\pi}{\beta} = \frac{2\pi}{\beta_1 - \frac{1}{2}\frac{d}{a_1}\,tang.\,\lambda};$$

but as the space occupied by the buckets is something considerable, we must take it into calculation, and thus make the division angle greater by an amount corresponding to an arc $\frac{s}{a_1}$, or we must take:

$$n = \frac{2\pi}{\beta_1 + \frac{s}{a_1} - \frac{d}{2\,a_1}\,tang.\,\lambda}.$$

If we introduce $tang.\ \lambda = \frac{S + D - F_0}{\frac{1}{2} d^2}$, and $F_0 = \frac{60\, Q}{n\, u\, e}$, then we get:

$$2\pi = n\left(\beta_1 + \frac{s}{a_1} - \frac{S + D}{a_1\, d} + \frac{60\, Q}{n\, u\, a_1\, d\, e}\right),$$

and, therefore, the required number of bucketst

$$n = \frac{2\,\pi\, u\, a_1\, d\, e - 60\, Q}{[\beta_1\, a_1\, d + s\, d - (S + D)]\, u\, e}.$$

If we put $d\, e = \frac{4\,.\,60\, Q}{2\,\pi\, u\, a_1}$, then we have more simply:

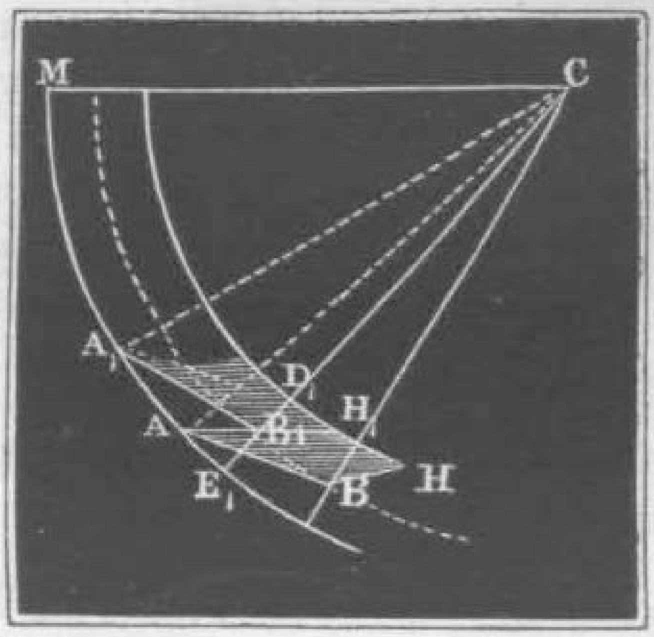
$$n = \tfrac{3}{2}\,.\,\frac{\pi\, a_1\, d}{\beta_1\, a_1\, d + s\, d - (S + D)},$$

and if we take $D = \frac{1}{2}\,\beta_1\, a_1\, d$, then:

$$n = \tfrac{3}{2}\,.\,\frac{\pi\, a_1\, d}{\frac{1}{2}\,\beta_1\, a_1\, d + s\, d - S}.$$

It is also easy to perceive that the angle of discharge λ is still more increased, when (as is represented in Fig. 204) the bucket immediately following that which is about to discharge, comes in contact with the surface of the water AH_1, in that bucket, with a *flat surface* instead of a *corner*; or, when the *wrist* of the bucket is not set in a radial direction, but in such a position, that, shortly before discharge commences, it is horizontal. In this case, the segment or triangle $S = A_1\, B_1\, D_1$, is increased by a triangle $B_1\, D_1\, H$, and hence $tang.\ \lambda = \frac{S + D - F_0}{\frac{1}{2} d^2}$,

Fig. 204.



and, therefore, also the angle of discharge λ becomes greater.

Iron buckets are always rounded at the corner B.

Example. What number of buckets should be put in an overshot wheel of 40 feet diameter, and 1 foot in width, giving $\beta_1{}^0 = 4^\circ$ or $\beta_1 = 0.06981$. $S = 24,5$ square inches $= 0,17014$ square feet, and the thickness of the buckets $s = 1$ inch $= 0,0833$ feet. According to our formula:

$$n = \tfrac{3}{2}\,\frac{\pi\,.\,19,5}{\frac{1}{2}\;0,06981\,.\,19,5 + 0,0833 - 0,17014} = \frac{29,25\,\pi}{0,5939} = 155,$$

which, for the sake of facility of division, we may take 152.

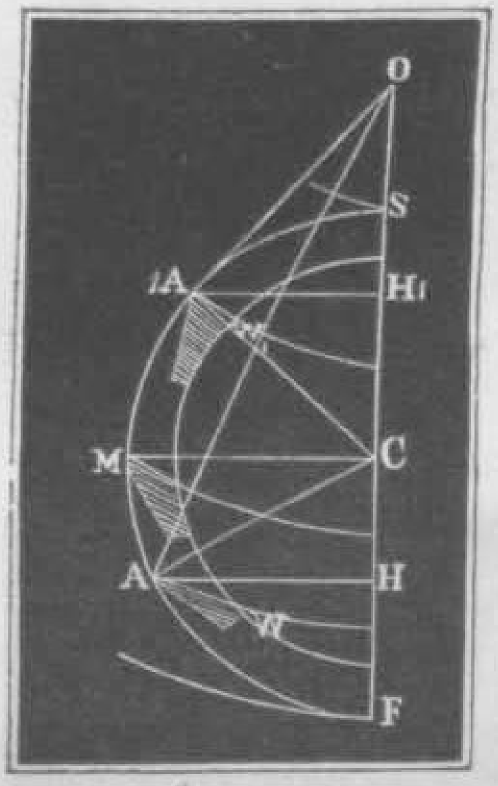
Remark. The construction of bucket shown in Fig. 204, has another advantage, viz.: that for it a less width of wheel is necessary, as it is impossible to make the wheel space = four times the capacity of the water space (see Vol. II. § 92). If we here introduce $S = \frac{1}{4}\, a_1\, \beta\, d_1 + \frac{1}{8}\, d^2\, tang.\ \lambda$, and as $S = \frac{1}{2}\, d^2\, tang.\ \lambda + F_0 - D$, and $D = \frac{1}{2}\, a_1\, \beta_1\, d$, we obtain: $tang.\ \lambda = \frac{\frac{3}{4}\, a_1\, \beta_1\, d - F_0}{\frac{3}{8}\, d^2}$, and hence:

$$2\,\pi = n\left(\beta_1 + \frac{s}{a_1} - \beta_1 + \tfrac{4}{3}\,.\,\frac{60\, Q}{n\, u\, a_1\, d\, e}\right), \text{ or } 2\,\pi = \frac{n\, s}{a_1} + \tfrac{4}{3}\,.\,\frac{60\, Q}{u\, a_1\, d\, e}.$$

If we neglect the thickness of the buckets, then $e = \frac{4}{3}\,.\,\frac{60\, Q}{2\,\pi\, u\, a_1\, d}$, or a much less width of wheel than was assumed at § 92. We see from this that we should approximate as nearly as possible to the limits of bucket construction, of which we have now been treating.

§ 98. *Effect of Centrifugal Force.*—For equal velocity of the circumference, small wheels make a greater number of revolutions than large; but the uniform motion of the machine, or the nature of the work to be done, as sawing, hammering, grinding, &c. &c., require a certain velocity of the wheel. Hence small wheels frequently make a great number of revolutions per minute. But at such high velocities, the *centrifugal force* of the water comes into play to such a degree that the inclination of the surface of the water in the buckets to the horizon is considerable, and, therefore, the discharge commences much earlier than would be the case if the wheel were moving slowly. We have found (Vol. I. § 274) that the surfaces of the water in the buckets are a series of cylindrical hollows, the common axis of which O, Fig. 205, runs parallel to the axis of the wheel, and lies at a height: $CO = k = \frac{g}{\omega^2} = g \cdot \left(\frac{30}{\pi u}\right)^2 = \frac{2850}{u^2}$ above the axis C of the wheel. This distance increases, therefore, *inversely* as the *square of the number of revolutions*, and becomes small for a great number of revolutions. Hence we at once perceive that the water-surface is horizontal only at the summit and at the bottom of the wheel, and that at a given point M above the centre of the wheel the deviation from horizontal is a maximum.

Fig. 205.



The deviation $HAW = AOC = \chi$ for any point A, at a distance $ACM = \lambda$ from the centre of the wheel, is:

$$tang.\ \chi = \frac{AH}{OH} = \frac{a\ cos.\ \lambda}{k + a\ sin.\ \lambda}.$$

For a point A_1 above M, λ is negative, and hence

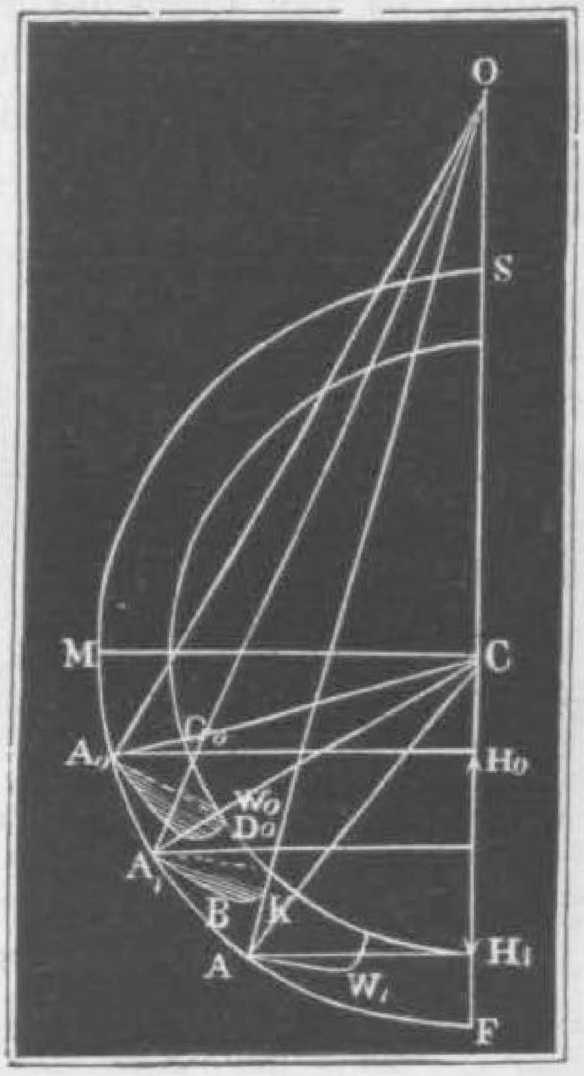
$tang.\ \chi = \frac{a\ cos.\ \lambda}{k - a\ sin.\ \lambda}$. If we lay off a tangent OA_1 from O, we have in the point of contact A_1, that point at which the deviation from horizontality is greatest, or where χ is a maximum, and $= \lambda$, $sin.\ \chi$ being, however, $= \frac{a}{k}$.

Example 1. For a wheel, making 5 revolutions per minute, $k = \frac{2850}{25} = 114$ feet, the radius a being $= 16$ feet, and the discharging angle $\lambda = 50°$. Then:
$tang.\ \chi = \frac{16\ cos.\ 50°}{114 + 16\ sin.\ 50} = \frac{10.285}{136,266}$, therefore. $\chi = 4°, 39'$, so that the surface of the water deviates in this case $4\frac{1}{3}$ degrees from the horizontal.

Example 2. For a wheel, making 20 revolutions, $k = \frac{2850}{400} = 7,125$. If, therefore, $a = 5$, $\lambda = 0°$, then $tang.\ \chi = \frac{5}{7,125}$, hence $\chi = 35°, 3'$. At an angle of $44°, 34'$ above the centre of the wheel, the deviation is as much as $44°, 34'$.

§ 99. If, now, we take into consideration the effect of centrifugal force, as is obviously necessary in the case of wheels revolving rapidly, the formula we above found for the arc of discharge must be replaced by others. Let A_0 (Fig. 206) be the point at which discharge commences, $MCA_0 = H_0A_0G_0 = \lambda$ the angle of discharge, $H_0A_0W_0 = A_0OC = \chi$ the depression of the water's surface below the horizon, or the angle $GA_0W_0 = \lambda + \chi$, and the $\Delta\, A_0GW_0 = \frac{1}{2} d \,.\, d\, tang.\, (\lambda + \chi) = \frac{1}{2} d^2\, tang.\, (\lambda + \chi)$. If we now put the contents of the segment $ABD_0 = S$, that of the $\Delta\, AGD = D_0$, and the section of the body of water $= F$, then $F_0 + \frac{1}{2} d^2\, tang.\, (\lambda + \chi) = S + D$, are, therefore

Fig. 206.



$$1.\ tang.\, (\lambda + \chi) = \frac{S + D - F_0}{\frac{1}{2} d^2}.$$

But
$$\frac{sin.\, AOC}{sin.\, OAC} = \frac{CA}{CO},\ i.\,e.,$$

$$\frac{sin.\, \chi}{sin.\, [90^\circ - (\lambda + \chi)]} = \frac{a}{k},$$

and hence follows:

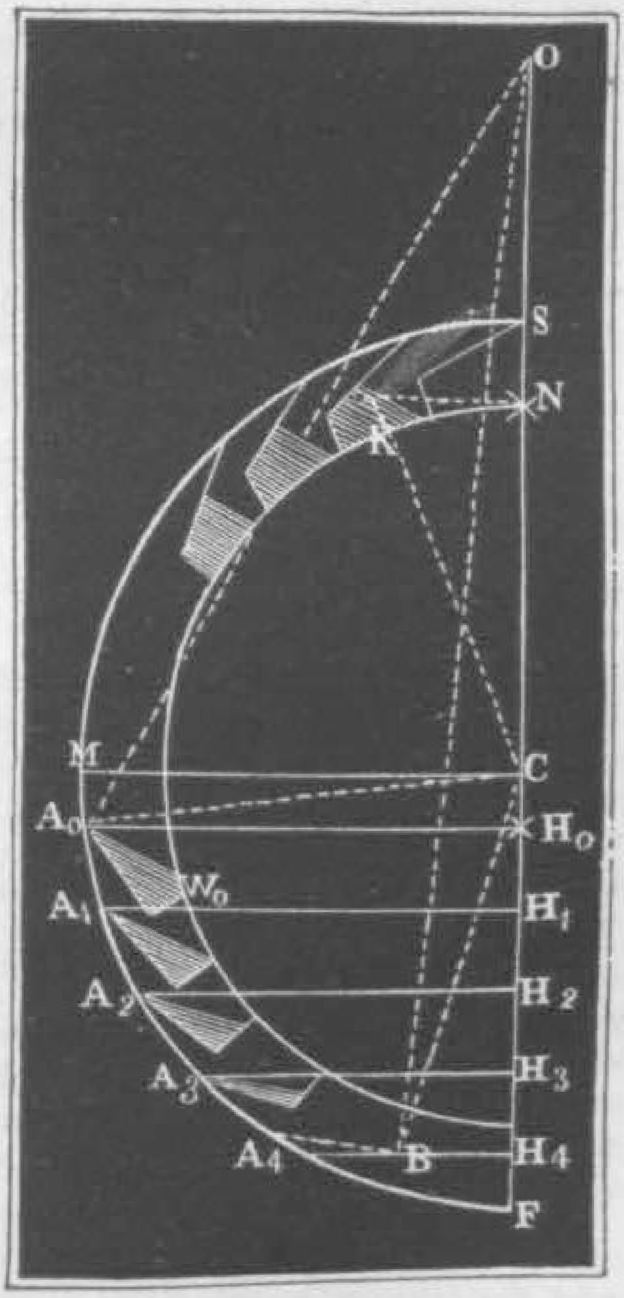
$$2.\ sin.\, \chi = \frac{a\, cos.\, (\lambda + \chi)}{k}.$$

When by the first formula, the value of $\lambda + \chi$, and by the second, the value of χ the depression, have been found, we obtain by subtraction of the two angles $\lambda = (\lambda + \chi) - \chi$.

At the end A_1 of the angle of discharge, the outer end of the bucket coincides with the water's surface A_1W_1, and, therefore, $CA_1W_1 = \lambda_1 + \chi$ at this point = the known angle δ depending on the form of bucket. Hence

$$sin.\, \chi_1 = \frac{a\, cos.\, \delta}{k} \text{ and } \lambda_1 = \delta - \chi_1,$$ that is, the angle by which the end of the arc of discharge deviates from the centre of the wheel.

Fig. 207.



If the height $H_0H_4 = h_4 = a\, (sin.\, \lambda_1 - sin.\, \lambda)$, Fig. 207, of the arc of discharge, be divided into 4 or 6 equal parts, and the filling of the bucket for corresponding positions be determined, we can again find the ratio

$x = \frac{Q_1}{Q} = \frac{F}{F_0}$ of the mean contents of the buckets during the discharge, to the contents before discharge commences, and so calculate the mechanical effect of the water in the arc of discharge. For this the above formula must be used inversely. In this case λ is given, hence:

$tang. \chi = \frac{a \cos. \lambda}{k + a \sin \lambda}$, and $F = S + D - \frac{1}{2} d^2 \, tang. (\lambda + \chi)$.

If the water does not fill the entire segment, or if $F < S$, or $\frac{1}{2} d^2 \, tang. (\lambda + \chi) > D$, then we must put:

$$F = \text{segment } ABD - \triangle ADK,$$

and in the case of straight bucketsh

$$F = S - \frac{1}{2} d_1^2 \cdot \frac{\sin. (\lambda + \chi - \delta_1) \sin. \delta}{\sin. (\lambda + \chi)},$$

in which d_1 is the diagonal AD and δ_1 is the angle DAC included between this and the radius CA.

Example. A small wooden water wheel (Fig. 207), 12 feet high, 1 foot depth of shrouding, and 4 feet wide, receives 1080 cubic feet of water, when making 17 revolutions per minute; required the mechanical effect produced by it. Here, $a = 6$, $d = 1$, $e = 4$, $a_1 = 5{,}5$, $Q = \frac{1080}{60} = 18$, $u = 17$; allowing 24 buckets,

$\beta^0 = \frac{360^\circ}{24} = 15^\circ$, and $F_0 = \frac{1080}{24 \cdot 17 \cdot 4} = \frac{45}{68} = 0{,}662$ square feet. If, again, $D = 0{,}652$,

and $S = 0{,}373$, then $tang. (\lambda + \chi) = \frac{0{,}373 + 0{,}652 - 0{,}662}{\frac{1}{2}} = 0{,}363 \times 2 = 0{,}726$,

therefore, $\lambda + \chi = 35^\circ, 59'$. But $CO = k = \frac{2850}{17^2} = 9{,}86$ feet, and hence

$\sin. \chi = \frac{6 \cos. 35^\circ, 59'}{9{,}86} = 0{,}4924$, hence $\chi = 29^\circ, 30'$, and $\lambda = 6^\circ, 29'$. Thus the discharge commences in this case at only $6\frac{1}{2}^\circ$ below the centre of the wheel. To find the point at which the discharge is complete, we have in the present case (in which water hangs in the bucket, although the water's surface touches the outer extremity of the bucket), to put in the formula $\sin. \chi_1 = \frac{a \cos \delta}{k}$, instead of a, the radius of the division circle $a_1 = 5{,}5$, and instead of δ, the angle formed by the outer element of the bucket and the radius, and which is here $79^\circ, 14'$. Hence: $\sin. \chi_1 = \frac{5{,}5 \cos. 79^\circ, 14'}{9{,}86}$, therefore, $\lambda = 5^\circ, 59'$, and the second angle of discharge $\lambda_1 = 79^\circ, 14' - 5^\circ, 59' = 73^\circ, 15'$. Hence, the height of the arc of discharge, $h_2 = a_1 \sin. \lambda_1 - a \sin. \lambda = 5{,}5 \sin. 73^\circ, 15' - 6{,}0 \sin. 6^\circ, 29' = 5{,}2666 - 0{,}6775 = 4{,}589$ feet. Dividing this height into 4 equal parts, we determine by delineation, by measurement, and calculation, the corresponding three intermediate values of F. The results arrived at are $F_1 = 0{,}501$, $F_2 = 0{,}409$, and $F_3 = 0{,}195$, and, therefore, the required ratio of the sectionsn

$$x = \frac{F}{F^0} = \frac{0{,}662 + 4\,(0{,}501 + 0{,}195) + 2 \cdot 0{,}409}{12 \cdot 0{,}662} = 0{,}537,$$

and the mechanical effect produced by the water during the descent of the arc of discharge: $L_1 = x \cdot h_1 Q\gamma = 0{,}537 \cdot 4{,}589 \cdot 18 \cdot 62{,}5 = 27{,}55$ feet lbs. If the water fell with a velocity of 20 feet, 20 degrees under the summit of the wheel, in such a direction that it deviated 25° from the tangent at the point of entrance, then the remaining effect of the water's pressuren

$L_2 = (5{,}5 \cos. 20^\circ + 6 \sin. 6^\circ, 29') \; 18 \cdot 62{,}5 = 5{,}854 \cdot 1120 = 6556$ feet lbs.

and the effect of impact, the velocity in the division circle v_1 being $\frac{11 \cdot \pi \cdot 17}{60} = 9{,}791$ feet is $L_3 = 0{,}031\,(20 \cos. 25^\circ - 9{,}791)\; 9{,}791 \cdot 18 \cdot 62{,}5 = 2{,}611 \times 1120 = 2974$ feet lbs, and hence the whole mechanical effect produced is:

$$L = L_1 + L_2 + L_3 = 12305 \text{ feet lbs.}$$

§ 100. *Friction of the Gudgeons.*—No inconsiderable portion of the mechanical effect of overshot wheels is lost in the mechanical effect absorbed by *friction on the gudgeons*. Let the weight of the water wheel, together with the water in the buckets, $= G$, the radius of the gudgeon $= r$, then the friction is $= f\,G$, and the velocity at the periphery of the axis $= v = \frac{\pi u r}{30}$, and hence the mechanical effect consumed by the friction of the gudgeons $= f\,G\,v = \frac{\pi u r}{30} f\,G$ $= 0{,}1047 \,.\, u f\,G\,r$. For well turned gudgeons on good bearings $f = 0{,}075$, when oil or tallow is well supplied, or $f = 0{,}054$, when a constant supply of best oil is kept up. In ordinary circumstances of the application of a black lead unguent, $f = 0{,}11$. The weight G of the wheel must be determined by admeasurement for each case. For wheels of 18 to 20 feet in height, the weight has been found to be from 800 to 1000 times the number of effective horse power in pounds. The wooden wheels of Freiberg, 35 feet in height, weigh, when saturated with water, nearly 44000 lbs. Being 20 horse power, this makes upwards of 2000 lbs. per horse power for the weight of the wheel.

The effective power L of a wheel increases as the weight of the wheel increases, as the proportion of the bucket filled $\varepsilon = \frac{Q}{d\,e\,v}$ increases, and as the number of revolutions u increases, so that, inversely, $G = \frac{\iota L}{\varepsilon u}$, in which ι is a co-efficient to be ascertained from experience. According to Rettenbacher, a small iron wheel, the buckets of which are filled $\frac{1}{3}$, the horse power being 6,3, $\iota = 3432$ lbs.

For the Freiberg wooden wheels of 20 horse power, $\iota = 2750$ lbs., so that, for a first approximation, we may use the formula: $G = 3000 \frac{L}{\varepsilon u}$ pounds.

The strength of the gudgeons depends on the weight of the wheel G, and thus the weight has a twofold influence on the friction (Vol. II. § 87). We have given the formula:

$2\,r = 0{,}048 \sqrt{G}$ inches $= 0{,}00045 \sqrt{G}$ feet, for the strength of gudgeons, and, therefore, we may here put $Gr = 0{,}00142 \sqrt{G^3}$, and hence the mechanical effect consumed by the friction at the gudgeons

$$= 0{,}1047\, uf h\; 0{,}00142 \sqrt{G^3} = 0{,}00015\, uf \sqrt{G^3}$$

Example. What amount of mechanical effect is consumed by the friction of a wheel of 25000 lbs. weight, with gudgeons of 6 inches diameter, the wheel making 6 revolutions per minute. Assuming $f = 0{,}08$, then $f\,G = 0{,}08 \times 25000 = 2000$ lbs., and the statical moment of this $= f\,G\,r = \frac{1}{4} \,.\, 2000 = 500$ feet lbs., and the mechanical effect $= 0{,}1047 n\; 6 \,.\, f\,G\,r = 314$ feet lbs.

Remark. The gudgeon friction of a water wheel may be increased or diminished according to the manner in which the mechanism for transmitting its power is applied to it. If, as in Fig. 208, the power P and the resistance Q act on the same side of the wheel, then the friction on the gudgeons is diminished by an amount equal to the resistance Q, so that the friction is less; but if the power and resistance work on opposite sides (as in Fig. 209), then the pressure on the gudgeons is increased by an amount equal to

Fig. 208.

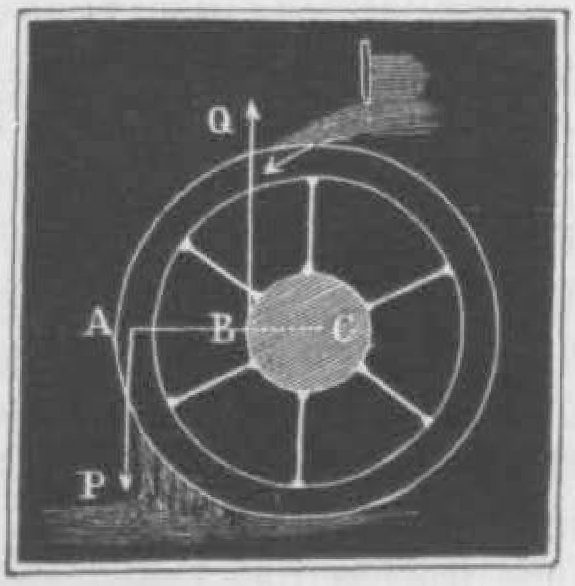
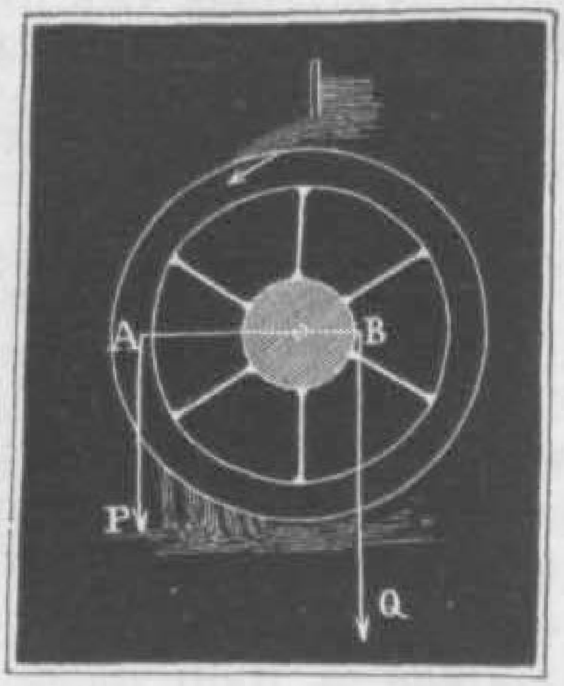


Fig. 209.



the resistance Q, and, therefore, the friction is as much greater as in the other case it was less. If in the former case the leverage *CB* of the resistance be made equal to the leverage of the power *CA*, so that the transmission is effected by a toothed wheel on the periphery of the water wheel, as shown in Fig. 171, then the effect of the power on the gudgeon is counterbalanced by the resistance (not if the pinion wheel be in the position shown in the diagram, but when this is placed *below the centre* of the wheel).

§ 101. *Total Effect.*—The total effect of an overshot water wheel may be put:

$$L = \left(\frac{(c_1 \cos. \mu - v_1)\, v_1}{g} + h_1 + x\, h_2\right) Q\,\gamma - f\,\frac{r}{a}\, Gv,$$

or when the water enters in a direction nearly tangential, and with a velocity equal to that of the wheel:

$$L = (h_1 + x\, h_2)\, Q\,\gamma - f\,\frac{r}{a}\, Gv.$$

Hence for a given weight of wheel, the *total effect*, as well as the mere water power effect, is a maximum when $v = 0$, or when the wheel revolves with the least possible velocity. This condition does not hold good in practice: for the dimensions and weight of water wheels depend on the power they give, and on their velocity, and are so much the greater, the greater the effect or power, and the less the velocity of the wheel.

§ 102. *Useful Effect of Wheel.*—Smeaton, Nordwall, Morin, and others, have experimented on the efficiency of overshot water wheels; but there is still room for further experimental inquiry, especially as to the efficiency of well-constructed large wheels; because the effect of these is not accurately known, and, as the author has had occasion to verify, their efficiency is generally *under estimated.*

Smeaton made experiments with a model wheel of 75 inches circumference having 36 buckets, and found for 20 revolutions per minute a maximum of useful effect of 0,74. D'Aubuisson mentions, in his work on hydraulics, that for a wheel of 11⅓ metres = 32 feet diameter, with a velocity of 8½ feet per second, the efficiency was found to be 0,76. Weisbach has tested the stamp-mill wheels at

Freiberg, generally 7 metres or 22′ — 9″ high, and 3 feet wide, having 48 buckets, and making 12 revolutions per minute, and found an efficiency of 0,78. The *pumping* and *winding* wheels of 35 feet diameter, making 5 revolutions per minute, give an efficiency of 0,80, and often higher. It is also quite ascertained that well-constructed wheels of greater diameter than the above, give 0,83 efficiency, the losses being 3 per cent. for shock at entrance, 9 per cent. by too early discharge, and 5 per cent. for gudgeon friction.

Small wheels always give a *less* efficiency than larger, not only because they make a greater number of revolutions, but because they have a smaller water arc. The most complete experimental inquiry on water wheels is that of M. Morin, "*Expériences sur les Roues hydrauliques à aubes planes, et sur les Roues hydrauliques à augets*, Metz, 1836."* Of these experiments we can here only allude to those made on three small-sized wheels. The first of these was a wooden wheel 3,425 metres = 10,6 feet diameter, with 30 buckets, and giving for a velocity of the circumference of 5 feet per second, an efficiency of 0,65, and the co-efficient $\nu = 0{,}775$. The second wheel was only 2,28 metres in diameter (7,47 feet)—of wood, with 24 curved plate buckets. With a velocity of 5 feet per second, the efficiency of this wheel was = 0,69, and the co-efficient of velocity or of fall $= \nu = 0{,}762$. The third was a wooden wheel for a tilt-hammer, 4 metres diameter (13 feet), with 20 buckets, and 1 metre of *impact* fall to the summit of the wheel. The velocity being 5 feet per second, the efficiency was 0,55 to 0,60, and the velocity being 11½ feet per second, its efficiency was not more than 0,40, which was further reduced to 0,25, when the velocity rose to 4 metres or 13 feet per second, for then the centrifugal force was such that the water could not properly enter the buckets.

Morin deduces from his experiments that for wheels of less than 6′ — 6″ diameter, having a maximum velocity of 6 feet per second at the periphery, and for wheels of more than 6′ — 6″ diameter, having a maximum velocity of 8 feet per second, the co-efficient ν of the *pressure fall* averages 0,78, and, therefore, the *useful effect* of these overshot wheels, exclusive of friction on the gudgeons, is:

$$Pv = \left(\frac{c\,(\cos.\,\mu - v)\,v}{g} + 0{,}78\,h\right) Q\,\gamma,$$

h being the height of the point of entrance above the foot of the wheel, or $0{,}78\,h$ represents the mean height of the arc holding water. This co-efficient $\nu = 0{,}78$ is, however, only to be used when the co-efficient ε representing the extent of filling of the buckets is under ½; it must be made 1,65 when ε amounts to nearly ⅔. In the case of wheels of great diameter ν is certainly higher. For the Freiberg wheels, for example, it is 0,9. Morin further deduces, that when wheels have a greater velocity of revolution than 6′ — 6″ per second, or if the buckets be more than ⅔ filled, a definite co-efficient ν for

[* The author was apparently unacquainted with the experiments made by a committee of the Franklin Institute.—AM. ED.]

the water-arc cannot be given, because, then, small variations or deviations in v and ε have considerable influence on the amount of the useful effect. It must, however, be remarked, that it is not the velocity, but the number of revolutions u (Vol. II. § 98), which determines this limit: for high wheels with 6′ — 6″ velocity at circumference, give a great and tolerably well-ascertained useful effect.

[*American Experiments.*—The most important of the deductions from the experiments on water wheels, made in 1829–30, by the Committee of the Franklin Institute, using a wheel 20 feet in diameter, and with a head and fall varying from 20¾ to 23 feet, may be stated as follows:—

1. "*In running a large overshot wheel to the best advantage*, 84 *per cent. of the power may be calculated upon for the effect.*"

2. "*The velocity of the overshot wheel bears a constant ratio for maximum effects to that of the water entering the buckets, this ratio being at a mean about* ,55 *or* $\frac{9}{16}$*ths.*"*

3. "*Without diminishing the ratio of effect to power more than* 2 *per cent., we may so arrange a high overshot wheel as to increase the velocity of its periphery from* 4½ *to* 6⅜*th, and probably even to* 7½ *feet per second.*"†

As the quantity of work done by a given wheel, when the ratio of effect to power is the same, must evidently depend on the velocity of the wheel, it must be advantageous to run the wheel with the highest velocity within which that ratio can be kept constant, or nearly so, that is, from 6 to 7 feet per second.

The ratio of effect to power with "centre buckets" was found to be .78 of that with "elbow buckets," owing to the water sooner leaving the former than the latter.

When air-vents are used they involve a loss of effect with centre buckets, but scarcely vary the action of elbow buckets.‡

4. With a wheel 15 feet in diameter "84 *per cent. of the power expended may, as before, be relied on for the effect,*" but when the *heads* bore to the *falls*, or heights of wheel, a proportion as great as 1 to 5 or 1 to 4, the ratio of effect to power was reduced as low as ,80 and even ,75.

An overshot wheel of 10 feet in diameter gave with heads of water above the gate varying from ,25 to 3,75 feet, a ratio decreasing from ,82 to ,67 of the power; and with a wheel 6 feet in diameter, the ratios, under like variations of the head of water above the gate, varied from ,83 to ,64. The same general law of a decrease of ratio of *effect* to *power*, according as the proportion of *head* to the

* [Mr. Elwood Morris (*see* Journal Franklin Institute Vol. IV., 3d series, p. 222) ascertained, by direct experiment on three excellent overshot flouring-mill wheels, with all the modern improvements, that, calculating by the whole head and fall, while they ran at their usual pace, and with everything in order, they required "788 *cubic feet of water falling one foot per minute, to grind and dress one bushel of wheat in an hour.*" This is an expenditure of power = 49,250 feet lbs. per minute = 1½ horse power.—Am. Ed.]

† Journ. Frank. Inst., 3d series, Vol. I., pp. 149 and 154.

‡ Ibid., p. 221.

§ Ibid., p. 223.

total head and fall increases, may be traced in the action of different wheels.

Thus the wheel having a diameter of
15 feet, and mean proportion of head to head and fall = ,063
gave ratio of effect to power = ,841
20 feet, and mean proportion of head to head and fall = ,067
gave ratio of effect to power = ,838
6 feet, and mean proportion of head to head and fall = ,072
gave ratio of effect to power = ,801
10 feet, and mean proportion of head to head and fall = ,079
gave ratio of effect to power = ,795.

A still further increase of proportion until the *head* was 45 per cent. of the *head and fall* gave, in the case of the 6 feet wheel, a ratio of effect to power only ,604. Indeed, it is easy to understand that all that head of water above the bottom of the bucket which exceeds what is necessary to give the water the same velocity as that of the wheel, can act only by creating impact, and, therefore, must be considered so much of a head destined to produce, not pressure, but percussion, and the co-efficient of effect of any water delivered under such increased head, must be undershot co-efficient, which experiment proved to be ,285.]

§ 103. *High-Breast Wheel.*—The overshot wheel frequently receives the water lower than the point we have above indicated, at a point somewhat nearer the centre of the wheel. These are called by the French *roues par derrière*, by the Germans *rückenschlägige Rädar*. The lead or water-course, or the pentrough, passes above the wheel in the case we have discussed. For high-breast wheels the pentrough is below the summit of the wheel, or the diameter of the wheel is greater than the total water-fall. In the overshot wheel, the wheel revolves in the direction in which the water is led on to it. In the high-breast wheel, it revolves in the opposite direction. High-breast wheels are erected more especially when the level of the water in the tail-race and pentrough are much subject to variation; because the wheel revolves in the direction in which the water flows from the wheel, and, therefore, *backwater* is less disadvantageous, and because penstocks or sluices can be applied that admit of an *adjustable* point of entrance of the water on the wheel, or of maintaining a given height between the water in the pentrough and in the tail-race; and even for different conditions of the water-course, the same velocity of discharge and of entrance of the water can be maintained. Penstocks, or sluices for these wheels are represented in Figs. 210 and 211, in which the shuttle *AB* is made to slide or fold as an apron, to open more or fewer apertures as required. In Fig. 210 *AB* is made concentric with the circumference of the wheel, in order that the aperture *A* may direct the water into the wheel for all positions of the apron. The motion of the sluice or apron is effected by the pinion and rack *AD* and *C*. In Fig. 211, the water flows over the top *A* of the sluice-board, which is adjusted in a manner similar to that above described. In order, however, that the water may come on to the wheel in a proper direction, a set of stationary *guide*-buckets *EF*, is laid between

Fig. 210.

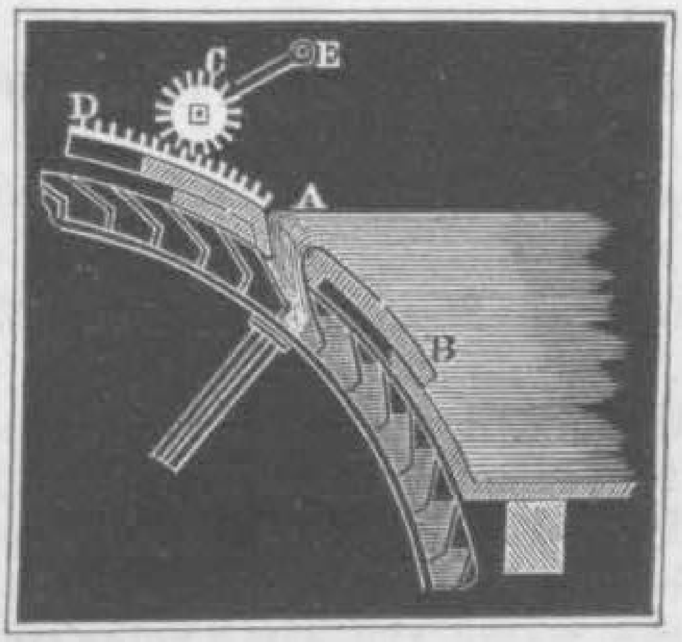
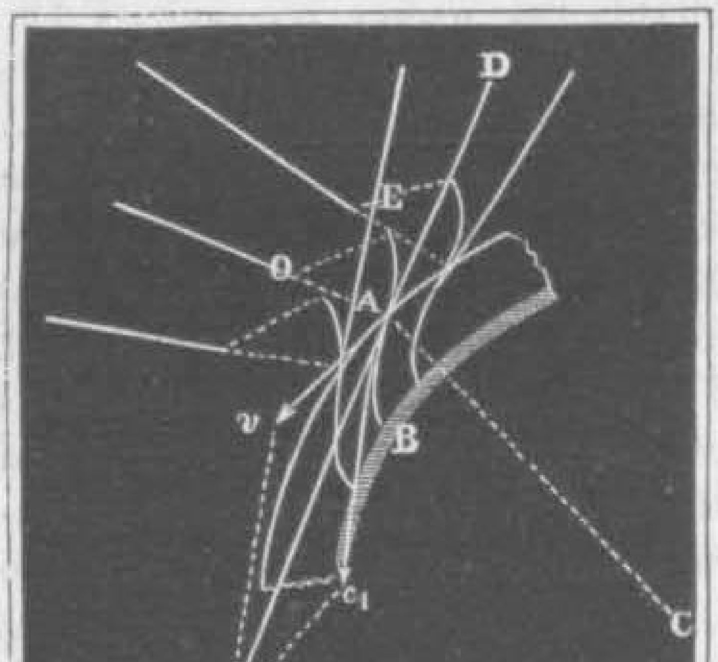


Fig. 211.

the wheel and the sluice-board, and this latter slides over them. The *guide-buckets* must have a certain position, that the water may not strike on the outer end of the wheel-buckets. If Ac_1, Fig. 212, be the direction of the outer element of the wheel-bucket, and if Av, represent in magnitude and direction the velocity of this element A, then, exactly as in Vol. II. § 92, the required direction Ac of the water entering the wheel is obtained by drawing vc parallel to Ac_1, and making Ac equal to c, the velocity of the water entering, as deduced from the height of the water's surface above A. If h be the depth of A below the surface of the pen-trough, then $c = 0,82 \sqrt{2\,gh}$ at least, as in the discharge through short additional tubes (Vol. I. § 323), and when the guide-buckets are rounded off on the upper side, then $c = 0,90 \sqrt{2\,gh}$. If the guide-curves be made straight, then they are to be laid in the direction CAD, but if curved buckets AE are adopted, which has the advantage of *gradually* changing the direction of the water's flow, then they are made tangents to AD at A, by raising AO perpendicular to AD, and describing an arc AE from O as a centre.

Fig. 212.



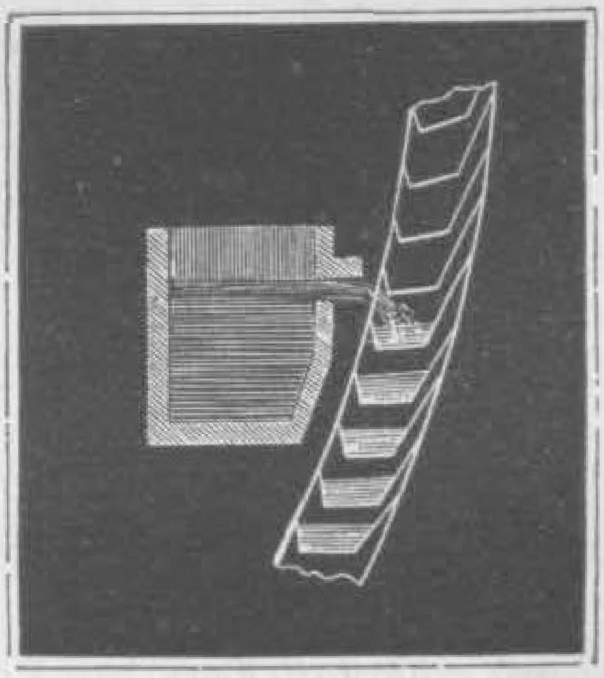
As for different depths, the pressure (h) is different, and the velocity due (c) is also different, the construction must be gone through separately for each guide-bucket. The velocity of entrance is usually 9 to 10 feet, and the velocity of the wheel is $\frac{1}{2}\,c$ to $\frac{5}{8}\,c$ at the most. The construction is to be gone through for the *mean* level of the water in the trough, that the deviation in cases of high and low water may not be excessive.

From this kind of sluice the air escapes less readily than in others, and, therefore, the sluice is made *narrower* than the wheel, or the wheel-buckets are specially *ventilated*, that is, the floor of the wheel is perforated with air-holes. It is not advisable to make the buckets too close, but rather to keep the water in the wheel by an apron, than by making the angle of discharge too great, for in this latter case the guide-buckets extend over too large an arc of the wheel, or become long and contracted, and so occasion loss of fall.

As to the efficiency of high-breast wheels, it is at least equal to that of the ordinary overshot. From the advantageous manner in which the water is laid on to them, it is not unfrequently greater than in overshot wheels having the same general proportions. For a wheel 30 feet high, having 96 buckets, the entrance of the water being at a point 50° from the summit, the velocity at circumference being 5 feet per second, and that of the water's entrance 8 feet, Morin found the efficiency $\eta = 0{,}69$, and the height of the arc holding water $= 0{,}78 \,.\, h$.*

§ 104. *Breast Wheels.*—These wheels are either ordinary bucket wheels, or they are wheels with paddles or floats confined by a stone curb or wooden mantle (Vol. II. § 83). As by a too early discharge of the water from the buckets, the greatest loss of fall or of mechanical effect takes place in the lower half of the wheel, it is evident, that *cæteres paribus*, breast wheels must have a smaller efficiency than overshot or high breast wheels. Upon these grounds the fall must be carefully preserved for breast wheels, or the water kept on the wheel to the lowest possible point. Hence the angle of discharge for their buckets is made great, or the water is even introduced from the inside of the wheel, as shown in Fig. 213, or, as is the better plan, the wheel is enveloped by a mantle or curb, and the buckets or *paddles* are set more or less radially. The curb must not be at more than from ½ an inch to an inch from

Fig. 213.



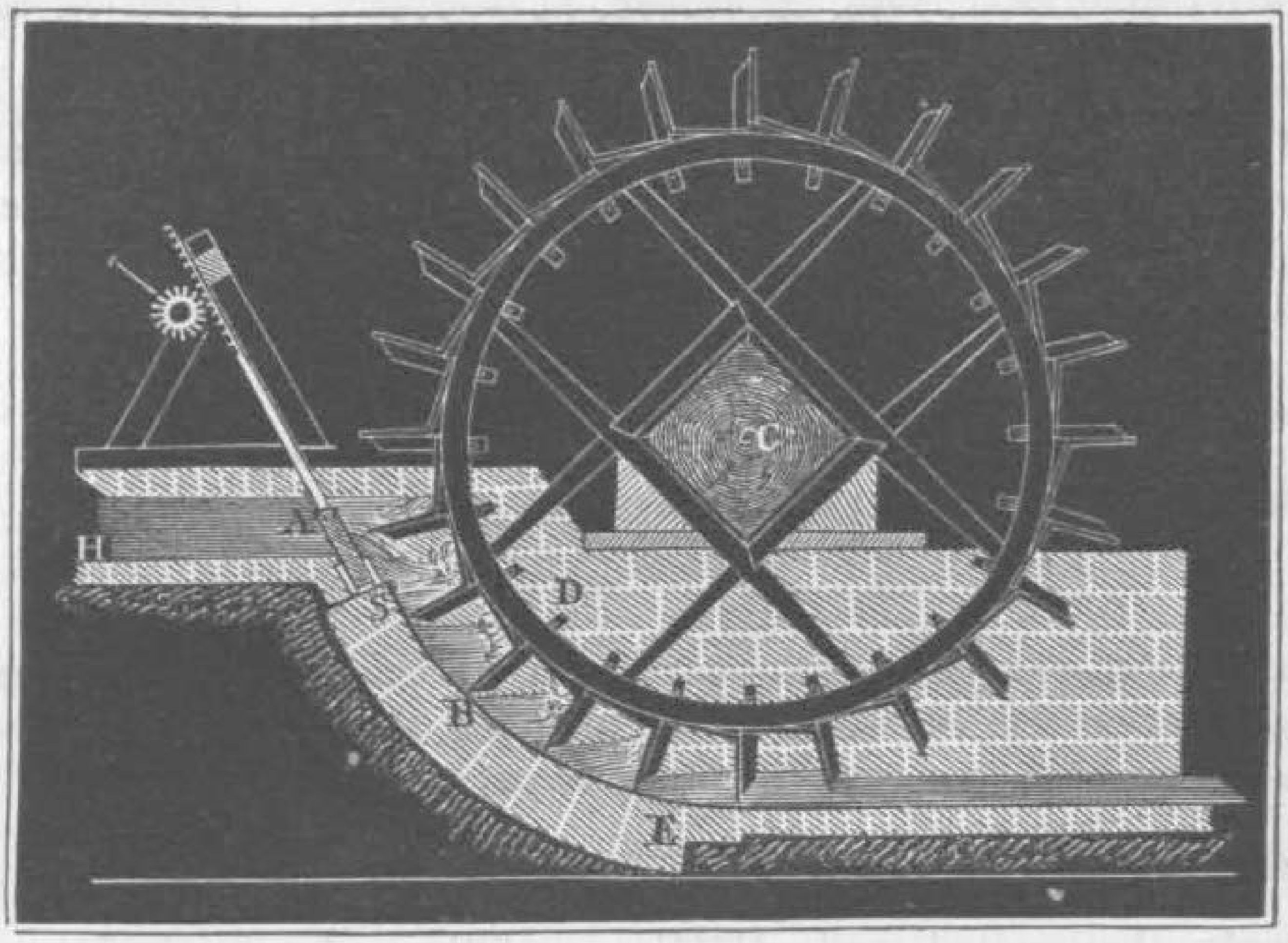
* [With a high breast wheel 20 feet in diameter, water let on 17 feet above the bottom of the wheel, under a head of 9 inches, the Committee of the Franklin Institute found that elbow buckets gave a ratio of effect to power of ,731 at a maximum, and centre buckets ,653. Admitting the water on at a height of 13 feet 8 inches, the elbow buckets gave ,658 and the centre buckets ,628. At 10,96 feet above the bottom of the wheel, the water produced on elbow buckets with a head of 4,29 feet, a ratio of ,544, and centre buckets ,329, with the gate 7 feet above the bottom of the wheel, and a head of 2 feet of water, this" low-breast" wheel gave a ratio of ,62 for elbow buckets and ,531 for centre buckets. At a height of 3 feet 8 inches above the bottom of the wheel, and one foot head above the bottom of the gate, elbow buckets gave ,555, centre buckets ,533.—AM. ED.]

the wheel, that as little water as possible may escape through the intermediate space. As the buckets or floats of wheels inclosed by a curb are not intended for holding water, they may be placed radially, but, in order that they may not *throw up* water from the tail-race, it is advisable to give the outer element of the float or bucket such a position that it may leave the water vertically. As to the number of floats, it is advantageous to have them numerous, not only because by this means the loss of water by the *play* left between the wheel and the mantle or curb is smaller, but, also, because, by putting them close together, the *impact*-fall is rendered less, and the *pressure*-fall increased. The distance between two floats is generally made equal to the depth of the shrouding d, or it is taken at from 10 to 15 inches, or one of the rules above given (Vol. II. § 88) may be applied for fixing the number of buckets. It is, however, essential that breast wheels be well ventilated, so that the air can escape outwards, because in them the stream of water laid on is nearly as deep as the whole distance between two buckets. Air-holes or slits must, therefore, be left in the floor to prevent the air from interfering with the water's entrance. This is so much the more necessary in these wheels, as they are usually arranged to be filled to $\frac{1}{2}$ at least, even $\frac{2}{3}$ of their capacity. Breast wheels are generally adopted for falls varying from 5 to 15 feet, and for supplies of from 5 to 80 cubic feet per second.

Remark. Theoretical and experimental researches on the subject of breast and undershot wheels, with the water laid on from the inside, have been made in Sweden, and are given in detail in a work entitled "*Hydrauliska Försök*, etc., of Lagerbjelm, of Forselles och Kallstenius, Andra Delen, Stockholm, 1822." Egen describes a wheel of this kind in his "*Untersuchungen über den Effect einiger Wasserwerke*, &c., Berlin, 1831." The efficiency of the wheel was not more than 59 per cent., although the fall was 13,42 feet. A wheel on the same model was erected in France, but only 6′—6″ in diameter. M. Mallet experimented on this wheel (see "*Bulletin de la Société d'Encouragement*, No. 282)," and found its efficiency 60 per cent. It would appear, therefore, as Egen justly observes, that these wheels are seldom to be adopted. They can have only a limited width, and cannot be so substantial as those receiving the water on the outside.

§ 105. *Overfall Sluices.*—The mode of *laying on* the water to breast wheels is very various. The overfall sluice, the guide-bucket sluice, and the ordinary penstock are in use. The water is seldom allowed to go on quite undirected. In the *overfall* sluices, shown in Figs. 214 and 215, the water flows over the *saddle-beam*, or *lip A* of the sluice; but that it may flow in the required direction, the lip is rounded, or a rounded guide-bucket AB, Fig. 215, is appended to it. This guide-bucket AB, Fig. 216, is curved in the form of the parabola, in which the elements of water lying deepest move; for if it were more curved, the water-layer would not lie to it, and if it were less curved, the width of the guide, and, therefore, the friction, would be unnecessarily increased, and the water would reach the wheel less in the direction of a tangent. According to the theory of discharge over weirs (Vol. I. § 317), if e_1 be the breadth of the weir, h_1 the height above the sill or lip HA, Fig. 216, and μ the coefficient of discharge, then $Q = \frac{2}{3} \mu\, e_1 h_1 \sqrt{2gh_1}$; but if the quantity

Fig. 214.



laid on Q, and breadth of the aperture e_1 (which is only 3 or 4

Fig. 215.

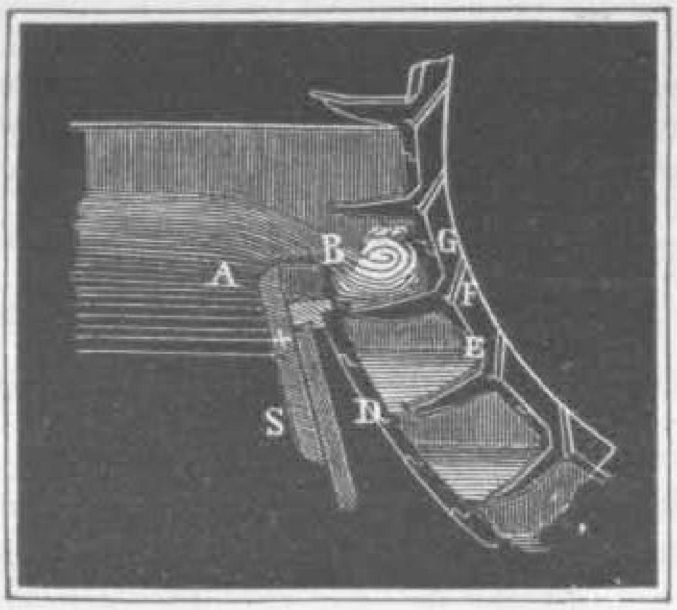
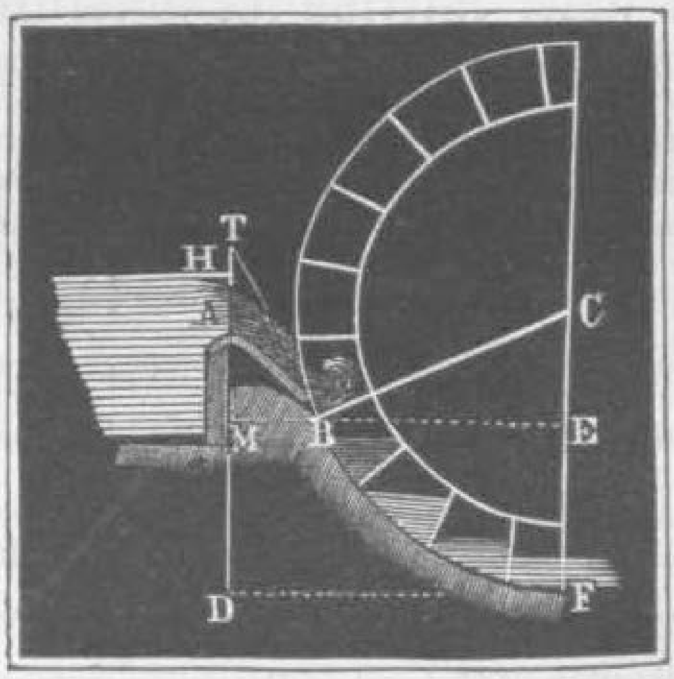


Fig. 216.



inches less than the width of the wheel e) be given, then the head for the discharge:

$$h_1 = \left(\frac{\frac{3}{2}\,Q}{\mu\, e_1 \sqrt{2g}}\right)^{\frac{2}{3}} = ,3093 \left(\frac{Q}{\mu\, e_1}\right)^{\frac{2}{3}}.$$

Again, the velocity c of the water entering the wheel at B is determined by its proportion $x = \frac{c}{v}$ to the velocity of the wheel, and, hence, the fall necessary for communicating this velocity:

$$HM = h_2 = \frac{c^2}{2g} = \frac{(x\, v)^2}{2g}, \text{ or,}$$

on account of absorption of fall by discharge, $h_2 = 1,1 \,.\, \frac{(x\, v)^2}{2g}$: x is generally made $= 2$, and, therefore, $h_2 = 4,4 \frac{v^2}{2g}$. From h_1 and h_2 we deduce the height AM of the lip of the guide-curve, $k = h_2 - h_1$, and if the total fall $HD = h$, there remains for the head available as weight on the wheel $MD = EF = h_3 = h - h_2$. Again, we have from the theory of projectiles, the angle of inclination TBM of the guide-curve's end to the horizon determined by the formula

$$k = \frac{c^2 \sin. \alpha^2}{2g}, \text{ therefore, } \sin. \alpha = \sqrt{\frac{k}{h_2}} = \sqrt{\frac{h_2 - h_1}{h_2}},$$

and the length or projection MB of the guide-curve is:

$$MB = l = \frac{c^2 \sin. 2\alpha}{2g} = h_2 \sin. 2\alpha.$$

Lastly, if the very desirable condition of bringing the water *tangentially* on to the wheel is to be fulfilled, the radius of the wheel $CB = CF = a$ is determined by the equation:

$$a(1 - \cos. \alpha = h - h_3, \text{ therefore, } a = \frac{h - h_2}{1 - \cos. \alpha}.$$

Inversely, the central angle $BCF = \alpha_1$ of the water arc is determined by $\cos. \alpha_1 = 1 - \frac{k - h_2}{a}$, and when the latter condition is not fulfilled, or α_1 is not made $= \alpha$, then the deviation of the direction of the water entering the wheel from the direction of the motion of the bucket on which it impinges: $\delta = \alpha_1 - \alpha$.

Example. Suppose for a breast wheel the quantity of water laid on by an overfall sluice, $Q = 6$ cubic feet, the total fall $h = 8$ feet, and the velocity of the periphery $= 5$ feet, also the ratio of the buckets filling $= \frac{2}{5}$, then for a depth of wheel $= 1$ foot, the requisite width on the breast $e = \frac{5}{2} \cdot \frac{Q}{dn} = \frac{5 \cdot 6}{2 \cdot 1 \cdot 5} = 3$ feet. And if the breadth of the aperture be made $2\frac{3}{4}$ feet, and $\mu = 0,6$, then the height at which the water stands $h_1 = 0,3093 \left(\frac{6}{0,6 \cdot \frac{11}{4}}\right)^{\frac{2}{3}} = 0,76$ feet. If we take $x = \frac{8}{5}$, then the fall necessary to generate the velocity c with which the water enters the wheel:

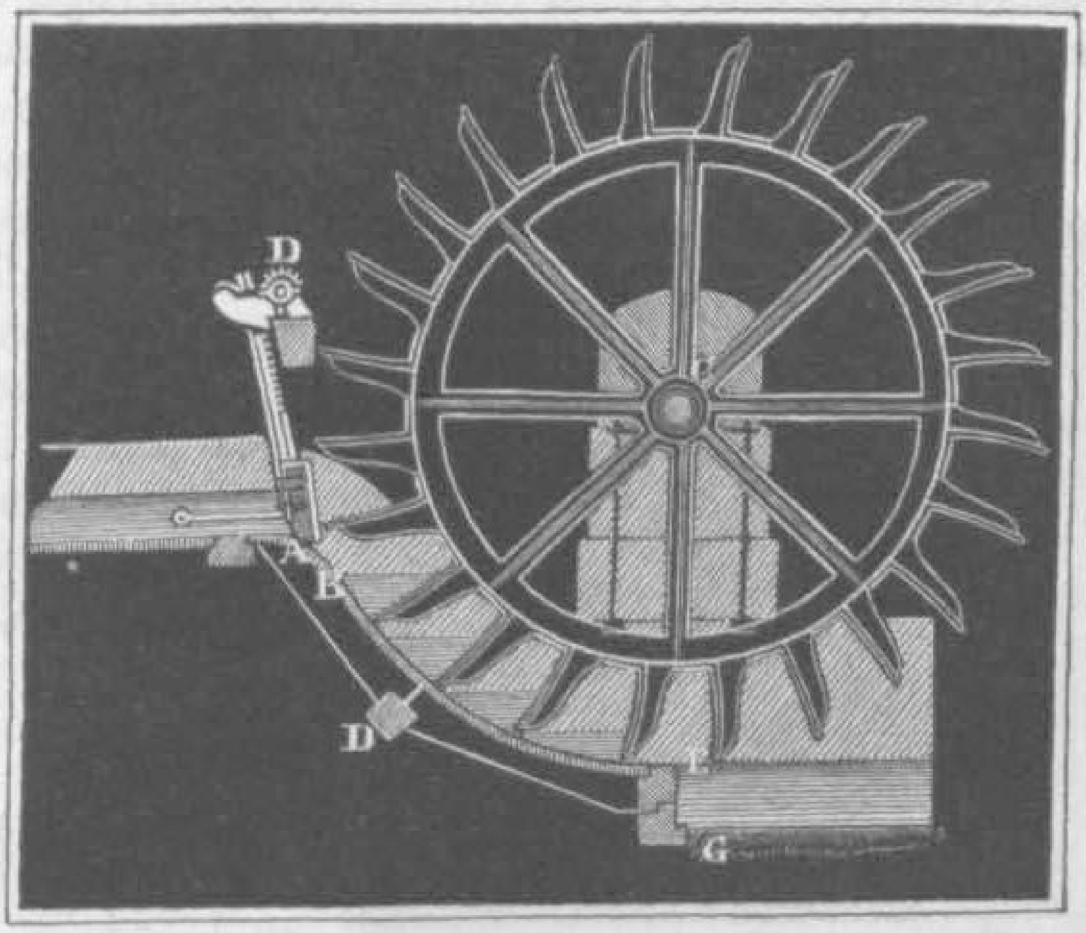
$c = \frac{8}{5} \cdot 5 = 8$ feet, $h_2 = 1,1 \cdot 0,0155 \cdot 8^2 = 1,1$ feet nearly, and, therefore, the height of the lip of the guide $k = 1,1 - 0,76 = 0,34$ feet $= 4,08$ inches. Again, for the angle of inclination of the end of the guide-curve: $\sin. \alpha = \sqrt{\frac{0,34}{1,1}} = 0,5539$, and hence $\alpha = 33°$, $38'$, and the *breadth* of the lip of the guide-curve $l = 1,1 \sin. 67°, 16' = 1$ foot nearly. That the water may enter the wheel tangentially, the radius of the wheel would have to be

$$a = \frac{h - h_2}{1 - \cos. \alpha} = \frac{8 - 1,1}{1 - \cos. 33°, 38'} = 41,06 \text{ feet};$$

but if we limit the size of the wheel to 25 feet diameter, or make $a = 12,5$, then the central angle α_1 of the water-arc is $\cos. \alpha_1 = 1 - \frac{8 - 1,1}{12,5} = 0,45$, or $\alpha_1 = 63°, 16'$, and the deviation of the direction of motion of the water from that of the wheel at the point of entrance: $\alpha_1 - \alpha = 63°, 16' - 33°, 38' = 29°, 38'$.

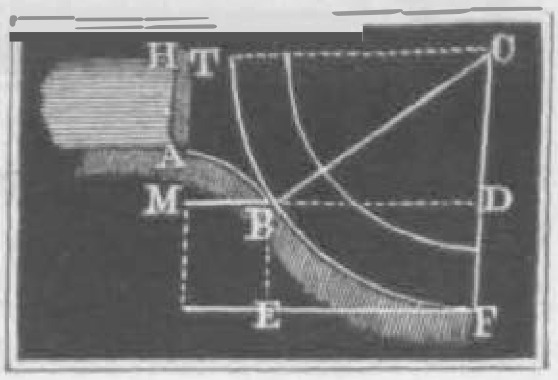
§ 106. *Penstock Sluices.*—Fig. 217 shows the manner of laying on the water on a breast wheel by the ordinary penstock. The sluice-board, which is placed as close to the wheel as possible, is made of such thickness and form at the lower edge, as obviates contraction. For the same reason the end AB of the bottom of the

Fig. 217.



pentrough or lead, is formed with a parabolic lip. The height $BE = DF = h_2$, Fig. 218, of the end of the curve depends on the total fall h, and on the velocity height

Fig. 218.



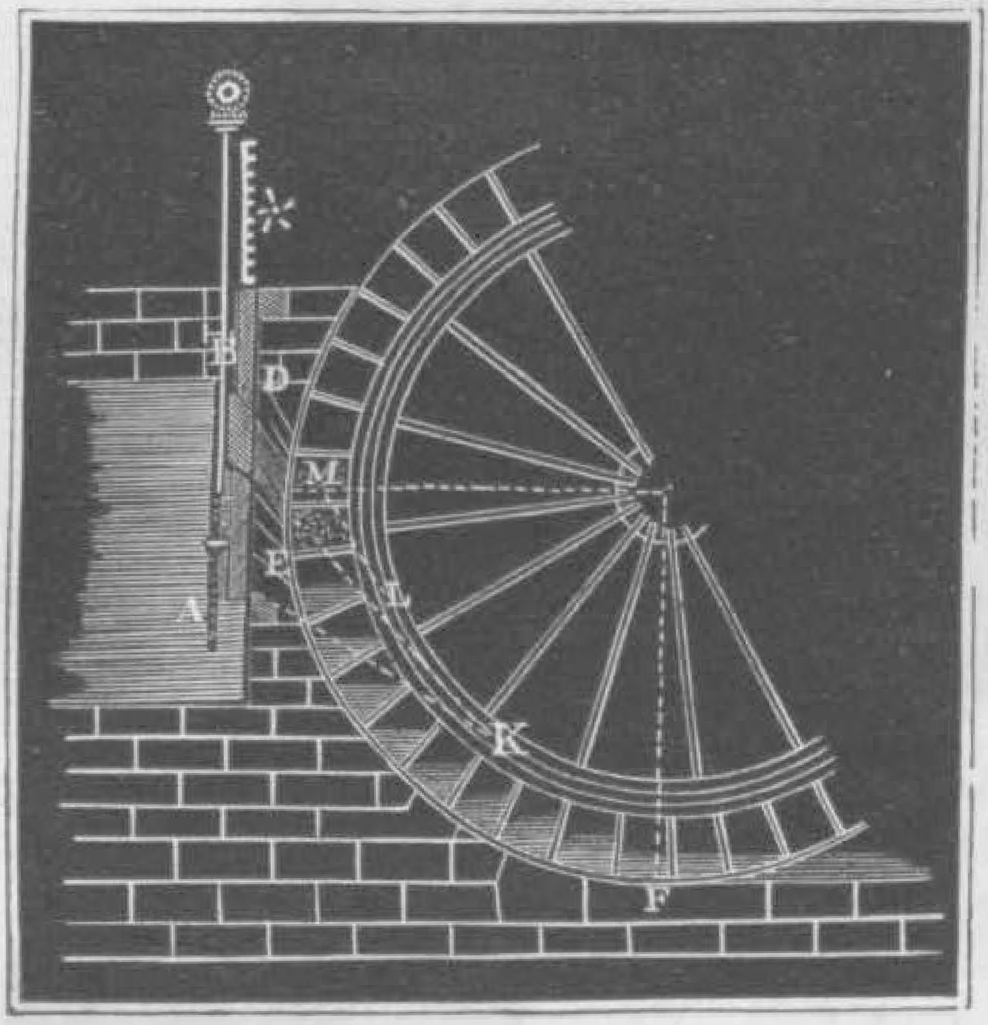
$$h_1 = 1{,}1 \cdot \frac{c^2}{2g} = 1{,}1 \cdot \frac{x^2 v^2}{2g},$$

and is determined by the formula $h_2 = h - h_1$: hence, as $\cos. \alpha = \frac{CD}{CB} = \frac{a - h_2}{a}$, the corresponding central angle $BCF = \alpha = 1 - \frac{h - h_1}{a}$. If, therefore, the water is to be taken on tangentially, the inclination TBM of the layer of water to the horizon is to be put $= \alpha$, and, hence, we determine the co-ordinates $MA = k$ and $BM = l$ of the apex of the parabola A by the formulas $k = \frac{c^2 \sin.\ \alpha^2}{2g}$, and $l = \frac{c^2 \sin.\ 2\alpha}{2g}$.

But it is not necessary to set the aperture exactly in the apex of the parabola, it may be placed in any other point of the parabolic arc, *provided that the axis of the aperture is tangential to the parabola* (Vol. II. § 93).

A third method of laying on the water, consists in a penstock with guide-curves, or buckets, Fig. 219. This arrangement is particularly applicable when the water in the pentrough is subject to great variation of level. The apparatus shown in Fig. 219 consists of two sluice-boards A and B, each of which can be separately adjusted, and thus, not only the *head*, but also the *orifice* of discharge, is regulated. It is not possible to lay the water on to the wheel tangentially by means of the guide-buckets DE; but we can approximate to within 20 to 30 degrees of this. The water flows out between the guides, according to the law for the *discharge through short additional tubes*, and, therefore, the co-efficient μ may be taken as ,82, or when the bottom of the sluice-board is accurately rounded on the inside $\mu = 0,90$. Hence the co-efficient of resistance is greater in this case than in overfall sluices, or in the ordinary penstock. Assuming $\mu = 0,85$, the height necessary for producing the velocity c is:

Fig. 219.



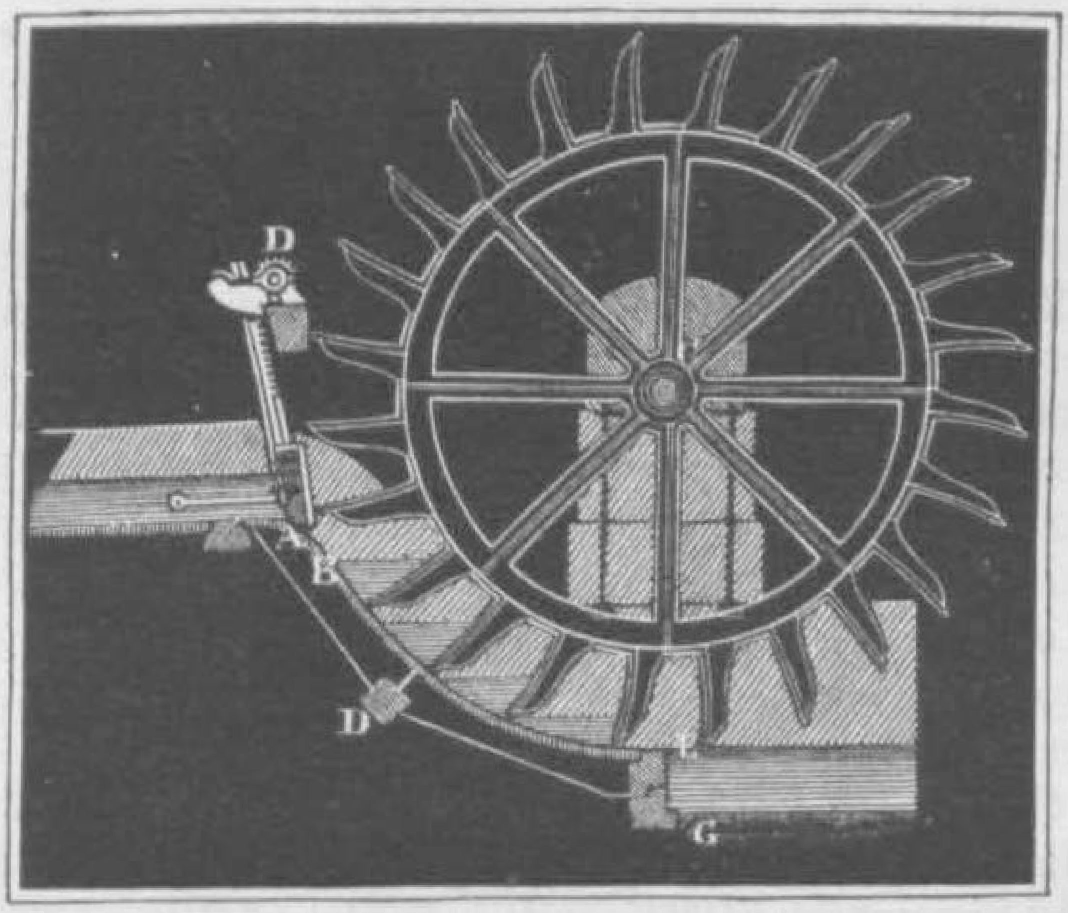
$$h_1 = \left(\frac{1}{0,85}\right)^2 \cdot \frac{c^2}{2g}$$

$= 1,384 \frac{c^2}{2g}$, and hence the portion of the total height h remaining in water-arc is: $h_2 = h - h_1 = h - 1,384 \frac{x^2 v^2}{2g}$. In the case of a variable supply of water, the arrangements are adapted to the average supply, by laying the outer end M of the centre guide-curve at the height h_2 above the foot of the wheel F. In order to place all the guides at the same angle of circumference as the wheel, the normal position of which is 3 inches from the guides, they are drawn tangential to a circle KL, concentric with the wheel, the position of which is determined by the direction EK of the first guide-curve.

§ 107. *Construction of the Curb or Mantle, and of the Wheel.*—The mantle or curb by which breast wheels are inclosed, in order to retain the water in them as long as possible, is formed either of masonry or of wood, and sometimes of iron. The object of the curb is the better fulfilled the less the play between the outer edge of the buckets and the cylindrical surface of the curb, as the water escapes by whatever free space is left. The play amounts to ½ an inch in the best constructed wheels; but it is not unfrequently as much as an inch, or even 2 inches. When the wheels are of wood and the curb likewise, a play of ½ an inch is an insufficient allowance, because the curb is apt to loose its symmetry, and then friction between the buckets and the curb might ensue. For iron wheels and stone curbs, the chances of deformation are small, and, therefore, a very small amount of play is sufficient. When the wheels fit too closely in their mantles, stray pieces of wood or ice floated on to the wheel may have very injurious consequences. It is, therefore, necessary to have a screen in front of the sluices to keep back all stray matters.

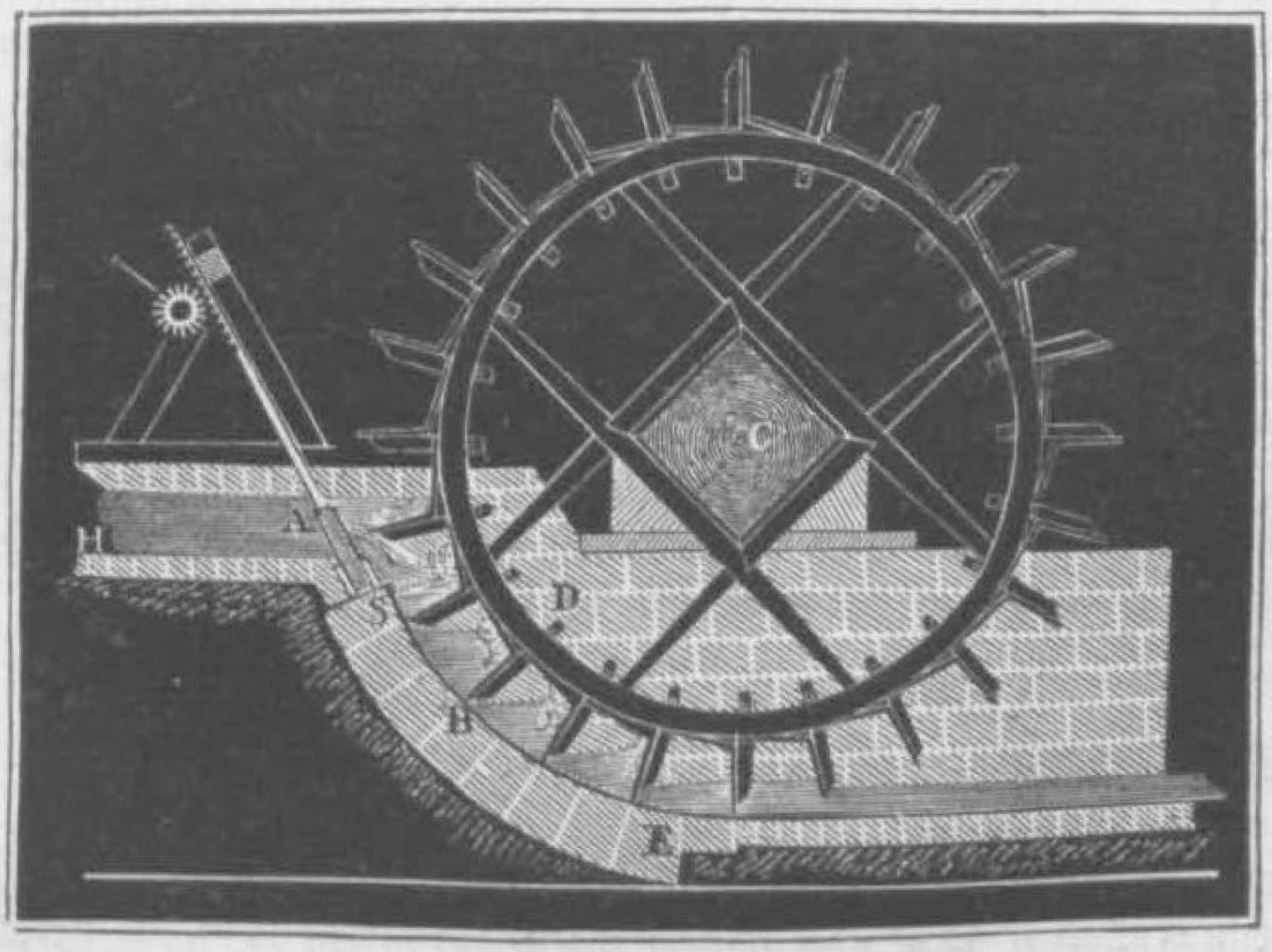
Stone curbs are constructed of properly selected and carefully dressed stone, or of brick, either being laid in good hydraulic mortar or cement. Wooden curbs *AE*, Fig. 220, are composed of beams

Fig. 220.



A, *D*, *E*, of larch or oak, carefully planked with deals curved as required. The bed of the curb is inclosed by side walls, so that lateral escape of water is prevented. If the water can flow off by the tail-race with the velocity with which it is delivered from the

Fig. 221.



wheel, then the curb may be finished flush with the bottom of the race, as at E, Fig. 221; but if the water flows with a less velocity, then the race should be cut out deeper, as at AE, Fig. 220, so as to avoid all risk of back water.

The construction of these wheels differs essentially from that of overshot wheels, in respect of the buckets of the latter forming cells, whilst in the former these are mere paddles or floats; and this gives rise to a different mode of connecting the buckets with the rim of the wheel. The Germans distinguish *Staberäder* and *Strauberäder*. The former have *shroudings* like the overshot wheels, to support the floats; in the latter, the floats or buckets are chiefly supported by short arms or cantilevers, which project radially from the circumference of the wheel. Fig. 219 is a wheel with shrouding, Figs. 220 and 221 are *Strauberäder*. Fig. 221 is a wooden wheel, and Fig. 220 an iron wheel. Very narrow float wheels have only a single narrow ring by which the floats are attached on to cantilevers. In wooden wheels the supports for the floats are passed between the two sides of a compound ring forming the shrouding. In iron wheels, on the other hand, they are either cast in one piece with the segments of the wheel, or they are bolted on to these. The buckets or paddles are usually of wood, and are nailed or screwed to the above-described supports. The floor of the wheel is placed on the *outside* of the rings or shrouding, and does not close the wheel completely, slits being left for the escape of the air, as is represented in Fig. 215, in which DE is the wheel-paddle, composed of two pieces, EF a piece of the bottom of the wheel, and G the air-slit or ventilator.

§ 108. *The Mechanical Effect of Curb Wheels.*—The mechanical effect produced by wheels hung in a curb or mantle consists, as in overshot wheels, of the mechanical effect produced by the impact, and that by the pressure or weight of water. The formula representing the efficiency of each is the same, save that the determination of the loss of water requires a different calculation, inasmuch as in the one case the water is lost by a gradual emptying of the buckets in the arc of discharge, and, in the case now in question, the water escapes through the space necessarily left between the wheel and the curb. We have, therefore, to determine how, and in what quantity, the water escapes through this space, and hence deduce the loss of effect to the wheel. If we put, as for overshot wheels, the velocity with which the water goes on to the wheel at the division line $b = c_1$, the velocity of the wheel in this line or circle $b = v_1$, and the angle $c_1 E v_1$, Fig. 222, between the directions of these velocities $= \mu$, then the effect of impact:

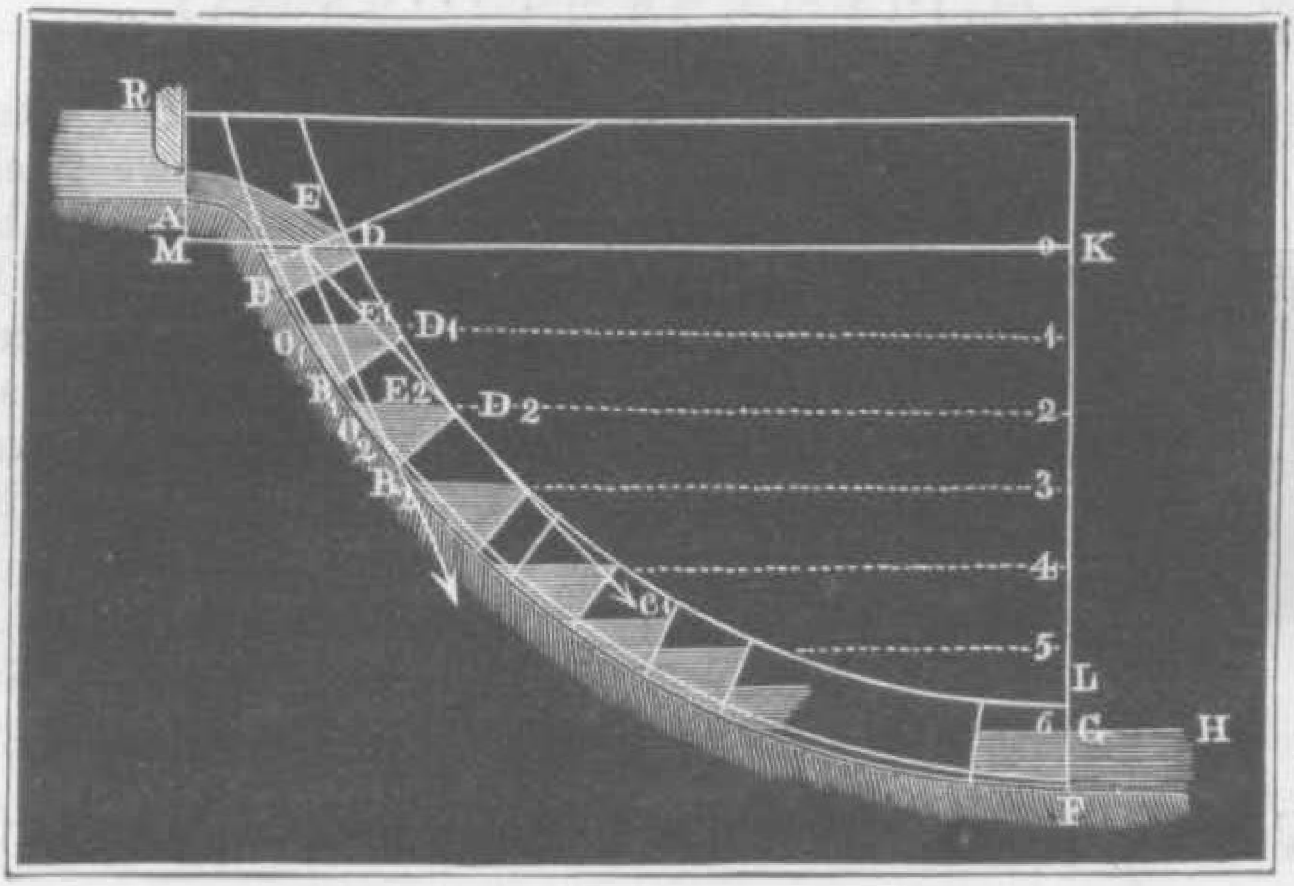
$$= \frac{(c_1 \cos. \mu - v_1) v_1}{g} . Q \gamma.$$

If, further, GK, the difference of level between the point of entrance E, and the surface of the water in the tail-race, $GH = h_1$, we have (neglecting loss of water through free space) the effect of the weight of the water $= h_1 Q \gamma$, and hence the total effect is, as before:

$$L = Pv = \left(\frac{(c_1 \cos.\mu_1 - v_1)\, v_1}{g} + h_1\right) Q\gamma.$$

From the theoretical effect given by the formula, we have to deduct the loss by the escape of water. This loss affects chiefly the effect

Fig. 222.



of the water's weight, as the water escapes uninterruptedly, as any given bucket or float BD descends successively to the position B_1D_1, B_2D_2, &c., to the lowest point FL. The free space forms an orifice of discharge through which the water escapes under a variable head BE, B_1E_1, B_2E_2. If we put e = the width of the wheel, and the breadth of the free space = s, the area of the orifice of discharge = se, and if we put for the *head* BE, B_1E_1, &c., z, z_1, z_2, &c., and ϕ for the co-efficient of discharge, then the quantity escaping in any instant of time t, through the free space:

$$Q_1 = \phi e s t \sqrt{2g\, z_1} + \phi e s t \sqrt{2g\, z_2} + \phi e s t \sqrt{2g\, z_3} + \ldots$$
$$= \phi e s t \sqrt{2g}\,(\sqrt{z_1} + \sqrt{z_2} + \sqrt{z_3} + \ldots)$$

If n be the number of positions of a float assumed, the mean escape from each cell:

$$= \phi e s t \sqrt{2g}\left(\frac{\sqrt{z_1} + \sqrt{z_2} + \ldots + \sqrt{z_n}}{n}\right),$$

or for one second:

$$= \phi e s \sqrt{2g}\left(\frac{\sqrt{z_1} + \sqrt{z_2} + \ldots \sqrt{z_n}}{n}\right).$$

But the whole of the cells, or the water-arc, corresponds to the fall h_1, and, hence, the loss of mechanical effect may be put:

$$L_1 = \phi e s \sqrt{2g}\left(\frac{\sqrt{z_1} + \sqrt{z_2} + \ldots + \sqrt{z_n}}{n}\right) h_1 \gamma.$$

There is a slight loss of water on each side of the wheel, as there must be a free space of 1 to 2 inches here. If we put arcs BO,

$B_1 O_1$, &c., of the curb covered with water, equal to l_1, l_2, &c., then the water escaping by the sides, as it were through a series of notches or small weirs:

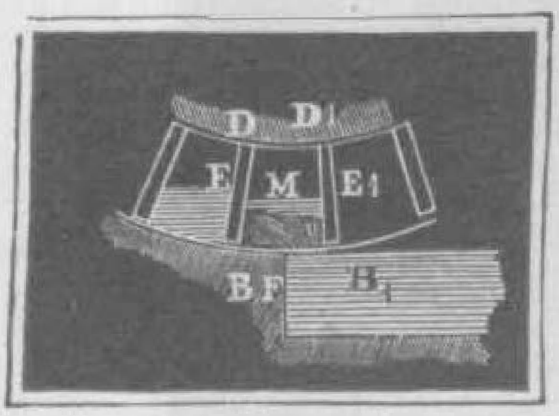
$$= 2 \cdot \tfrac{2}{3} \phi s l_1 \sqrt{2g z_1},\ 2 \cdot \tfrac{2}{3} \phi s l_2 \sqrt{2g z_2},\ \text{\&c.},$$

and, therefore, the corresponding loss of mechanical effectw

$$L_2 = \tfrac{4}{3} \phi s \sqrt{2g} \left(\frac{l_1 \sqrt{z_1} + l_2 \sqrt{z_2} + \ldots l_2 \sqrt{z_n}}{n} \right) h_1 \gamma.$$

§ 109. *Losses.*—There is a still further loss of effect, when the surface of the water in the lowest cell does not correspond with the surface of the tail-race, as represented in Fig. 223. For in this case the water flows from the cell $BD\,D_1B_1$, as soon as the float BD passes the end of the curb F, and acquires a velocity due to the height $FM = h_2$, in addition to the velocity v of the wheel. This height h_2 is variable, but its mean value is evidently $\frac{1}{2}\, h_2$, and, therefore, the head to which the velocity of the water flowing from the wheel is due is not $\frac{v^2}{2g}$, but $\frac{v^2}{2g} + \frac{1}{2}\, h_2$. We have already deducted the loss due to the height $\frac{v^2}{2g}$ in estimating for impact, and we have, therefore, only $\frac{1}{2}\, h_2\, Q\, \gamma$ to deduct from the effect found. From this we see, that a sudden fall from the end of the circle should be adopted only in cases where back-water is to be feared.

Fig. 223.



There are still other sources of loss of effect in breast wheels, such as the friction of the water on the curb, and the resistance of the air to the motion of the floats; but these are comparatively of slight importance.

§ 110. *Formula for Total Effect.*—We shall now give the formula for the total effect of breast wheels, leaving out of consideration the loss of effect by escape of water at the sides, as also the loss from friction of the water and resistance of the air; but allowing for the escape through the free space between the wheel and curb, and for the friction of the gudgeons. The formula will then stand thus:

$$L = Pv = \frac{(c_1 \cos.\, \mu - v_1)\, v_1}{g}\, Q \gamma + h_1 Q \gamma - \phi e s w h_1 \gamma - f G \frac{r}{a} v,$$

in which w is substituted for the mean velocity of discharge:

$$\sqrt{2g} \left(\frac{\sqrt{z_1} + \sqrt{z_2} + \ldots + \sqrt{z_n}}{n} \right),$$

and r for the radius of the gudgeons. Hence:

$$L = \left[\frac{(c_1 \cos.\, \mu_1 - v_1)\, v_1}{g} + h_1 \left(1 - \frac{\phi e s w}{Q} \right) \right] Q \gamma - \frac{r}{a} f G v.$$

or, introducing ε the co-efficient of the bucket's filling, generally $= \frac{1}{2}$, and $\frac{e}{Q} = \frac{1}{\varepsilon d v}$, we have:

$$L = \left[\frac{(c_1\, cos.\, \mu_1 - v_1)\, v_1}{g} + h_1 \left(1 - \frac{\phi\, s\, w}{\epsilon\, d\, v_1}\right)\right] Q\gamma - f\, G\, \frac{r}{a}\, v.$$

If we put the total fall, measured from the surface of the water in the pentrough to the surface of the tail-race $= h$, then, instead of h_1, we may introduce $h - 1{,}1 \,.\, \frac{c_1^2}{2g}$, and then:

$$L = \left[\frac{(c_1\, cos.\, \mu - v_1)\, v_1}{g} + \left(h - 1{,}1 \,.\, \frac{c_1^2}{2g}\right)\left(1 - \frac{\phi s w}{\epsilon d v_1}\right)\right] Q\gamma - f G \frac{r}{a}\, v.$$

In order to find the value of the velocity of entrance c_1, for which the effect is a maximum, we have only to consider when the expression

$$\frac{c_1\, v_1\, cos.\, \mu}{g} - 1{,}1 \,.\, \frac{c_1^2}{2g}\left(1 - \frac{\phi\, s\, w}{\epsilon\, d\, v_1}\right)$$

is a maximum. Putting $\frac{2\, v_1\, cos.\, \mu}{1{,}1\left(1 - \frac{\phi\, s\, w}{\epsilon\, d\, v_1}\right)} = k$, then the expression to be made a maximum becomes $kc_1 - c_1^2$. But we know from Vol. I. § 386, that this becomes a maximum for $c_1 = \frac{k}{2}$, and, therefore, it is evident that the effect will be a maximum when the velocity of entrance of the water $c_1 = \frac{v_1\, cos.\, \mu}{1{,}1\left(1 - \frac{\phi\, s\, w}{\epsilon\, d\, v_1}\right)}$.

If, from the necessarily small value of μ, we put $cos.\, \mu = 1$, and assume also, that there is no loss in the discharge from the sluice, then $c_1 = \frac{v_1}{1 - \frac{\phi\, s\, w}{\epsilon\, d\, v_1}}$; and hence we perceive that the velocity of entrance must be made greater than the velocity at the circumference of the wheel, and this so much the more as the free space w is greater. The loss by escape of water is not, as an average, more than 10 to 15 per cent., or $\frac{\phi\, s\, w}{\epsilon\, d\, v_1} = \frac{1}{10}$ to $\frac{3}{20}$, and, therefore, the velocity of entrance of the water $c_1 = \frac{10}{9}\, v_1$ to $\frac{20}{17}\, v_1$. In practice, however, c_1 is made $= 2\, v_1$, or the wheel revolves with *half the velocity the water has acquired at entering the wheel*, because in this way the loss is not much, and the water's entrance is more easily regulated.

If we introduce $c_1\, cos.\, \mu = 2\, v_1$, or $v_1 = \frac{1}{2}\, c_1\, cos.\, \mu$, into the equation above found, we get:

$$L = \left[\frac{v_1^2}{g} + \left(h - 1{,}1 \,.\, \frac{4\, v_1^2}{2g\, cos.\, \mu^2}\right)\left(1 - \frac{\phi\, s\, w}{\epsilon\, d\, v_1}\right)\right] Q\gamma - f \frac{r}{a}\, G\, v,$$

$$= \left[-\left(\frac{2{,}2}{cos.\, \mu^2} - 1\right)\frac{v^2}{g} + h\left(1 - \frac{\phi\, s\, w}{\epsilon\, d\, v}\right)\right] Q\gamma - f \frac{r}{a}\, G\, v.$$

From the factor $\left(1 - \frac{\phi\, s\, w}{\epsilon\, d\, v}\right)$, we learn that the maximum effect does not take place here when $v = 0$; for even when $v = \frac{\phi\, s\, w}{\varepsilon\, d}$ the whole effect of the water's weight is lost by the water escaping through the free space.

§ 111. *Efficiency of Breast Wheels.*—Morin has made a number of experiments on breast wheels of good construction. He has compared his experimental results with those of the theoretical formula:

$$Pv = \left(\frac{(c\, cos.\, \mu - v)\, v}{g} + h_1\right) Q\gamma,$$

and has found a tolerable agreement between the two to subsist, when the formula is corrected by a co-efficient κ, or if we put:

$$Pv = \kappa\left(\frac{(c\, cos.\, \mu - v)\, v}{g} + h_1\right) Q\gamma.$$

One wheel on which Morin experimented was of cast iron, with wooden buckets placed obliquely to the sluice, and turning in a close-fitting iron curb. The wheel was 21 feet 4 inches in diameter, and 4′ — 10″ wide; the fall was 5′ — 6″, there were 50 floats, and the velocity of revolution was from 3′ — 4″ to nearly 8 feet per second, the velocity of the water from a well-constructed sluice being from 9 feet 2″ to 10 feet 6″. The co-efficient κ was found to be about 0,75, and the efficiency, including the friction of the axles, nearly 0,60. A second iron wheel, experimented upon by M. Morin, was hung in a well-fitting sandstone curb. It was 13 feet diameter, and 13 feet wide. There were 32 floats, the fall being 6 feet 6″. So long as the speed of the wheel did not differ more than from 47 to 100 per cent. from that of the water entering it, that is within speeds varying from 1′ — 8″ to 6 feet, the co-efficient κ remained nearly constant, viz.: = 0,788, and the efficiency of the wheel was = 0,70. A third wheel was almost entirely of wood, and hung in a close-fitting curb. Its height was 20 feet, and it had 40 floats. Worked with a common sluice, the co-efficient κ = 0,792, and with an *overfall sluice* this rose to 0,809. The efficiency, however, was in the first case only 0,54, and in the second 0,67. If from these results we adopt a mean value, we get for breast wheels with penstock sluicew

$$L = 0{,}77\left(\frac{(c\, cos.\, \mu - v)\, v}{g} + h_1\right) Q\gamma,$$

and for those with overfall sluice:

$$L = 0{,}80\left(\frac{(c\, cos.\, \mu - v)\, v}{g} + h_1\right) Q\gamma,$$

from which, however, the mechanical effect consumed by the friction of the axles has to be deducted. The greater efficiency of the overfall-sluiced wheels arises from the water entering more slowly than in the case of the penstock, and, hence, there is no loss by impact.

It follows, besides, from Morin's experiments, that the efficiency diminishes if the water fills more than from $\frac{1}{2}$ to $\frac{2}{3}$ of the space between the floats, and that the efficiency does not vary much for variations of the angular velocity of the wheel from $1' - 8''$ to $6' - 6''$ per second.

Egen made experiments on a breast wheel 23 feet in diameter, and $4\frac{1}{2}$ feet wide. There were two peculiarities in this wheel. The 69 well-ventilated buckets, were constructed exactly as in overshot wheels; and the sluice was in two divisions, of which, according to the state of supply of water, the upper or under one could be drawn. Although the mantle fitted very closely, the efficiency of this wheel was, at best, only 0,52, and as an average, with 6 cubic feet water per second, and 4 revolutions per minute, the efficiency was only 0,48.

Experiments with a breast wheel are described in the "*Bulletin de la Société Indust. de Mulhouse*, L. XVIII." The wheel was of wood, 5 metres or 16,4 feet in diameter, and 13 feet wide, made in three divisions on 2 centre shroudings. The curb started from a parabolic saddle-beam 8 inches in height, and the water was laid on by an overfall sluice 8 inches high. Thus the velocity of the water was about 8,8 feet, and the angular velocity of the wheel from 5 feet to $6' - 6''$. The buckets were filled from $\frac{1}{3}$ to $\frac{2}{3}$, and the efficiency increased as the buckets were more filled. When the buckets were quite filled, the efficiency was 0,80; when half-filled, it was 0,73; and with less water, it was only 0,52. The experiments on the efficiency of the wheel for different degrees of filling of the buckets, were easily and precisely made in this case, from the circumstance that the water could be laid on each division of the wheel separately.*

Example. Required the calculated proportions of a breast wheel, being given $Q = 15$ cubic feet per second, $h = 8\frac{1}{2}$ feet, and the velocity of revolution 5 feet. We shall assume the depths of the floats or of the shrouding to be 1 foot, and suppose the buckets to be filled to $\frac{1}{2}$ their contents. The width of the wheel is, hence, $e = \frac{2Q}{d\,v} = \frac{30}{1.5} = 6$ feet. Assume also that the water enters with double the velocity of rotation, then $c = 2 \,.\, 5 = 10$ feet, and the fall required to generate the velocity

$$h_1 = 1{,}1 \frac{c^2}{2g} = 1{,}1 \,.\, 0{,}00155 \,.\, 100 = 1{,}705 \text{ feet.}$$

Deducting this impact fall from the total fall, there remains for the height of the curb, or for the fall during which the water's weight alone acts, $h' = h - h_1 = 8{,}5 - 1{,}705 = 6{,}795$ feet. We shall adopt a large wheel, that the water may not fall too high into it. Making the radius $a = 12$ feet, and the radius of the division line $= 11.5$ feet. The water revolving with the velocity of the wheel, we shall suppose to be carried to the bottom of the curb, as represented in Fig. 224. The central angle α of the curb EG, or the angle by which the points of entrance of the water E deviates from lowest point F, is

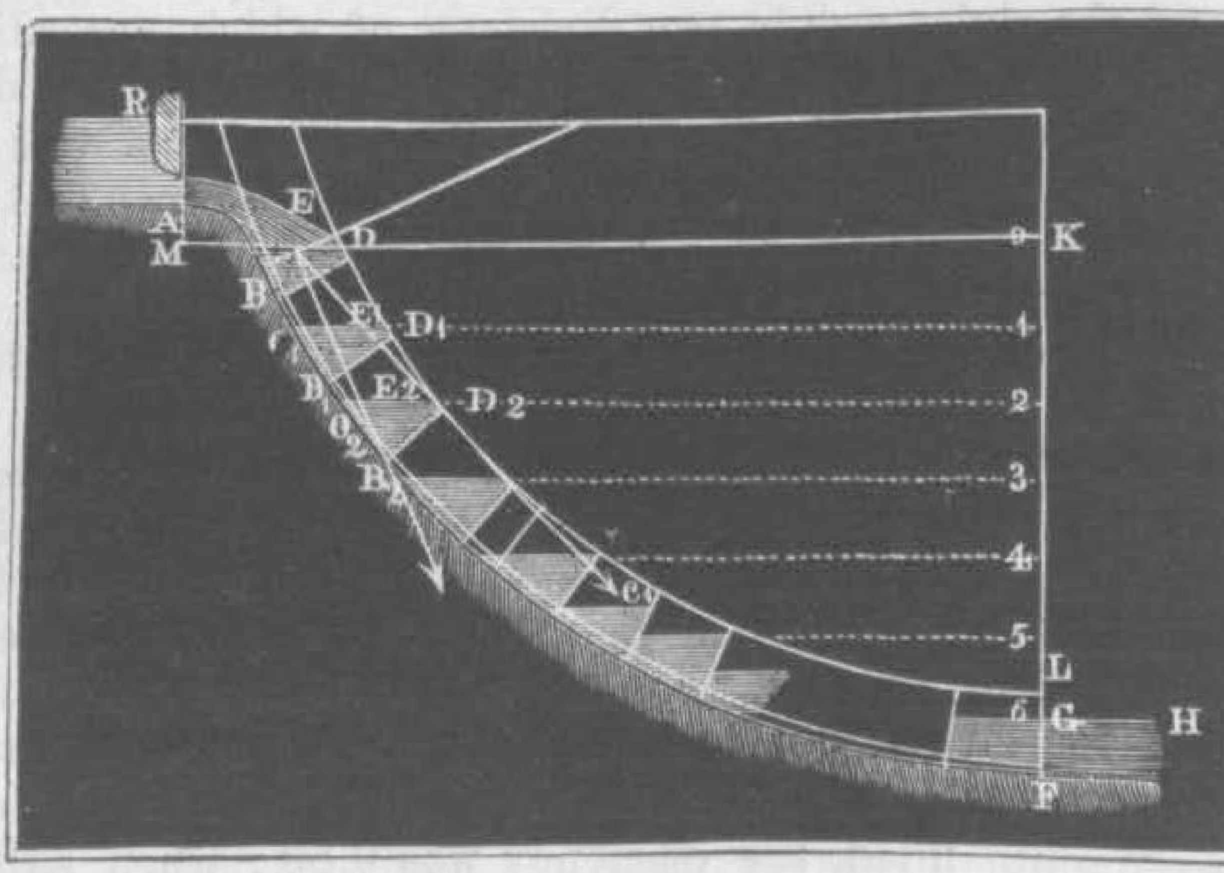
$$\cos.\, \alpha = 1 - \frac{h_2}{a_1} = 1 - \frac{6{,}795}{11{,}5} = 0{,}4092,$$

and, hence, $\alpha = 65^\circ, 50'$. We shall assume that the direction Ec_1 of the water deviates 20 degrees from the direction Ev_1 of the wheel's motion at the division

* [For the efficiency of breast wheels, with elbow and centre buckets, see above (p. 182, note). The Franklin Institute committee found, with a 15 feet wheel and elbow buckets, taking the water 10,46 feet above the bottom of the wheel, an efficiency from 612 to 677, and laying in on 7 feet from the bottom, the efficiency varied, with different *heads*, from ,568 to ,631.—AM. ED.]

circle, and refer the velocity of 5 feet, in like manner, to the division or *pitch* circle. We then have the co-ordinates of the summit of the parabolic saddle, $AM = k = \frac{c_2 \sin. (45^\circ\ 50')^2}{2g} = 0{,}8$ feet, and $ME = l = \frac{c_2}{2g} \sin. 91^\circ\ 4' = 1{,}55$ feet; according to which dimensions the construction of Fig. 224 has been carried out. The height of the water

Fig. 224.



AR above the sill is $h_1 - k = 1{,}705 - 0{,}8 = 0{,}905$, and if we put the height of the orifice $= x$, $Q = \mu e x \sqrt{2g\left(0{,}905 - \frac{x}{2}\right)}$

$$\therefore x = \frac{Q}{\mu e \sqrt{2g\left(0{,}905 - \frac{x}{2}\right)}} = \frac{15}{0{,}9 \,.\, 6 \,.\, 8{,}02 \sqrt{0{,}905 - \frac{x}{2}}}$$

$$= \frac{5}{14{,}43 \sqrt{0{,}905 - \frac{x}{2}}} = \frac{0{,}35}{\sqrt{0{,}905 - \frac{x}{2}}}\text{: and, hence, } x = 0\,.\,4 \text{ feet.}$$

The theoretical effect of this wheel is $L =$

$$\left(\frac{(c \cos. \mu - v)\, v}{g} + h_2\right) Q \gamma = \left(\frac{(10 \cos. 20^\circ - 5)\, 5}{32{,}2} + 6{,}795\right) . 15 \,.\, 62{,}25 = 7244 \text{ feet}$$

lbs., and the whole available effect is $8{,}5 \times 933$ feet lbs. $= 7930$ feet lbs. We have now to deduct the loss by the escape of water through the free space between the curb and the wheel. Assuming the *play* to be 1 inch $= \frac{1}{12}$ feet, then the area of the slit by which the water can escape is $\frac{1}{12} \,.\, 6 = \frac{1}{2}$ square feet. In order now to find the mean velocity w, with which the water passes through this aperture, the height of curb KG is to be divided into 6 equal parts, and the position of the buckets for each point so found, accurately delineated, as is done in Fig. 224, and the heads or pressures measured. Commencing at the top, we have: $z = 0{,}80$, $z_1 = 0{,}80$, $z_2 = 0{,}80$, $z_3 = 0{,}80$, $z_4 = 0{,}67$, $z_5 = 0{,}48$ and $z_6 = 0$. From this we have the mean of the square roots of these quantities $=$

$$\frac{\frac{1}{2} \,.\, 0{,}894 + 0{,}894 + 0{,}894 + 0{,}818 + 0{,}693 + 0}{6} = 0{,}7736,$$

and, hence, the mean velocity of escape of the water $= 8{,}02 \times 0{,}7736 = 6{,}188$. The mechanical effect corresponding to this is $L_1 = \phi\, e\, s\, w\, h_1\, \gamma$, in which $\phi = 0{,}7$ the coefficient of discharge, therefore,

$$L = 0{,}7 \,.\, \tfrac{1}{2} \,.\, 6{,}188 \times 6{,}79 \times 62{,}25 = 916 \text{ feet lbs.}$$

The loss by escape at the *sides* of the wheel may be calculated by the formula given § 108. It will be found $= 180$ feet lbs. So that the total loss by the escape of water $= 1096$ feet lbs; deducting this from 7244 feet lbs., there remain 6148 feet lbs.

effective. The escape of water in this wheel we see involves a loss of 15 per cent. of the mechanical effect of the fall. By the friction of the water and the resistance of the water, the loss is about 160 feet lbs., or about 2½ per cent. There remains, therefore, 5988 feet lbs.

If now we take the weight of the wheel $G = \frac{3000\ L}{\epsilon u}$, ϵ, the ratio of filling the buckets, being ½, we have $u = \frac{30\ .\ 5}{\pi n} = \frac{25}{2\pi} = 4$ and $L = \frac{5988}{550} = 11$ lbs. feet nearly; then, the weight of the wheel $= \frac{3000 \times 12}{\frac{1}{2}\ .\ 4} = 1650$ lbs. Hence, the radius of the gudgeons $r = 0{,}002 \sqrt{8250} = 0{,}182$ feet, and from this we get the mechanical effect absorbed by friction $= \frac{r}{a} \varphi G v = \frac{0\ .\ 19}{11\ .\ 5}\ .\ 0{,}1\ .\ 16500\ .\ 5 = 136$ feet lbs. Making this farther deduction, there remains 5852 feet lbs. = 10,6 feet lbs., and, lastly, the *efficiency* of the wheel $\eta = \frac{5852}{7930} = 0{,}74$.

§ 112. *Undershot Wheels.*—Undershot wheels usually hang in a channel made to fit as closely to the wheel as possible, so that water may not escape without producing its effect. Hence, the application of a channel having a curb concentric with the wheel is considered better than a straight channel tangential to the wheel. The curb allows of some of the effect of the weight of the water being availed of. The calculation for such a wheel as is represented in Fig. 225, when the curb AB embraces 3 to 4 floats at least, is identical with these for the breast wheels last considered. The rules for construction of undershot wheels, correspond too with those for breast wheels. The floats are usually put in radially; but sometimes they are inclined upwards towards the sluice, that they may carry no water up with them on the opposite side. These floats are not unfrequently composed of two equal pieces AD and BD, Fig. 226, so that the angle $ADB = 100°$ to $120°$. This arrangement allows of ample openings being left in the *flooring* of the wheel, without fear of the water flowing through the sluice. The cells or buckets are allowed to fill from one-half to two-thirds of their capacity, or $\epsilon = \frac{1}{2}$ to $\frac{2}{3}$. To prevent overflow of the water inwards, or, in order to have greater capacity, the depth of the wheel, *i. e.*, of the shrouding, is made from 15 to 18 inches. The laying on the water *tangentially* is more rarely done than in breast wheels. The sluice-board is *inclined* in order that the sluice-

Fig. 225.

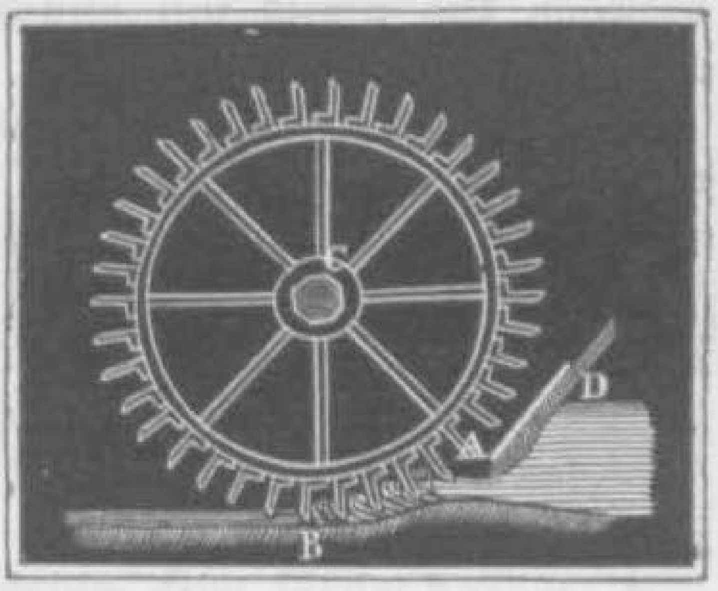
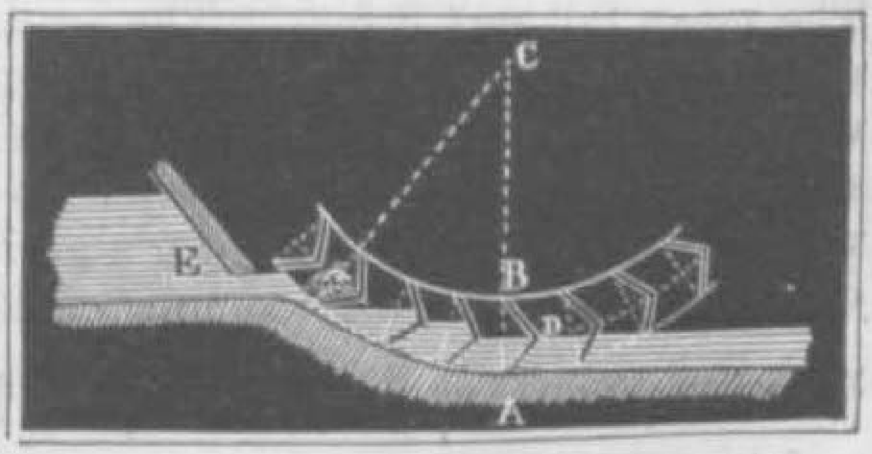


Fig. 226.



aperture may lie as close to the wheel as possible. The lower edge should be rounded off to prevent *partial contraction* of the vein of water.

§ 113. The effect of undershot wheels is less than that of breast wheels, the fall available as weight being greater in the latter. The *half* of the fall is necessarily lost when it acts by impact, whereas the loss by escape of water acting by its weight on those wheels does not amount to $\frac{1}{4}$ of the whole. Experiment has satisfactorily established this. The wheel with which Morin experimented was 19′.6″ in diameter, $5\frac{1}{2}$ feet wide, and had 36 radial floats. The sluice was inclined at an angle of $34\frac{1}{2}^{\circ}$ to the horizon, and the sluice-aperture was $2\frac{1}{2}$ feet back from the commencement of the curved course. The total fall was 6′—3″, and the head on the sluice-aperture 4′—7″. There was, therefore, a fall of about 1′—8″ through which the water's weight acted. The velocity of the circumference of the wheel was from 6′—6″ to 13′—0″: and the velocity of the water on reaching the wheel from 16 to 18 feet. As long as $\frac{v}{c}$ did not exceed 0.63, the efficiency η was 0.41 as a mean: but when $\frac{v}{c}$ varied between the limits 0.5 and 0.8, then the mean efficiency η was only 0.33.

Retaining our former notation, we have, for the effect of this wheel, exclusive of friction of gudgeons,

$$Pv = 0{,}74\left(\frac{(c-v)\,v}{g} + h_1\right) Q\,\gamma,$$

in the first case; and

$$Pv = 0{,}60\left(\frac{(c-v)\,v}{g} + h_1\right) Q\,\gamma$$

in the second.

A second wheel with which Morin experimented, was about 13 feet high, 2′—8″ wide, 11,8 inches deep, and had 24 floats. The water was laid on by a vertical sluice, and reached the wheel through a straight course 2′—8″ long. This channel and the curb were of sandstone, and the free space left amounted to only 0,2 of an inch. The mean fall was 3 feet. The head of water on the sluice-aperture varied from 6 to 18 inches. Experiments were made at various velocities of rotation. For small velocities the efficiency was very small. For the mean velocity of 5 feet it was a maximum, and, when the velocity of the water's arrival on the wheel was not much different from this, a maximum efficiency 0,49 was obtained. For ratios of velocities $\frac{v}{c} = \frac{1}{4}$ and $\frac{v}{c} = \frac{4}{4}$, the mean value of η was found to be, as for the former wheel 0,74. Hence

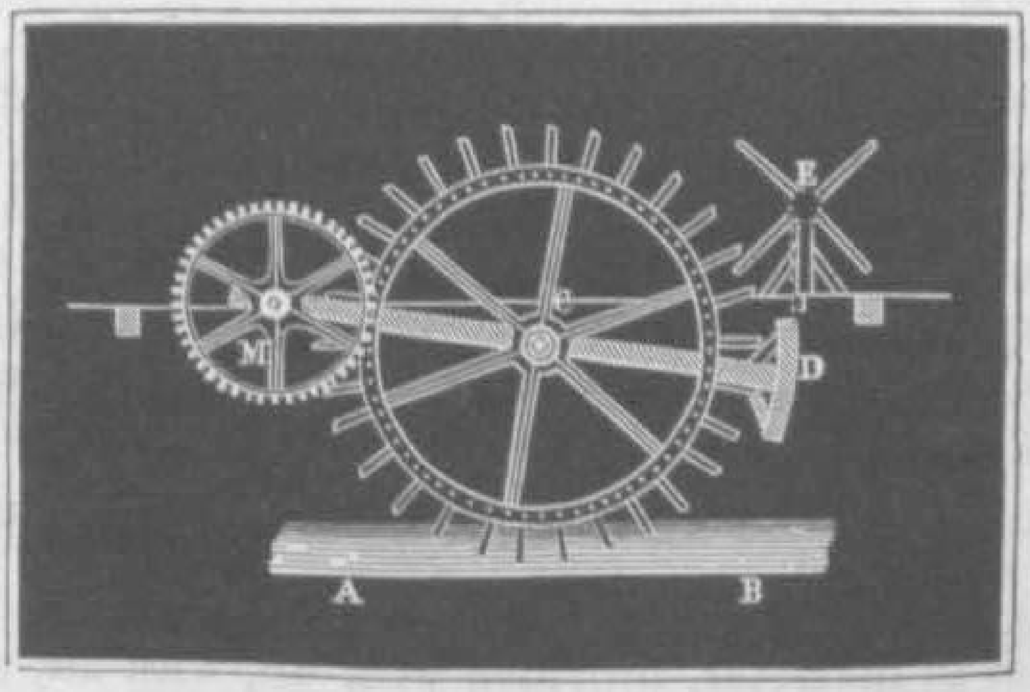
$Pv = 0{,}74\left(\frac{(c-v)\,v}{g} + h_1\right) Q\,\gamma$, is the formula for this case also.

Morin puts together the results of his experiments on wheels confined in mantles or curbs as follows. Wheels in which $h_1 = \frac{1}{4}h$ $\kappa = 0{,}40$ to $0{,}45$. When $h_1 = \frac{3}{8}h$, $\kappa = 0{,}42$ to $0{,}49$. When $h_1 = \frac{3}{8}h$, $\kappa = 0{,}47$, and when $h_1 = \frac{3}{4}h$, $\kappa = 0{,}55$.

Example. Required, the effect of an undershot wheel, 15 feet in diameter, and making 8 revolutions per minute. The fall 4 feet, and the quantity of water 20 cubic feet per second, $v = \frac{\pi u a}{30} = \frac{\pi \cdot 8 \cdot 15}{60} = 6{,}283$ feet, and supposing the velocity of the water to be double this, then the pressure of the water in front of the sluice, or what we have termed the *impact*-fall $= 1{,}1 \frac{c^2}{2g} = 1{,}1 \times 0{,}0155 \times 12{,}56^2 = 2{,}689$ feet, and there therefore remains as fall, through which the water acts by its weight, $h_1 = 4 - 2{,}689 = 1{,}311$ feet, and hence the theoretical effect $= (0{,}031 \cdot 6{,}283^2 + 1{,}311)\ 20 \cdot 62{,}5 = (1{,}263 + 1{,}311)\ 1245 = 3264$ feet lbs. In this case, $h_1 = \frac{1{,}311}{4}\ h = 0{,}33\ h$, and, therefore, the co-efficient α may be taken 0,42, and hence the effect $L = 0{,}42 \alpha\ 3264 \alpha = 1370{,}8$ feet lbs. from which, however, the gudgeon-friction has to be deducted.

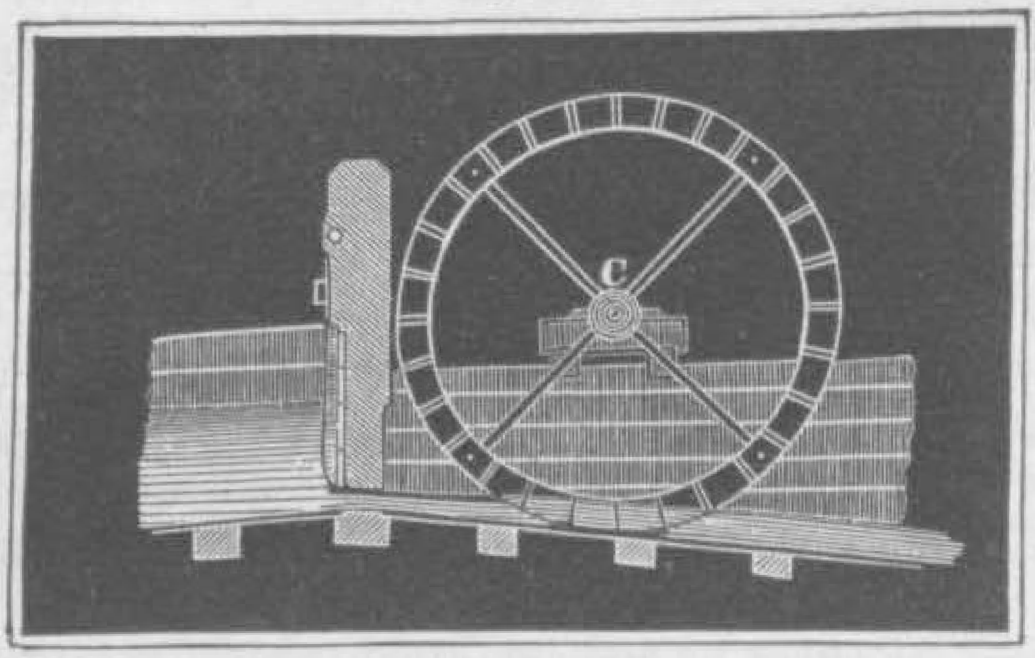
§ 114. *Wheels in Straight Courses.*—When the undershot wheel is hung in a straight course, the effect is a minimum; because the water produces its effect by impact alone, and a considerable quantity escapes unused. These wheels are only adopted for falls of less than 4 feet, and where *water power is of value* the Poncelet-wheel, or turbines, are now invariably preferred. They are made from 12 to 24 feet in diameter, with 24 to 48 floats, usually radial, but sometimes placed with a slight inclination towards the sluice. The breadth or depth of the floats should be made about three times the thickness of the layer of water coming through the sluice, because the water in contact with the wheel retains only 35 to 40 per cent. of the velocity of the water before impact, when the greatest effect is produced; and, therefore, the stream of water flowing along as the wheel revolves is $2\frac{1}{2}$ to 3 times the thickness of the water coming from the sluice. The depth of the sluice-aperture is usually 4 to 6 inches, and thus the floats are made from 12 to 18 inches deep for the above reason. The straight course in which undershot wheels are suspended may be either horizontal as in AB, Fig. 227,

Fig. 227.



or inclined, as in Fig. 228. That as little water as possible may escape unemployed, the space between the wheel and the course must be reduced to 1 or 2 inches at most. And hence, it is better to lay the course with a slight curvature, the floats being made so numerous, that there are always 4 or 5 floats submerged.

Fig. 228.



The penstock is set with an inclination to bring the orifice of discharge as near to the wheel as possible, and to avoid contraction. To prevent back-water, the course is made to drop suddenly some inches, at the point where the water quits the wheel. Besides this provision, arrangements for raising and depressing the wheel, or the water-course, are adopted.

Fig. 227 represents a *lift* for the wheel (called in German *Ziehpanster*). The axle M of the lever MD coincides with that of the wheel, so that the connection between the driving wheel and pinion may not be altered in raising or depressing the water wheel. All these arrangements are, however, rendered unnecessary by the adoption of the turbine, instead of undershot wheels, in all cases in which the water is liable to much variation.

§ 115. *Useful Effect of Undershot Wheels.*—Experiments on the useful effect of undershot wheels, with straight courses, have been made, but only on models, by De Parcieux, Bossut, Smeaton, Lagerhjelm, &c.

The experiments of Smeaton and Bossut are the best.* The results of the experiments are satisfactorily in agreement with each other, and confirm the theory. The mechanical effect evolved by these wheels was ascertained in all the experiments, by raising a weight by means of a cord passed round the axle of the wheels. Smeaton's experiments were made with a small wheel 75 inches in circumference, having 24 floats, each 4 inches wide, and 3 inches deep. The general conclusion at which Smeaton arrived is, that for the velocity ratio $\frac{v}{c} = 0{,}34$ to $0{,}52$, the maximum useful effect amounts to 0,165 to 0,25. Bossut's experiments were made with a wheel, 3 feet in diameter, provided with 48, with 24, and with 12 buckets, 5 inches wide, and 4 to 5 inches deep. Bossut found, as

* [See foot note and reference next page.—AM. ED.]

theory indicates, that with 48 floats, the efficiency is greater than with 24, and with 24 greater than with 12; and he deduced from his experiments that about 25° of the wheel's circumference, or $\frac{25}{360} \cdot 48 = \frac{10}{3}$, or more than 3 floats should be in the water at the same time. From Bossut's experiments on the wheel with 48 floats, a somewhat greater efficiency results than is indicated by Smeaton's experiments, and this may probably be attributed to the greater proportional number of buckets in Bossut's model.* The mean result of the two sets of experiments gives the effect of such wheels, friction not taken into account

$$L = 0{,}61 \frac{(c - v)\, v}{g} Q\gamma = 1{,}19\, (c - v)\, v\, Q \text{ feet lbs.}$$

This formula will only apply on the scale of practice, when the play allowed between wheel and course *is not greater than* $1\frac{1}{2}$ *inch.* Instead of Q we have Fc, in which F is the arc of the float dipping into the water; and hence we have the formula given by Christian in his "*Mécanique industrielle*," substituting 0,76 for 0,61.

$$L = 0{,}76\, F\gamma \cdot \frac{(c - v)}{g} c v = 1{,}48\, (c - v)\, F c v \text{ feet lbs.}$$

From the experiments extant, it follows also, that the maximum effect is produced for the velocity ratio $\frac{v}{c} = 0{,}4$, as indicated by theory. For greater velocities this ratio is somewhat less, and for large bodies of water the ratio is somewhat greater.

§ 116. *Partition of Water Power.*—A given *fall* of water is often divided between several wheels, not only because a single wheel would have cumbrous dimensions, but more especially for the sake of working different machines or tools independently, avoiding the coupling connections with one source of power. The question may arise as to a division of height of *fall*, or, as to partition of the quantity of water. As a general rule, we may assume that for wheels on which the water acts by its weight, a partition of the *quantity* of water, and for wheels on which the water acts by impact, a partition of the height of fall is to be preferred; for we have seen that the efficiency of overshot wheels of great diameter, is greater than that of overshot wheels of smaller diameter, or even than breast wheels; and, on the other hand, it is manifest that the loss of effect by impact, and by the escape of water through the wheels, is less when these wheels are placed one behind the other, than when they are put side by side, because the velocity due to the height corresponding to the loss of effect $\frac{(c - v)^2}{2g}$ (Vol. I. § 387), and the ratio $\frac{s}{d_1}$ of the free space to the depth of water, is less than in the latter case. For breast wheels in a curb, on which the water acts

* [The ratio of effect to power, obtained by the Committee of the Franklin Institute, was found to vary from ,266 to ,305, and the average is set down at ,285. See Journal Franklin Institute, 3d series, Vol. II., p. 2, for July 1841.—Am. Ed.]

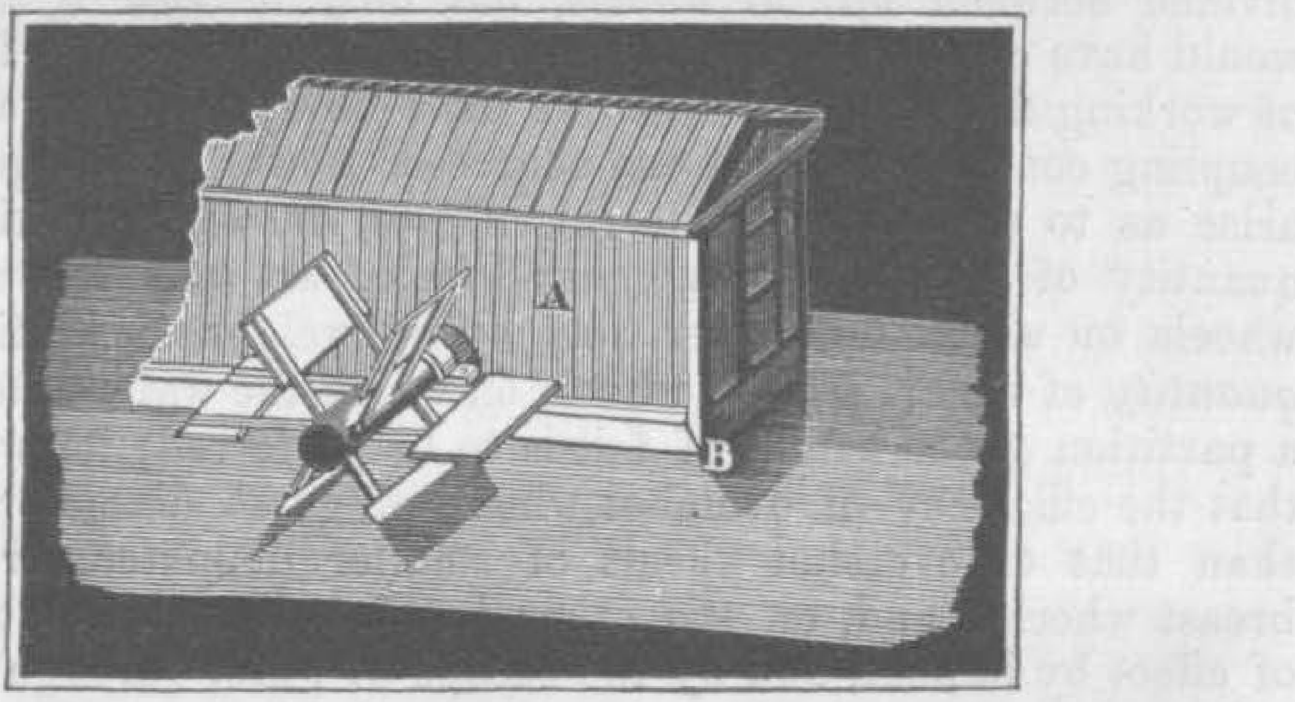
by its weight and by impact, and in which the loss of water depends mainly on $\frac{s}{d_1}$, there is no general rule for the preference of one mode of partition over the other, and the circumstances of each case must determine our choice.

§ 117. *Floating-mill Wheels.*—Wheels suspended on two boats, or barges, conveniently moored in a river, are undershot wheels without curb or *limited* course of any kind. These wheels are supported either on two boats, one of which contains the mill machinery, or one end of the axle rests on a boat, the other resting on piles driven in on shore, in which case the mill machinery is kept on shore.

The construction of boat-mill wheels differs from that of ordinary undershot wheels, inasmuch as they have no shrouding, the floats being attached directly to the arms. These wheels are made from 12 to 15 feet in diameter, and have generally only 6 or 7 floats, although 10 to 12 would constitute a better wheel. The floats are made long and very broad, that they may catch a large stream of water, for, the velocity being usually small, the *vis viva* depends in a great measure on the *mass*. Floats of 6 to 18 feet in length, and 2 feet to 30 inches broad, are usual. The floats are inclined at angles of from 10° to 20° to the radius, and dip to about one-half their breadth into the stream.

Fig. 229 represents a boat-mill (Fr. *moulin à nef;* Ger. *Schiff-*

Fig. 229.



mühle). *A* being the mill-house on the barge *B*, and *C* a wheel with 6 floats, the axle of which passes through the mill-house, and projects as far on the opposite side of the boat as the one seen in the figure does on this. The mill gear is within the house.

The effect of boat-mill wheels is less than of wheels hung in a confined course, for two reasons, viz.: the water not only escapes by the sides, and under the floats, but a considerable quantity passes through the wheel without coming into action, from the small number of buckets that dip into the water.

§ 118. The theoretical effect of a freely suspended water wheel may be represented, as for undershot wheels, by the formula

$$L = Pv = \frac{(c - v)\,v\,c}{g} F \gamma,$$

c and v being the velocities of the water and of the wheel, and F the area of the part of the float dipped, neglecting the damming up of the water upon it. This expression has to be multiplied by a co-efficient allowing for the loss of water.

§ 119. Experiments on the effect of these wheels have been made by De Parcieux, Bossut, and Poncelet, but principally on models.

Bossut's model wheel was 3 feet in diameter, had 24 floats, 6½ inches wide, dipping 4½ inches into the water. The velocity of the water was 6 feet per second. The result of these experiments gives $\mu = 1{,}37$ to $1{,}79$ as the co-efficient, by which the formula $L = \frac{(c - v)\,v\,c}{g} F \gamma$ is to be multiplied, and $\mu = 0{,}877$ to $0{,}706$ as the co-efficient for the formula $L = \frac{(c - v)\,v\,c}{g} F \gamma$ (*see* D'Aubuisson, "Hydraulique," § 352). The limits of the values of the co-efficients are nearer each other, in this latter case, than in the other, which was to be expected, as, from the number of buckets, the second formula is most applicable. The number of buckets should be such that 2 at least are in the water, and then the latter formula with the mean co-efficient $\mu = 0{,}8$, will apply, or,

$$L = 0{,}8 \frac{(c - v)\,c\,v}{g} F \gamma = 1{,}55\,(c - v)\,c\,v\,F \text{ feet lbs.}$$

Poncelet's observations, made on three boat-mill wheels on the Rhone, agree with this. These wheels were 8 to 10 feet long, and the floats dipped 2′ — 3″ to 2′ — 9″ into the water flowing with a velocity of from 4 to 6½ feet per second. Poncelet cites an experiment by Boistard, and one by Christian, both of which confirm the accuracy of this formula.

Bossut's experiments, in exact accordance with theory, show that the maximum effect is obtained when $v = 0{,}4\,c$, and Poncelet's experiments on the Rhone boat wheels also give: $\frac{v}{c} = 0{,}4D$

Introducing $v = 0{,}4\,c$ into the above formula, the useful effect becomes:

$$L = 0{,}8\,\frac{0{,}6 \cdot 0{,}4\,c^3}{g} F \gamma = 0{,}192 \frac{c^3}{g} F \gamma = 0{,}384 \frac{c^2}{2g} Q \gamma,$$

and, hence, the *efficiency* $\eta = 0{,}384$.

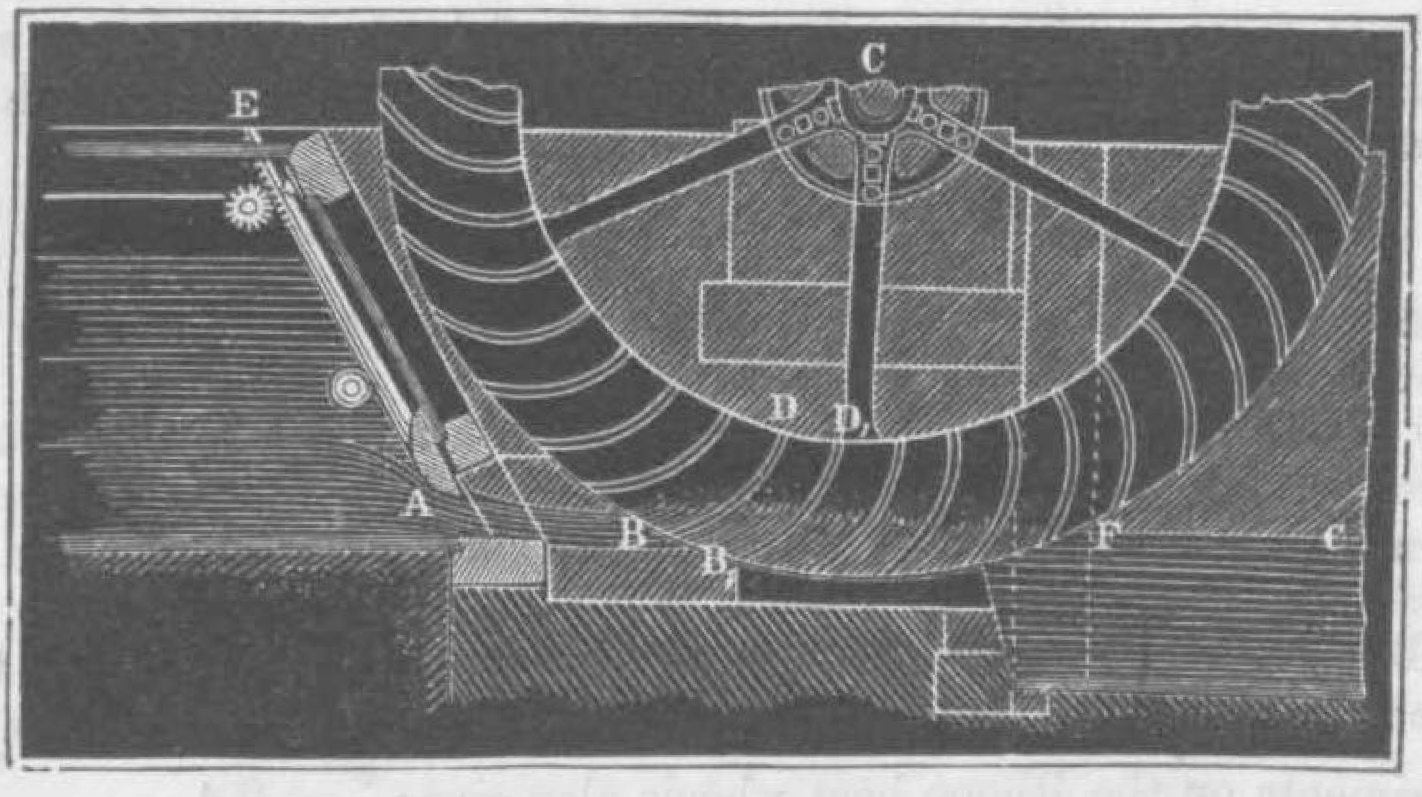
De Parcieux's experiments were specially directed to ascertaining the best position for the floats. The result was that an inclination of 60° to the stream is the best.

Remark. There has long been a doubt as to which of the two formulas $L = \frac{\mu\,(c - v)^2\,v}{g} F \gamma$ and $L = \frac{\mu_1\,(c - v)\,c\,v}{g} F \gamma$ is the more correct. The one is known as *Parent's* formula, the other as *Borda's*. Now, although for a wheel in an unconfined stream acting on the floats, all the water going through the wheel does not assume the velocity of the floats, yet, considering the great extent of the floats' surface, it may certainly be presumed that the greater part of the water on impinging, takes the velocity of the floats, and, hence, the greater accordance between experiments in Borda's formula is explained. Parent's formnla is founded on the assumption that the impact is proportional to the height due to the relative velocity $c - v$. (*Compare* Vol. I. § 392, where the force of impact is given $= 1{,}86 \frac{c^2}{2g} F\gamma$ when $v = 0$.)

§ 120. *Poncelet's Wheels.*—If the floats of undershot wheels be curved so that the stream of water runs along the concave side, pressing upon it without impact, the effect produced is greater than when the water impinges at nearly right angles against straight buckets.

Poncelet introduced these wheels. They are of very advantageous application for low falls under 6 feet, because their effect is much greater than that of undershot wheels with or without a curb. For greater falls, breast wheels with a well-formed circle excel them, and as their construction is more difficult, they are not applied for greater falls than 6 feet. Poncelet has treated of these wheels in a special work, entitled "*Mémoire sur les Roues hydrauliques à aubes courbes, mues par-dessous*, Metz, 1827." Fig. 230 represents the

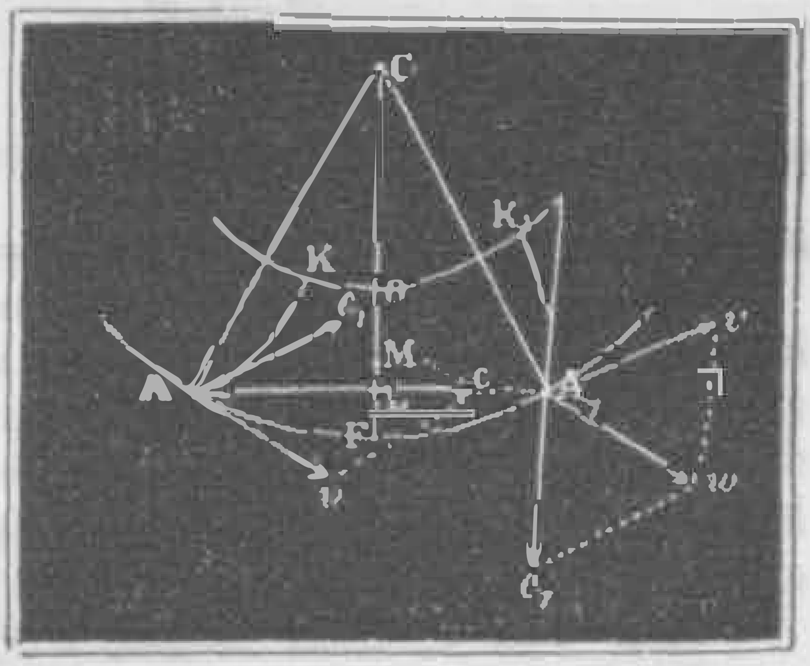
Fig. 230.



general arrangement of these wheels: AE is an inclined sluice-board; AB is the stream of water entering the wheel at the buckets BD and B_1D_1. FG is the surface of the tail-race. In order that nearly all the water may come into action, the wheel must have very little play in the water-course, and to prevent partial contraction, the under side of the sluice-board is rounded off: also, to prevent loss of *vis viva* by friction in the channel, the aperture of the sluice is brought close to the wheel. The first part of the course AB is inclined at $\frac{1}{10}$ to $\frac{1}{15}$. The remainder of the course, which embraces the length occupied by three buckets at least, is accurately curved concentrically with the wheel, and at the end of it, a sudden dip of 6 inches is made, and the tail-race should also be widened to guard against any liability to back-water on the wheel. Poncelet wheels have been constructed from 10 to 20 feet in diameter, and with 32 to 48 floats of sheet iron or of wood. Wooden floats are composed of staves, like a barrel, the outer edge being sharpened, or provided with a sheet iron edge piece. Sheet iron floats are, however, much more suitable, as good construction is an essential feature in this wheel. The sluice is not drawn more than 1 foot in any case, and for falls of 5 to 6 feet, an aperture 6 inches high, or less, is arranged for.

§ 121. *Theory of Poncelet's Wheels.*—To obtain the maximum effect from these wheels, the water must go on to the buckets without impact. If $Ac = c$ (Fig. 231) be the velocity of the water going on to the wheel, and $Av = v$, the velocity of the periphery of the wheel, we then have in the side $Ac_1 = c_1$ of the parallelogram $A\,v\,c\,c_1$, the velocity of the water in reference to the wheel, both in magnitude and direction. If, therefore, we put the curved float AK tangential to Ac_1, the water will begin to ascend along it without the least shock, with the velocity c_1. If we put the angle

Fig. 231.



$c\,A\,v$ by which the direction of the water deviates from that of the circumference of the wheel, or the tangent $Av = \delta$, we have for the relative velocity of the water beginning its ascent on the floats $= c_1 \sqrt{c^2 + v^2 - 2\,c\,v\,\cos.\,\delta}$, and for the angle $v\,A\,c_1 = \epsilon$, by which it deviates from the circumference of the wheel, or from the tangent Av, we have $\sin.\,\epsilon = \dfrac{c\,\sin.\,\delta}{c_1}$.

The water ascends on the float with a retarded velocity, and partakes of the velocity of rotation v of the wheel at the same time. Having ascended to a certain height, its relative velocity is lost, and it descends with an accelerated velocity, so that at last it arrives at the outer extremity A_1 with the same velocity c_1 with which it commenced its ascent. If we combine the relative velocity $A_1\,c_1 = c_2$, after the water leaving the wheel at A, with the velocity of the circumference $A_1\,v = v$ as a parallelogram of the velocities, we have in the diagonal $A_1 w = w$ the absolute velocity of the water leaving the wheel. This velocity is

$$w = \sqrt{c_1^2 + v^2 - 2c_1\,v\,\cos.\,\epsilon},$$

and, therefore, the mechanical effect, retained by the water, and lost for useful effect, is

$$L_1 = \frac{w^2}{2g}\,Q\,\gamma = \left(\frac{\sqrt{c_1^2 + v^2 - 2c_1\,v\,\cos.\,\epsilon}}{2g}\right)Q\,\gamma.$$

If, now, we deduct this loss from the amount of effect $\dfrac{c^2}{2g}\,Q\,\gamma$ inherent in the water before its entrance on the wheel, we have the following expression for the theoretical effect of the wheel:

$$L = \left(\frac{c^2}{2g} - \frac{w^2}{2g}\right) Q\gamma = \left(\frac{c^2 - w^2}{2g}\right) Q\gamma = \left(\frac{c^2 - c_1^2 - v^2 + 2 c_1 v \cos. \varepsilon}{2g}\right) Q\gamma,$$

or, as $c^2 = c_1^2 + v^2 + 2c_1 v \cos. \varepsilon$, $\therefore L = \frac{2 c_1 v \cos. \varepsilon}{g} \cdot Q\gamma$, or,

$c_1 \cos. \varepsilon = \sqrt{c_1^2 - c^2 \sin. \delta^2} = \sqrt{c^2 \cos. \delta^2 + v^2 - 2 c v \cos. \delta} = c \cos. \delta - v$, and, if we put this in the above expression, we have:

$$L = \frac{2 v (c \cos. \delta - v)}{g} Q\gamma.$$

We easily perceive that the effect is a maximum when $v = \frac{1}{2} c \cos. \delta$, and then $L = \frac{c^2 \cos. \delta^2}{2g} Q\gamma$. Also, the loss of mechanical effect is *null*, or the whole mechanical effect available, or $L = \frac{c^2}{2g} Q\gamma$ is got from the water when $\cos. \delta = 1$, or when $\delta = 0$.

Although it is not possible to make the angle of entrance $\delta = 0$, it follows from this that δ should not be a large angle—not more than 30°, if a good effect is desired, and it is also manifest that the velocity of rotation of the wheel should be only a little less than half the velocity of the water going on to the wheel, that the efficiency may be the greatest.

§ 122. The vertical height LO, to which the water ascends on the floats, would be $\frac{c_1^2}{2g}$ if the wheel were at rest, but as it has a velocity of rotation v, a centrifugal force arises, acting nearly in the same direction as gravity, and giving rise to an acceleration p, which may be represented by $\frac{v_1^2}{a_1}$, if a_1 be the mean radius CM, and v_1 the mean velocity of the wheel's shrouding, or the velocity of the point M. We have then:

$$(g + p) h_1 = \frac{c_1^2}{2}, \text{ or } \left(g + \frac{v_1^2}{a_1}\right) h_1 = \frac{c_1^2}{2},$$

and hence, the height of ascent in question $h_1 = \frac{c_1^2}{2\left(g + \frac{v_1^2}{a_1}\right)}$.

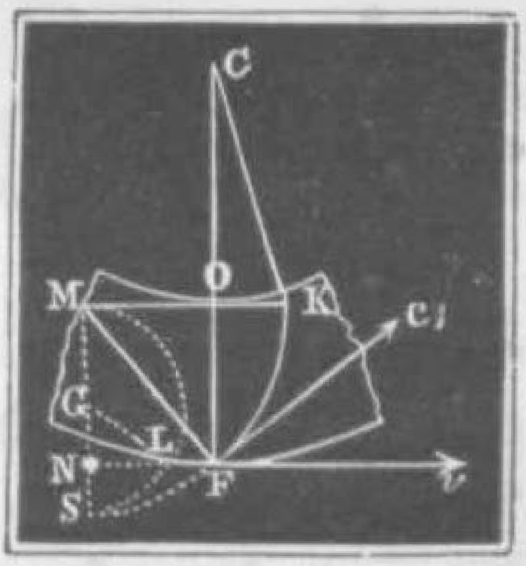
In order that the water may not pass over the top at O, it is necessary that the shrouding should have a certain depth $FO = d$, which is determined by the equation $d = LO + FL = h_1 + CF - CL$ $= h_1 + a - a \cos. ACF = \frac{c_1^2}{2\left(g + \frac{v_1^2}{a_1}\right)} + a\,(1 - \cos. \lambda)$, where λ is the angle ACF by which the point of entrance of the water on the wheel deviates from the lowest point of the wheel F. The thickness of the stream of water d_1 is to be added to this, because the particles in the upper stratum must rise so much higher than those of the lower stratum on the assumption of a mean velocity. The depth of the shrouding is, therefore,

$$d = d_1 + \frac{c_1^{\,2}}{2\left(g + \frac{v_1^{\,2}}{a_1}\right)} + a\,(1 - cos.\,\lambda).$$

The width of the wheel is equal to the width of the stream of water; or, $e = \frac{Q}{d_1\,c}$. If the capacity $d\,e\,v_1$ of the wheel be made $1\frac{1}{2}$ times that of the water laid on, then we have the equation $d\,v_1 = \frac{3}{2}\,d_1\,c$ to $2\,d_1\,c$, and hence the thickness or depth of the stream laid on $= d_1 = \frac{1}{2}\,\frac{d\,v_1}{c}$ to $\frac{2}{3}\,\frac{d\,v_1}{c}$. Another important circumstance in reference to these wheels is the determination of the points of entrance and exit of the water, that is the water arc $A A_1$, which it is best to set off in two equal portions on each side of the lowest point of the wheel F. The length of this arc depends on the time necessary for the ascent and descent of the water on the floats. To find this, we must know the form and dimensions of the floats. If the time $= t$, then we may put $A A_1 = 2\,\lambda\,a = v\,t$, and hence the points on either side of F, at which the water enters and quits the wheel, are at a distance $= \lambda = \frac{v\,t}{2a}$.

§ 123. In order that the water, when it has reached the highest point K, Fig. 232, may not run over, but fall back along the float, the inner end of the float K must not overhang the float when in the mean position FK; but, on the other hand, that the float may not be too long, the end K of the float must not cut the inner circumference of the shrouding at too acute an angle. Hence, it is best to give the inner end of the float a vertical position, when the float is in its mean position. Adopting a cylindrical form of float, we get the centre of the circular arc, its section, by drawing MF perpendicular to Fc_1, and OM horizontal. From the depth of shrouding $FO = d$, we have the radius:

Fig. 232.



$$FM = KM = r = \frac{d}{cos.\,\varepsilon},$$

ε being the angle $MFO = c_1\,F\,v$.

The time required for the ascent and descent of the water on the arc FK may be found in the same manner as the time of oscillation of a pendulum, by substituting for the accelerating force of gravity the sum $g + \frac{v_1^{\,2}}{a_1}$ of this acceleration, and that of centrifugal force. This time may be found exactly by the method given Vol. I. § 246, by putting here, as there, the instant of time required to move through a small space:

$$\tau = \left(1 + \frac{h(1 + cos.\, \phi)}{8r}\right)\sqrt{\frac{r}{g}} \cdot \frac{\phi}{2n}.$$

In order to find the time for ascent and descent in the arc FK, we have to substitute for ϕ the central angle MGL, which may be determined from the angle $c_1 Fv = FMS = \epsilon$, and the radius $MF = MS = r$, by the formula:

$$cos.\, \phi = -\frac{NG}{LG} = -\frac{MN - MG}{MG} = -\frac{r\, cos.\, \epsilon - \frac{1}{2} r}{\frac{1}{2} r}$$

$$= -(2\, cos.\, \epsilon - 1), \text{ or } sin.\, \tfrac{1}{2}\phi = \sqrt{cos.\, \epsilon}.$$

We have now the time t_1 required for describing the whole arc FK, by adding together all the values of the expression:

$$\tau = \left(1 + \frac{h}{8r}(1 + cos.\, \phi)\right)\sqrt{\frac{r}{g}} \cdot \frac{\phi}{2n},$$

when for $cos.\, \phi$ we substitute in succession:

$$cos.\, \frac{\phi}{n},\ cos.\, \frac{2\phi}{n},\ cos.\, \frac{3\phi}{n} \ldots cos.\, \frac{n\phi}{n}. \quad \text{But}$$

$$cos.\, \frac{\phi}{n} + cos.\, \frac{2\phi}{n} + cos.\, \frac{3\phi}{n} + \ldots + cos.\, \frac{n\phi}{n} = \frac{sin.\, \frac{\phi}{2}\, cos.\, \frac{\phi}{2}}{\frac{\phi}{2n}}$$

$= \dfrac{sin.\, \phi}{\frac{1}{n}\phi}$, and hence $t_1 = \left[\frac{\phi}{2} + \frac{h}{8r}\left(\frac{\phi}{2} + \frac{\phi}{2n}\right.\right.$ times the sum of all

cosines from 0 to $\left.\left.\phi\right)\right]\sqrt{\frac{r}{g}} = \left[\frac{\phi}{2} + \frac{h}{8r}\left(\frac{\phi}{2} + \frac{1}{2}\, sin.\, \phi\right)\right]\sqrt{\frac{r}{g}}.$

If we also consider that the whole height of fall, or the diameter $MS = h$, that $MF = r$, and that $g + \frac{v_1^2}{a_1}$ is to be substituted for g the force of gravity, the whole time for the rise and fall of the water on the arc FK is

$$t = 2\, t_1 = [\phi + \tfrac{1}{8}(\phi + sin.\, \phi)]\sqrt{\frac{r}{g + \frac{v_1^2}{a_1}}},$$

and the length of the water arc AA_1 (Fig. 231), is:

$$b = 2\lambda a = v\, t = \left[\phi + \frac{(\phi + sin.\, \phi)}{8}\right] v \sqrt{\frac{r}{g + \frac{v_1^2}{a_1}}}.$$

§ 122. We have now to derive rules for the arrangement and construction of Poncelet's wheels from these data. We can only assume the height of fall h, the quantity of water Q, and the number of revolutions u of the wheel, as given, and from this we have to deduce the velocity of rotation v, the radius of the wheel a, the depth of shrouding d, the width of wheel e, and the angles δ, ϵ, λ, and the velocity c_1 of the water at the beginning of its ascent. If we attentively consider the formulas above found, we perceive that they do not admit of a direct solution of the problem, but that the method of gradual approximation must be adopted.

If we lay on the water in a horizontal direction, the deviation δ of the direction of the water-stream from the periphery of the wheel is equal to the distance λ of the point of entrance from the foot of the wheel. In the first place, we may put, as an approximation, the velocity of the water entering the wheel: $c = \mu \sqrt{2gh}$, and from this again, the velocity of rotation of the wheel $v = \frac{1}{2} c$, as also the initial velocity of the ascending water $c_1 = \frac{1}{2} c$, we have hence also an approximate value of the radius $a = \frac{30v}{\pi u}$, and the same for the depth of shrouding $d = \frac{c_1^2}{2g} = \frac{1}{4} \cdot \frac{c^2}{2g}$, and, hence, also, we obtain an approximate value for the length of the water arc, if we put in the last formula of the preceding paragraph:

$$\phi = \pi, \ \frac{\phi + \sin.\phi}{8} = 0, \text{ and } r = d, \text{ then:}$$

$$2 \lambda a = \pi v \sqrt{\frac{d}{g + \frac{v^2}{a}}}, \text{ and, therefore,}$$

$$\lambda = \frac{\pi v}{2a} \sqrt{\frac{d}{g + \frac{v^2}{a}}} = \frac{\pi^2 u}{60} \sqrt{\frac{d}{g + \frac{v^2}{a}}}.$$

With the assistance of this approximate value of $\lambda = \delta$, the calculations must be repeated, using the more exact formulas, and taking for the depth of the water-stream d_1 an appropriate value of from 3 to 12 inches, according to circumstances. The head or pressure is then only $h - d_1$, and hence the velocity of the water entering the wheel is: 1. $c = \mu \sqrt{2g\,(h - d_1)}$, that of the wheel. 2. $v = \frac{1}{2} c \cos.\delta$. Again, the radius of the wheel 3. $a = \frac{30\,v}{\pi u}$; for the angle ϵ made by the circumference of the wheel with the end of the float,

4. $\cot g.\,\epsilon = \cot g.\,\delta - \frac{v}{c \sin.\delta} = \frac{1}{2} \cot g.\,\delta$, or $tang.\epsilon = 2\ tang.\,\delta$;

and the initial velocity of the water rising on the float.

5. $c_1 = \frac{c \sin.\delta}{\sin.\epsilon} = \frac{v}{\cos.\epsilon}$; and if, instead of $\frac{v_1^2}{a_1}$, we put $\frac{v^2}{a}$, the depth of shrouding,

6. $d = d_1 + \dfrac{c_1^2}{2\left(g + \frac{v^2}{a}\right)} + a\,(1 - \cos.\lambda)$:

and hence again we have the width of the wheel:

7. $e = \frac{Q}{d_1\, c}$, and the radius of the curvature of the floats:

8. $r = \frac{d}{cos.\ \epsilon}$, and the angle ϕ:

9. $sin.\ \frac{1}{2}\phi = \sqrt{cos.\ \epsilon}$, and lastly the length of the water arc,

10. $$b = 2\lambda a = \left(\phi + \frac{\phi + sin.\ \phi}{8}\right) v \sqrt{\frac{r}{g + \frac{v^2}{a}}};$$

and from this the accurate value of:

11. $$\lambda = \left(\phi + \frac{\phi + sin.\ \phi}{8}\right) \frac{\pi u}{60} \sqrt{\frac{r}{g + \frac{v^2}{a}}}.$$

Even after these values have been found, the calculations may be repeated on the more accurate foundations.

Example. It is required to ascertain the general proportions of a Poncelet undershot wheel. *Given.* the height of fall 4,5 feet, the quantity of water 40 cubic feet per second. If we make the radius $a = 2h = 9$ feet, and allow the thickness of the stream $d_1 = \frac{1}{6}\, h = 0,75$ feet, and further, $\mu = 0,90$, then the velocity of discharge $c = 0,9\, \sqrt{2g\,(h - d_1)}$ $= 0,9 \times 8,02\, \sqrt{3,75} = 7,218 \times 1,936 \times 14$ feet; and, therefore, the velocity of the wheel, as also the initial velocity of the water, is approximately $v = c_1 = \frac{1}{2}\, c = 7$ feet. Hence the depth of shrouding is, nearly,

$d = \frac{1}{4} \cdot \frac{c^2}{2g} + d_1 = \frac{1}{4} \cdot 3,04 + 0,75 = 1,51$ feet, and the arc $\lambda = \delta = \frac{\pi \cdot 7}{2 \cdot 9} \sqrt{\frac{1,51}{32,2 + \frac{7^2}{9}}}$ $= 0,24$, and the angle λ° corresponding $= 14^\circ$, for which, however, we shall take 15°. If we now introduce this value of δ, we get, more accurately, $v = \frac{1}{2}\, c\ cos.\ \delta = 7\ cos.\ 15^\circ$ $= 6,762$ feet, and hence, the number of revolutions $u = \frac{30\, v}{\pi\, a} = 7,17$. It follows, therefore, that $tang.\ \epsilon = 2\ tang.\ \delta = 2t.\ 0,26795 = 0,53590$, $\therefore \epsilon = 28^\circ\ 11\frac{1}{4}'$, and, therefore, $c_1 = \frac{67,62}{cos.\ 28^\circ,\ 11\frac{1}{4}'} = 7,67$ feet. Again, we have the depth of shrouding $d = 0,75 + 9$ $(1 - cos.\ 15^\circ) + \frac{7,6^2}{2\,(32,2 + \frac{1}{8} \cdot 6,76^2)} = 1,845$ feet; and the width of the wheel $e = \frac{40}{0,75t.\ 14} = 3,80$ feet. The radius of curvature of the floats $r = \frac{1,845}{cos.\ 28^\circ,\ 11\frac{1}{4}'}$ $= 2,093$ feet, and $sin.\ \frac{1}{2}\phi = \sqrt{cos.\ 28^\circ,\ 11\frac{1}{4}'}$ $\therefore \frac{1}{2}\phi^\circ = 69^\circ,\ 51\frac{1}{2}'$, and $\therefore \phi^\circ = 139^\circ,\ 43'$, $\phi = 2,4385$, $sin.\ \phi = 0,6466$; and, lastly,

$$\lambda = \left(2,4385 + \frac{2,4385 + 0,6466}{8}\right) \times \frac{6,76}{18} \sqrt{\frac{2,093}{36,52}} = 2,824 \times 0,3697 \sqrt{\frac{2,093}{36,52}}$$

$= 0,2499$, and $\lambda^\circ = 14^\circ,\ 19'$, for which $14\frac{1}{2}^\circ$ would be substituted in the actual construction of the wheel, so that the length of the water arc, or the length of the concentric curb $b = 2\,\lambda\, a = 18 \cdot 0,253 = 4\frac{1}{2}$ feet, or $2\frac{1}{4}$ feet on each side of the lowest point of the wheel.

§ 123. ***Experiments with Poncelet's Wheels.***—Poncelet himself instituted experiments on the useful effect of his water wheels. These are minutely detailed, and their results ascertained in his work above cited.

The first experiments were made with a model wheel of 20 inches diameter. It was of wood, had 20 floats, about $\frac{1}{10}$ of an inch thick, $2\frac{3}{4}$ inches deep, and 3 inches wide. The greatest effects were produced when the velocity of the wheel = 0,5 that of the water, as indicated by theory, and then the efficiency was 0,42 to 0,56, the former when the water stream was kept thin, the latter when this was increased, or the cells of the wheel better filled. Reckoning the efficiency by the height due to the velocity of the water, and not by the actual fall, the effect rises to 0,65 to 0,72.

Poncelet afterwards experimented on a water wheel erected on his principle, measuring the effect by means of a friction brake, and the results are very much the same as those obtained from the model. The wheel was 11 feet in diameter, and had 30 plate-iron floats of $\frac{1}{8}$ inch thickness. The shroudings, arms, and axle of the wheel were of wood. The shrouding was 14 inches deep, and 3 inches thick, the distance between them, or width of the wheel, 28 inches. For a mean head of 4′ — 4″, and 8 inches depth of water stream, the ratio of the velocities being 0,52, the efficiency came to 0,52, which gives 0,60, when the height due to the velocity, instead of the total fall, is made the basis of calculation. Poncelet makes the following deductions from his series of experiments.

The best velocity ratio $\frac{v}{c}$ is 0,55;* but this may vary between 0,50 and 0,60 without material diminution of the useful effect. For falls of 6′ — 6″ to 7′ — 6″, the efficiency $\eta = 0,5$, for falls of 5 feet to 6′ — 6″, the efficiency $\eta = 0,55$, and for falls of less than 5 feet $\eta = 0,60$.

The useful effect may, therefore, be represented, in the first case, by:

$Pv = 0,96\,(c - v)\,v\,Q$ ft. lbs., in the second:

$Pv = 1,06\,(c - v)\,v\,Q$ ft. lbs., and in the third:

$Pv = 1,15\,(c - v)\,v\,Q$ ft. lbs.

Poncelet gives the following general rules for the construction and arrangement of his wheels, deduced from his experiments. The distance between 2 floats, at their outer extremity, should not exceed 8 to 10 inches, and the radius of the wheel should not be less than 3′ — 4″ (1 metre), nor more than 8′ — 2″ ($2\frac{1}{2}$ metres). The axis of the water stream should meet the periphery of the wheel at an angle of 24° to 30°, and be inclined about 3° to the horizon. The offset at the end of the curb should be sufficient to insure the water's free escape from the wheel, and the space left between the wheel and the curb would not exceed $\frac{3}{8}$ inch.

According to the experiments, the efficiency increases with the depth of the water stream laid on, and, therefore, *cæteris paribus*, as the filling of the cells. Further experiments prove that the degree of filling of the cells is an important element in the question.

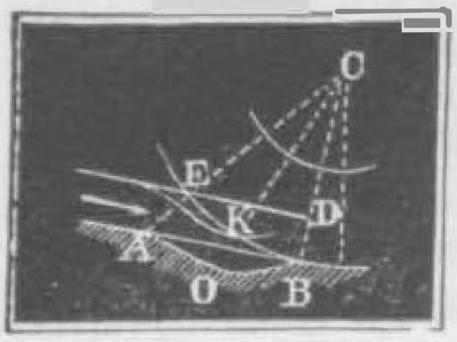
§ 124. *Recent Experiments.*—Morin has quite recently instituted

---

* [This is the same ratio as that found by the Committee of the Franklin Institute for the velocity of an overshot wheel with elbow buckets.—AM. ED.]

experiments with three wooden and one iron wheel, constructed on Poncelet's principle, using the friction-brake. They were made with the special object of testing the advantages of a curvilinear course for laying on the water, proposed by M. Poncelet; as also for the purpose of getting more exact information as to the influence of the relative dimensions of the wheels, for in several wheels that have been erected according to Poncelet's rule, it is found that, when the deviation from the mean velocity is considerable, the water overruns the floats. (See *Comptes Rendus*, 1845, t. XXII.) As to the curved water course, its object was to lay the whole of the water on to the wheel without impact, and not the top or bottom stratum only. When the water stream is straight $ABED$, Fig. 233, the upper layer of water DE meets the periphery of the wheel, as also the float, at a different angle from that at which the lower stratum does; so that if one enters without impact, the other cannot do so. If, however, we hollow out the bottom of the course as AOB, the water stream comes upon a smaller arc BK, and the difference in the direction of the periphery of the wheel and the layers of water is less, and, therefore, the impact is less than when the water stream embraces the arc BE.

Fig. 233.



The three wooden wheels were respectively $5' - 3''$, $8' - 3''$, and $10' - 3''$ in diameter. The diameter of the iron wheel was $9' - 3''$. The buckets were of sheet iron. The first three wheels were 16 inches wide, and the other was 32 inches. The depth of shrouding was 30 inches. It was found that wooden wheels, having very little inertia, moved unsteadily, and hence arose a loss of water. The smallest wheel revolved very unsteadily, and for a fall of 18 to 22 inches, the cells being at least half filled, the efficiency was 0,485. Had the weight of the wheel been greater, its efficiency would probably have been 0,55. The second wheel, having a fall of 30 inches, gave an efficiency = 0,60 to 0,62. The third wheel was used to make experiments on different lengths of floats. It appeared that for a fall of 22 inches, a length of 17 inches, and for a fall of 28 inches, a length of 24 inches, is too little. Poncelet's curved lead was adapted to this wheel, and it was found that the efficiency was increased, and also that the degree to which the cells are filled, might he made $\frac{2}{3}$ without inconvenience.

The experiments with the iron wheel were instituted with falls of 4 feet to $4\frac{1}{2}$ feet, and of 3 feet, the wheel being free from back-water, and with a fall of 15 inches, the wheel being *in* back-water. For sluice-openings of 6 inches, 8 inches, 10 inches, and 11 inches, the maximum efficiency was 0,52, 0,57, 0,60, and 0,62 respectively, and for variations in the number of revolutions between the limits of 12 to 21, 13 to 21, 11 to 20, and 12 to 19, the efficiency did not differ more than $\frac{1}{18}$, $\frac{1}{14}$, $\frac{1}{12}$, and $\frac{1}{9}$ from the maximum values. From the results of these experiments, it follows that, for wheels with the *hollow* water-lead, the effect is expressed by the formula:

$$Pv = 0,871\left(\frac{c^2 - w^2}{2g}\right) Q\gamma.$$

Also, that the best velocity ratio $\frac{v}{c} = 0,5$ to $0,55$. That the same effect is produced, whether the water in the race be 5 inches below, or 8 to 10 inches above the bottom of the wheel—that the efficiency falls as low as 0,46, if the wheel be in back-water to the depth of half the depth of the shrouding. The main advantage of the new form of lead is, that the wheel may vary its velocity of rotation within wider limits, without material diminution of the efficiency. Morin considers that, for falls of 3 feet to 4 feet, a breadth of shrouding equal to the half of the radius is a good proportion to adopt, and that the capacity of the wheel should be double that corresponding to the water to be laid on, *i. e.*, the co-efficient of filling $\epsilon = \frac{Q}{dev}$ should be made $= \frac{1}{2}$.*

Remark. It would thus appear that the capacity of the wheel treated in our last example is too small, and that it would have been better to have made $d_1 = 0,5$ feet, and $e = 5,71$ feet.

§ 125. *Small Wheels.* — Some other vertical water wheels have been applied, besides the systems we have now discussed. Very small wheels of 2 or 3 feet diameter, are moved by the pressure or impact of water.

D'Aubuisson describes, in his "Hydraulique," small impact wheels *ACB*, Fig. 234, with falls of 6 to 7 metres, often to be met with in the Pyrenees. These wheels are from 7 to 10 feet in diameter, and have 24 hollowed floats. Their effect is about ,73 of that of an overshot wheel of the same fall. The effect of such a wheel may be calculated by the theory of breast wheels above given, for these wheels are nothing more than breast wheels with a great impact fall and small height, during which the water can act by its weight. To prevent the spilling of the water, the wheels are hung in a curb with close-fitting sides. Such wheels may be very neatly made of iron, and are to be found in North Wales. This kind of wheel is very commonly employed at the forges in the Alps.

Fig. 234.

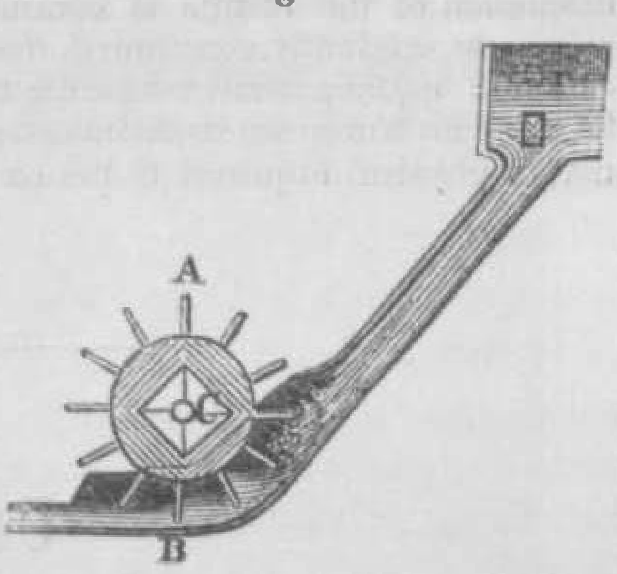


Fig. 235.

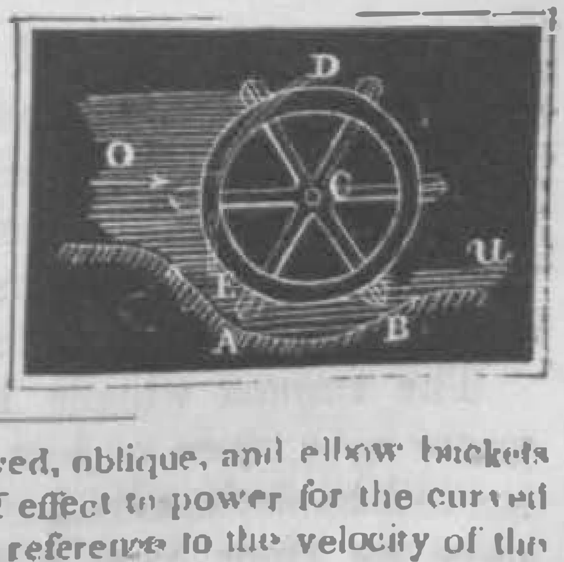


Fig. 235 represents a wheel erected by Mr. Mary, and described in the "Tech-

* [The Committee of the Franklin Institute tried curved, oblique, and elbow buckets successively on the same wheel. They found the ratio of effect to power for the curved buckets nearly equal to that for elbow buckets, while in reference to the velocity of the wheel they are much inferior. Elbow buckets gave 5.6, curved 4.2, and oblique 3.7 feet per second velocity of wheel.—Am. Ed.]

nologiste, Sept., 1845." The water here works chiefly by pressure. Belanger experimented with the wheel, and reported an efficiency of 0,75 to 0,85 for a velocity of 4 feet per second. The wheel consists of a shrouding of plate iron, 13 inches wide and 5 inches deep, and 7′ — 6″ in diameter, and having six elliptical floats strengthened by ribs.

The curb is made to fit very accurately, and sheet iron fenders, fitting close to the wheel, prevent the water in the lead from escaping into the race. The power with which such a wheel revolves, is, of course, the product of the weight of water, measured by the difference of level in the lead and in the race, by the area of the float.

Literature. The literature treating of vertical water wheels is very extensive; but there are few works upon the subject worthy of much attention, as the most of them give very superficial and even erroneous views of the theory of these wheels. Eytelwein, in his "Hydraulik," treats very generally of water wheels. Gerstner, in his "Mechanik," treats very fully of undershot wheels. Langedorf's "Hydraulik" contains little on this subject. D'Aubuisson, in his work "Hydraulique à l'usage des Ingénieurs," treats very fully of overshot wheels. Navier treats water wheels in detail in his "Leçons," and in his edition of "Belidor's Architecture Hydraulique." In Poncelet's "Cours de Mécanique appliquée," the theory of water wheels is briefly, but very clearly, set forth. In the "Treatise on the Manufactures and Machinery of Great Britain," P. Barlow has given details on the *construction* of water wheels, but has not entered into the theory of their effects, &c. Very complete drawings and descriptions of good wheels are given in Armengaud's "Traité pratique de Moteurs hydrauliques et à vapeur." Nicholson's "Practical Mechanic," contains some useful information on this subject. The most complete work hitherto published on vertical water wheels is Redtenbacher's "Theorie und Bau der Wasserräder, Manheim, 1846." Poncelet's and Morin's Memoirs have been already cited.

[The experiments of the Franklin Institute are contained in the Journal of that institution for 1831–2 (vols. 7, 8, & 9), and for 1841. In the last mentioned volume, the discussion of the results is commenced, but has not yet been completed. The committee, as originally constituted, does not appear to have given its attention to the application of mathematical reasoning to the observations made and experiments performed. Subsequent European experiments have consequently, in this respect, occupied the attention of physical inquirers to the exclusion of the American.—AM. ED.]

CHAPTER V.

OF HORIZONTAL WATER WHEELS.

§ 126. IN horizontal water wheels, the water produces its effect either by *impact*, by *pressure*, or by *reaction*, but never directly by its weight. Hence, horizontal water wheels are classified as impact wheels, hydraulic pressure wheels, and reaction wheels. These wheels are now very commonly designated by the generic term *turbines* (Ger. *Kreiselräder*).

The *impact* wheels have plane or hollow pallets, on which the water acts more or less perpendicularly. The *pressure* wheels have curved buckets, along which the water flows, and the *reaction* wheels have as their type a close pipe, from which the water discharges