# Modeling of Soil Profile Produced by a Single Sweep Tool

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## ABSTRACT

An improved mathematical model of soil profile by tillage with a single sweep was developed and validated using experimental data from soil bin tests. This model was based on a soil displacement model developed in a previous study. The parameters involved in the soil displacement model were soil properties, tillage depth, tillage speed, and tool parameters. Tool parameters include dimensions, rake angle, and sweep angle. The model parameters describing soil profile included ridge height, ridge spacing, furrow depth, and the width of soil distribution. The improved model was validated with experimental results from two different soil bins filled with soils with different soil textures. Tillage tools were the same, a single 325-mm-wide sweep. The sweep was operated at 100-mm deep and speeds of 5, 7.5, and 10 km/h. Predicted and measured soil profiles had the same trend with respect to tillage speed. Results showed that the model had maximum error of 16% for all parameters predicted. The maximum error of this improved model was reduced from previous error of 40% for the original model developed in a previous study to 16%.

**Keywords**: Soil, profile, tillage, model, soil disturbance

## 1. INTRODUCTION

Soil profile or soil redistribution after tillage operation is important in several aspects such as incorporating manure and crop residues and protecting soil from wind and water erosion. The study of soil profile and soil redistribution by tillage has progressed slowly due to its complexity which involves many factors, such as soil types and properties, types of tillage tools and their operational parameters. Dowell et al. (1988) conducted a study with sweeps and found that the ridge height and the lateral distance increased with increasing travel speed. Hanna et al. (1993) also indicated that higher speed and larger rake angle of sweep resulted in more soil movement creating higher ridges. Sharifat and Kushwaha (1999) studied soil lateral movement under different soil conditions with a sweep and a furrow opener at speeds from 5 to 8 km/h, and they concluded that different tools created different geometries of soil profiles; the parameters of soil profile were also affected by tillage speed, soil bulk density, and soil moisture content. They proposed an index of lateral soil movement, but no mathematical models were developed. McKyes (1985) described the results of a soil disturbance study, and indicated that the shape, width and rake angle of tools strongly influence transporting and mixing of soil particles; soil throwing to the sides of a tool varied with the square of tillage speed. It is imperative to develop physically-based mathematical models and to quantitatively describe the soil profile after tillage operation. These models will assist in new tool design for a variety of purposes.

A mathematical model of soil movement by a single sweep was developed (Liu et al., submitted for publication), and it was validated with results from a soil bin and field experiments. The parameters involved in this model were tillage depth and speed, soil properties, geometric parameters of a sweep, and surface straw. Soil forward displacement and lateral displacement can be calculated using this model.

Based on the model of soil displacement, a mathematical model of soil profile by tillage with a single sweep was proposed (Liu, 2005) as shown in Figure 1. The average lateral displacement calculated with the model of soil displacement was used as the index of lateral soil movement ($b_0$). The profile of a ridge ($A_1TA_2$) was simplified as an isosceles triangle. Point *C* is associated with the central point of right wing of the sweep's two wings. Point *D* is the average lateral displacement of soil. The parameters describing the soil profile include ridge spacing ($2b_0$), ridge height ($h$), width of soil distribution ($2b$), furrow depth ($d_1$), and ridge angles ($\varphi = \varphi_i = \varphi_o$). The ridge angles were also assumed as constant. Consequently, the modeled ridge or the isosceles triangle will "shift" to a wider area with increasing tillage speed, and the ridge height and the furrow depth change correspondingly. Compared to the measured results, the error of this model was around 20% when tillage speed was 5 km/h; but, much higher errors resulted at a speed of 10 km/h. The ridge height had the largest error, 40%, among all the parameters studied. This large error was not acceptable.

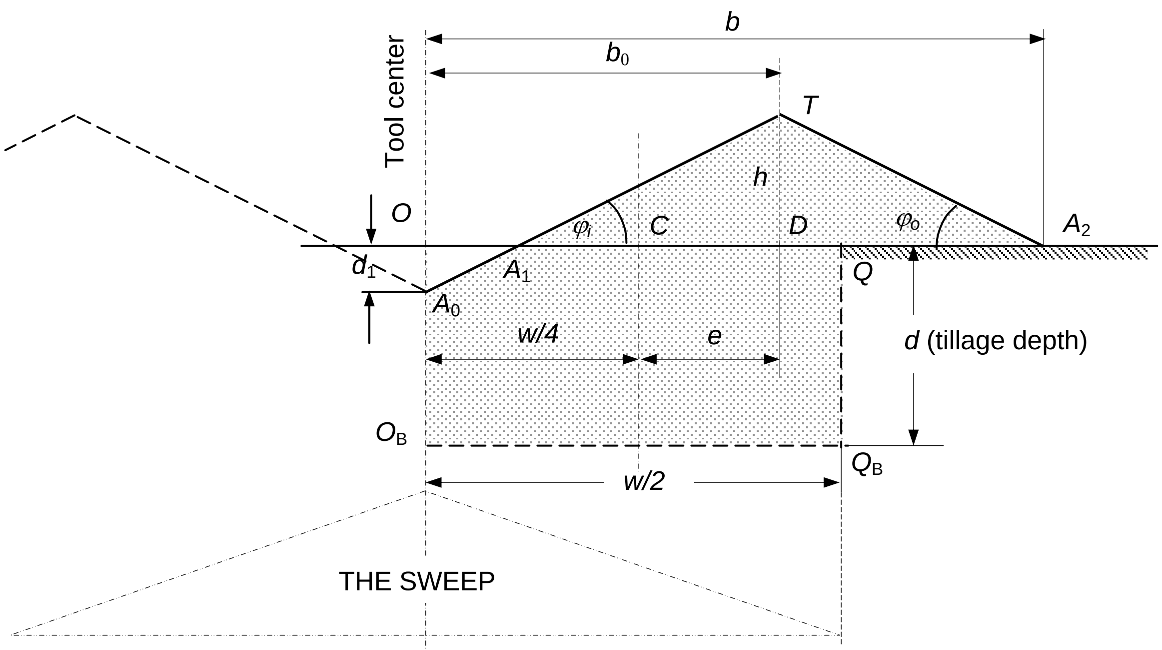


Figure 1. Geometric relations of a soil profile after tillage with a single sweep (*d*: tillage depth; *w*: tool width; *e*: average lateral soil displacement).

The error of this model was mainly caused by the assumption of isosceles triangular shape of a ridge. Sharifat and Kushwaha (1999) measured soil profile in a soil bin using the same sweep (McKay 50-12K) with the one used in this study. The results indicated that the soil profile could be described as an isosceles triangle if the tillage speed was 5 km/h. However, these results indicated that the outer ridge angle $\varphi_o$ tended to be smaller than the inside ridge angle with increasing tillage speed. The assumption of a isosceles triangle was not appropriate. Therefore, the goal of this study was to improve the model of soil profile after tillage with a sweep. The objectives of this study were:

a. To study the effect of operating speed of the soil profile by a single sweep tool in a soil bin;
b. To improve the mathematical model of the soil profile with a single sweep tool proposed in a previous study; and,
c. To validate the improved model with data from soil bin tests.

## 2. EXPERIMENTAL STUDY

### 2.1 Material and Methods

#### 2.1.1 Soil Bin and Tillage Tool

An experiment was conducted in an indoor soil bin (1.75-m width and 12.2-m length) located in the Department of Agricultural and Bioresource Engineering at the University of Saskatchewan, Canada. The maximum travel speed of the tool carriage was 16 km/h. The bin was filled with a sandy loam soil (sand 78%, clay 10%, silt 12%) to a 300-mm depth. Soil processing equipment included a rotary tiller for loosening soil, a water sprayer for controlling soil moisture content, a sheep-foot roller and a plain roller for packing soil. Tillage tool was a 325-mm-wide sweep (McKay 50-12K) as shown in Figure 2.

Figure 2. A 325-mm-wide sweep (McKay 50-12K) used in this study.

#### 2.1.2 Experimental design

The experimental variable was tillage speed. Tillage depth was kept constant at 100 mm for all the runs. Three tillage speeds, 5, 7.5, and 10 km/h were selected to get comparable results with data obtained in previous studies. After the tillage operation at each speed, five cross-sections along the direction of tool travel were randomly selected, and the soil profile of each cross-section was measured. These five sections were treated as replications. Soil bulk density was 1280 kg/m$^3$ resulting from the packing process, and the moisture content was 12% (w.b.).

### 2.1.3 Measurements and Measuring Device

Soil profiles were measured using a laser line scanner (AccuRange 4000, Acuity Research Incorporated, Menlo Park, CA 94025), which consisted of a rail, a laser sensor, and a stepping motor. The scanner was mounted on the rear frame of the tool carriage. The laser sensor was powered by the stepping motor to move along the lateral direction of the tool travel. Prior to measurements, a piece of cubic wood was placed on the level soil surface to calibrate the laser sensor. The scanner measured the distance from the laser sensor to the soil surface stopping at any given location. To acquire accurate data and save experimental time, steps of 10-mm and 20-mm were used. The 10-mm step was used at the areas of ridge tops and furrow bottoms, and the 20-mm step was used in the remaining areas.

Using measured soil profile data, the parameters of soil profile, namely $b$, $b_0$, $h$ and $d_1$ were determined using the average values of left and right ridges, Figure 3. The ridge angles $\varphi_1$ and $\varphi_o$ were determined with linear regression of all measured points between $C$ and $F$ and between $F$ and $G$.

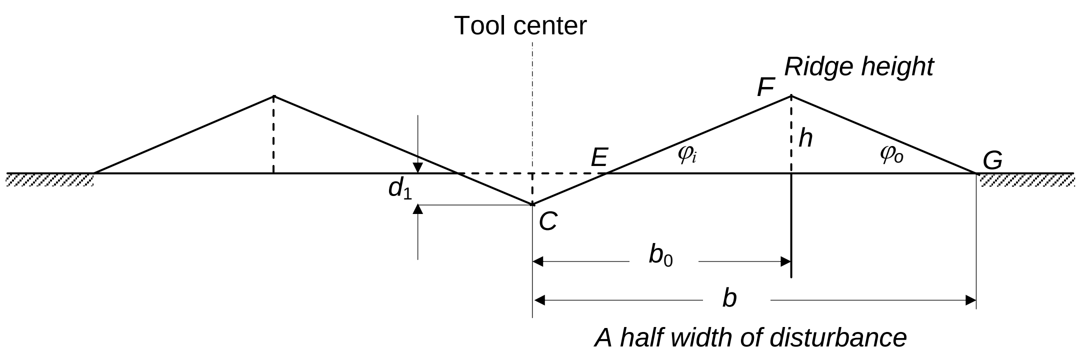


Figure 3. Geometric parameters used to determine soil profile after tillage

## 2.2 Results and Discussion

Measured soil profiles with five replications at three speeds are shown in Figures 4 through 6. Results indicated that soil was redistributed in a wider and flatter area with increasing tillage speed, and the ridge height slightly decreased, but not significantly (P – 0.05). The increase in tillage speed resulted in more soil being tossed away and an increased furrow depth ($d_1$ in Table 1). Hence, the width of soil disturbance became wider. The ridge spacing ($2b_0$) and the width of soil distribution ($2b$) were significantly increased with increasing tillage speed (Table 1). The inside ridge angle ($\varphi_i$) was significantly decreased when tillage speed increased to 7.5 km/h; but, it did not differ with that at tillage speed of 10 km/h. The outside ridge angle ($\varphi_o$) was reduced when increasing tillage speed and it was smaller than $\varphi_i$. As a result, the ridge tended to be flattened at higher tillage speeds.

Figure 4 shows that the ridge profile is very close to an isosceles triangle at the speed of 5 km/h. However, there is no evidence to suggest the assumption of an isosceles triangle at the speeds of 7.5 and 10 km/h (Figures 5 and 6). This indicated that the model developed in the previous study was not suitable for tillage speed higher than 5 km/h.

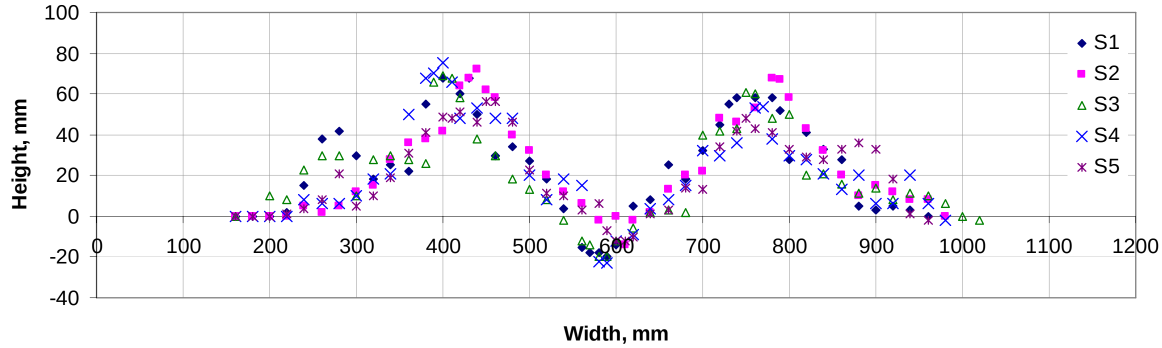


Figure 4. Measured soil profile of a single sweep operated at 100 mm deep and 5 km/h speed.

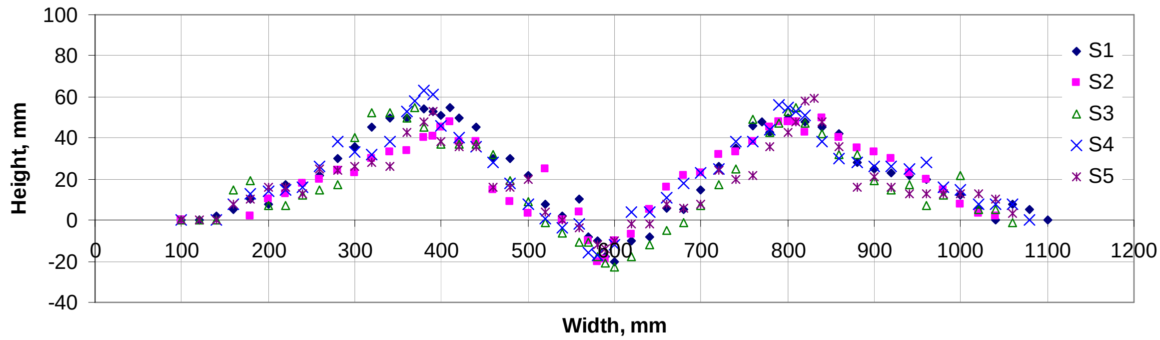


Figure 5. Measured soil profile of a single sweep operated at 100 mm deep and 7.5 km/h speed.

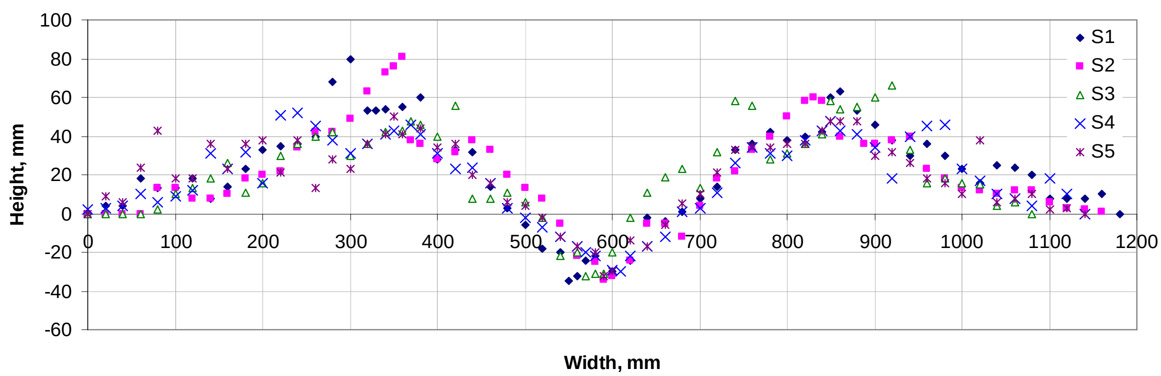


Figure 6. Measured soil profile of a single sweep operated at 100 mm deep and 10 km/h speed.

Table 1. Some parameters measured from the experiment

| Speed | $b_0$ | $b$ | $h$ | $d_1$ | $\varphi_i$ |
|---|---|---|---|---|---|
| km/h | mm | mm | mm | mm | degree |
| 5 | 172 (13[1]) a[2] | 388 (22) a | 63 (8) a | 14 (2) a | 24 (1) a |
| 7.5 | 214 (20) b | 472 (21) b | 54 (4) a | 19 (3) a | 18 (2) b |
| 10 | 250 (25) c | 564 (38) c | 58 (12) a | 32 (2) b | 19 (2) b |

[1] Standard deviation of ten replications;

[2] Results followed with the same letters in the same column show no significant difference detected by ANOVA Duncan test at significant level of 0.05

# 3. MODEL DEVELOPMENT

## 3.1 Modeling the Soil profile of a Single Sweep

The mathematical model of soil displacement developed in previous study (Liu, 2005) provided a method of calculating soil displacement by simplifying soil movement into three phases: forced, projectile, and rolling and sliding. The process of soil redistribution by a sweep was modified when compared to the previous model described in Figure 1. The improved model assumed soil aggregates landed at location *F* (Figure 7), and then began rolling and/or sliding. However, aggregates rolling and sliding was not always equal for all soil aggregates while forming a ridge. It was noted that the soil rolling and sliding displacement described here designates the lateral average displacement of all soil aggregates. At the beginning, the soil surface was flat and relatively smooth; soil clods could roll or slide easily. Once some soil aggregates were distributed over, the soil surface became rough. Consequently, the distance of aggregates rolling and sliding was reduced by the rough surface; the rolling and sliding displacement at the bottom layer of a ridge had a full range, but it was zero at the ridge top. Aggregates rolling and sliding displacement was considered linearly distributed between the top and bottom of a ridge. As a result, the asymmetrical part (*FGH* in Figure 7) in the profile triangle was caused by aggregates rolling and sliding. Triangle *EFH* corresponds to the profile, isosceles triangle $A_1TA_2$, in Figure 1.

A trench was formed between two ridges, and the two ridges were symmetrical to the tool center. The ridge shown in Figure 7 has two bottom angles: $\varphi_i$ and $\varphi_o$. The angle $\varphi_i$ is called the inside ridge angle and the $\varphi_o$ is referred to as the outside ridge angle. The parameters describing a soil profile include the ridge spacing ($2b_0$), the width of soil distribution ($2b$), the ridge height ($h$), and the furrow depth ($d_1$).

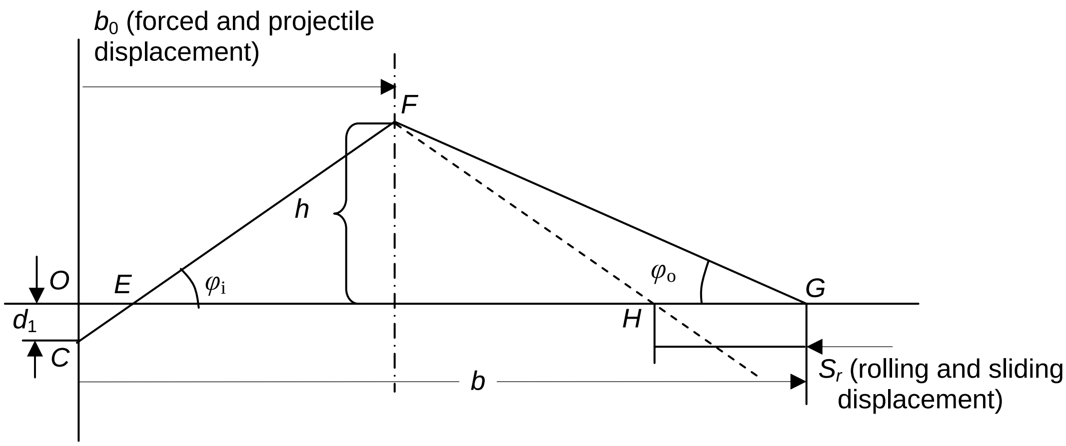


Figure 7. Improved geometric relations of soil profile after tillage operation with a sweep tool ($S_r$: average lateral displacement of soil rolling/sliding).

**3.2 Hypotheses**

We assumed that the soil volume is expanded after the tool operation. This is illustrated as follows. The soil mass in the tool region $OO_BQ_BQ$ in Figure 1 is assumed of a number of cubes with an edge length of *a* (Figure 8). When the tool passes, these cubes are randomly redistributed, and each cube occupies a space of a ball with the diameter of $\sqrt{2}a$. All the cubes are rearranged as a stack of balls. The spaces between the balls are eliminated. Due to each cube occupies an extra space, the redistribution of these soil cubes results in a volume expansion. To model the soil profile as a two dimensional problem, the soil expansion is considered in a plane perpendicular to the tool travel. In this case, the cubes are squares in the plane. The stack of balls becomes a stack of circles. Therefore, the extra area occupied by each square is:

$$\Delta A_0 = \left(\frac{\pi}{2} - 1\right)a^2 \quad (1)$$

Total number of cubes is $\frac{0.5w}{a} \cdot \frac{d}{a}$, the total area expansion (*ΔA* shown in Figure 8) is then:

$$\Delta A = \frac{1}{2}\left(\frac{\pi}{2} - 1\right)wd \quad (2)$$

where *w* is tool width; *d* is tillage depth.

The expanded area is distributed in the ridge area *EFG* (Figure 7). In addition, the soil located in the area of *OCE* is also contributed to the ridge area. That is, the expanded area *ΔA* plus area *OCE* are equal to the area of *EFG*.

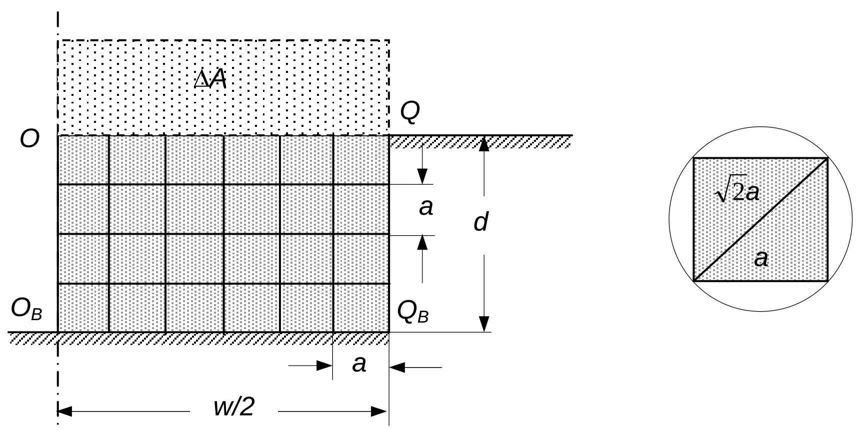


Figure 8. Assumption of soil expansion after a tool operation.

**3.3 Analytical Model**

The soil displacements to be used in determining soil profiles can be calculated using following equations (Liu, 2005).

$$S_f = d\,\frac{\cos(\beta - \xi) - \cos\beta}{2\sin\beta} + \frac{w}{4}(1 - \cos\xi) \quad (3)$$

$$S_p = y - y_0 = \frac{v_y}{g}\left(\sqrt{v_z^2 + 2z_0 g} + v_z\right) \tag{4}$$

$$S_r = \frac{v_y^2}{2\mu g} \tag{5}$$

$$\begin{cases} v_y = \left\{\left(\frac{\sin\alpha}{\sin\beta}v_t + \frac{\sqrt{2}}{2}\alpha v_t\right)\sin(\beta-\alpha) + \frac{\sqrt{2}}{2}\xi v_t\right\}\cos(\beta-\xi) \\ v_z = \left\{\left(\frac{\sin\alpha}{\sin\beta}v_t + \frac{\sqrt{2}}{2}\alpha v_t\right)\sin(\beta-\alpha) + \frac{\sqrt{2}}{2}\xi v_t\right\}\sin(\beta-\xi) \end{cases} \tag{6}$$

According to the simplified ridges, relationships of various parameters used in the model shown in Figure 7 are derived as Equations 7 to 11. The final format of the model does not include the dimension of the soil clod though it was assumed in the hypothesis.

$$b_0 = w/4 + S_f + S_p \tag{7}$$

$$h = \frac{1}{2}\left(\sqrt{(2b_0 + S_r)^2 + (2\pi - 4)wd\cot\varphi_i + 4b_0^2} - (2b_0 + S_r)\right)\tan\varphi_i \tag{8}$$

$$d_1 = b_0 \tan\varphi_i - h \tag{9}$$

$$b = b_0 + h\cot\varphi_i + S_r \tag{10}$$

$$\varphi_o = \tan^{-1}\left(\frac{h}{h\cot\phi_i + S_r}\right) \tag{11}$$

where,

| | |
|---|---|
| $x_0$, $y_0$, $z_0$ | the coordinates of the center point on the rear edge of one wing, m |
| $S_f$ | lateral component of forced displacement, m |
| $S_p$ | lateral component of projectile displacement, m |
| $S_r$ | lateral average of rolling and sliding displacement, m |
| $d$ | tillage depth, m |
| $g$ | gravitational acceleration, m/s$^2$ |
| $w$ | tool width, m |
| $v_t$ | tool travel velocity, m/s |
| $v_y$ | lateral component of velocity of soil moving, m/s |
| $v_z$ | vertical component of velocity of soil moving, m/s |
| $\alpha$ | sweep rake angle, rad |
| $\beta$ | soil failure angle, $\beta = 45° - \phi/2$, rad |
| $\phi$ | soil internal friction angle, rad |
| $\xi$ | lateral incline angle of the sweep, or incline angle to $y$-axis, rad, and |
| $\mu$ | frictional coefficient between soil clods and soil surface. |

## 4. MODEL VALIDATION

### 4.1 Model Calculation

To calculate a soil profile, a Cartesian coordinate was established to represent the sweep and calculate soil lateral displacement. The sweep shown in Fig. 2 is simplified as a tetrahedron (*ABGC*), which is symmetrical about the *x*-axis, and placed in the coordinates as shown in Figure 9. The sweep bottom (*ABG*) is located at the *x*-*y* plane; the origin of the coordinates is placed at the tool centerline and passing the vertical plane of the rear edge of the tool. The rake angle is represented by $\alpha$, and angle $\xi$ denotes the lateral incline of a wing. Coordinate *x* represents the direction of tool travel or forward movement, *y* is a direction of lateral movement, and *z* is the vertical direction. A soil block slides from point *D* on the front edge of the sweep to the rear edge and is projected at point *P*. Given the coordinates of any point on rear edge *CB* or *CG*, its displacement can be calculated using Equations 8 to 11. The displacement of the center point of rear edge *CB* was calculated and treated as soil average displacement in this paper.

Using the predicted soil lateral displacement, the soil profile of a single 325-mm-wide sweep was determined with Equations 3 to 7. Equation 7 shows that one of two ridge angles is required to determine the soil profile. To verify the hypothesis in Section 3.2, measured inside ridge angle ($\varphi_i$) values listed in Table 1 were used as inputs of the profile calculation.

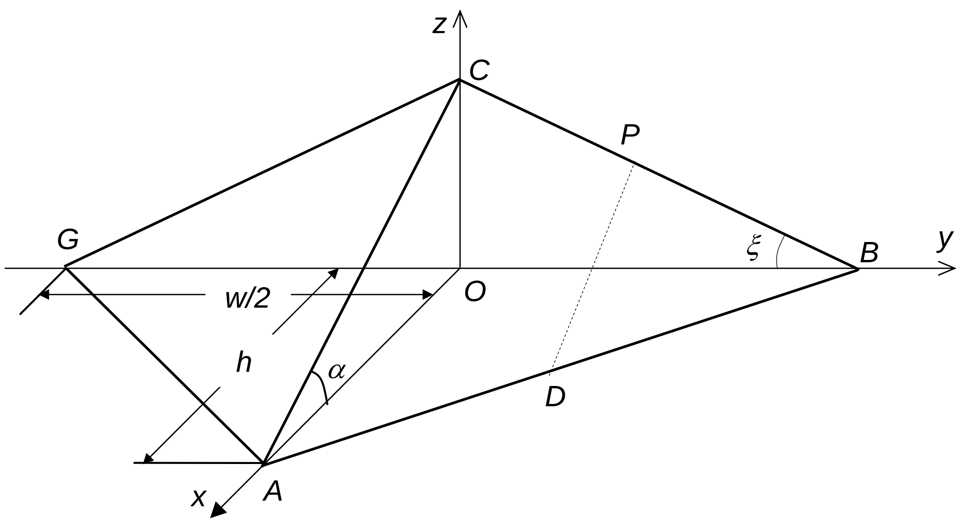


Figure 9. A Cartesian coordinate and the simplified sweep tool.

### 4.2 Validation Using Results of Soil Bin Test

Measured soil profile in the soil bin test was used to validate the model of soil profile. Predicted parameters and measured values of the soil profile are given in Table 2 for comparisons. Experimental results indicated that the width of soil distribution significantly increased with increasing tillage speed. The measured ridge height tended to decrease if increasing tillage speed though there was no significant difference detected. The predicted ridge height also tended to decrease with increase in tillage speed. The comparison showed that the model of soil profile correctly predicted the width of soil distribution (2*b*), and its relative error was 14% or lower for the speeds of 5, 7.5 and 10 km/h. Predicted ridge height *h* and ridge spacing $2b_0$ had maximum error of 10% and 3% respectively. For the furrow depth $d_1$, the maximum relative error of 16% occurred at tillage speed of 5 km/h. The changes of soil profile at different tillage speeds were determined using the model and were found to be the same as measured in the soil bin tests. It could be concluded that the soil profile model worked well in the speed range of 5 to 10 km/h. These results also verified the hypothesis of soil volume expansion made in Section 3.2.

### 4.3 Validation Using Previous Experimental Data

The soil profile model was also validated with results measured in another soil bin experiment. The soil bin was located in the Department of Biosystems Engineering at the University of Manitoba. Tillage tool was identical with the sweep shown in Figure 2. The sweep was operated at 5, 7.5, and 10 km/h speeds and 100-mm deep. The soil was loamy (fine) sand with 16.3% moisture content and 1250 kg/m$^3$ bulk density. The parameters of soil profile were recalculated according to the soil conditions. Predicted and measured results are listed in Table 3. The inside ridge angle ($\varphi_i$) was estimated with the ridge angle of that soil, 25º which was used as $\varphi_i$ in the calculation for all three tillage speeds. The comparison indicated that the maximum error in the three parameters of soil profile was 11% for all three tillage speeds. However, the maximum error of the old model shown in Figure 2 was 40%. This indicates that the model proposed in this paper is improved.

Table 2. Comparison of predicted and measured parameters of soil profile of a single 325-mm-wide sweep in the soil bin. Soil: (coarse) sandy loam, tillage depth: 100 mm

| | Tillage speed (km/h) | 5 | 7.5 | 10 |
|---|---|---|---|---|
| | Measured | 388 *a* [2] | 472 *b* | 564 *c* |
| | (Standard deviation[1]) | (22) | (21) | (38) |
| *b* (mm) | Predicted | 334 | 419 | 504 |
| | Relative error (%) | 14 | 11 | 11 |
| | Measured | 63 *a* | 54 *a* | 58 *a* |
| | (Standard deviation) | (8) | (4) | (12) |
| *h* (mm) | Predicted | 61 | 53 | 52 |
| | Relative error (%) | 3 | 2 | 10 |
| | Measured | 172 *a* | 214 *b* | 250 *c* |
| | (Standard deviation) | (13) | (20) | (24) |
| $b_0$ (mm) | Predicted | 168 | 208 | 251 |
| | Relative error (%) | 2 | 3 | 0.4 |
| | Measured | 14 a | 19 a | 32 b |
| | (Standard deviation) | (2) | (3) | (2) |
| $d_1$ (mm) | Predicted | 11 | 17 | 31 |
| | Relative error (%) | 16 | 9 | 3 |

[1] Measured results were calculated from five replicates for $d_1$, Ten replications for other parameters;
[2] Results followed with the same letters in the same row show no significant difference was detected by ANOVA with Duncan test at significant level of 0.05

Table 3. Comparison of predicted and measured (Liu, 2005) parameters of soil profile of a single 325-mm-wide sweep in the soil bin. Soil: loamy (fine) sand, tillage depth: 100 mm

| | Tillage speed (km/h) | 5 | 7.5 | 10 |
|---|---|---|---|---|
| *B* (mm) | Measured | 358 *a*[2] | 412 *b* | 488 *c* |
| | (Standard deviation[1]) | (31) | (14) | (18) |
| | Predicted | 320 | 395 | 489 |
| | Relative error (%) | 11 | 4 | 0.2 |
| *H* (mm) | Measured | 72 *a* | 69 *a* | 69 *a* |
| | (Standard deviation) | (10) | (12) | (42) |
| | Predicted | 72 | 73 | 75 |
| | Relative error (%) | 4 | 5 | 10 |
| $B_0$ (mm) | Measured | 172 *a* | 222 *b* | 258 *b* |
| | (Standard deviation) | (2.1) | (13.2) | (10.8) |
| | Predicted | 169 | 211 | 255 |
| | Relative error (%) | 2 | 5 | 1 |

[1] All measured results were calculated from six replicates;
[2] The same letters followed the measured results in the same row show no significant difference was detected by ANOVA with Duncan test at significant level of 0.1

## 5. DISCUSSION

The model of soil profile for a single sweep was developed and validated with experimental results from two different soil bin experiments. The model included the speed and depth of tool operation, but the validation was completed at a constant depth only.

The importance of this model is its application in sustainable crop productions. One example is straw burial. If the purpose of tillage operation was only to bury all the straw, it can be accomplished by adjusting tool spacing to 2$b$. To partially bury straw, the tool spacing can be arranged between $w$ and 2$b$ depending on the amount of straw to be kept on the soil surface.

For sweep type manure injectors, manure is injected at the sweep bottom. This model can be applied to this case to predict the depth of manure burial, which is an indicator of odor control. The depth of manure burial, tillage depth minus a furrow depth $d_1$, varies if tillage speed changes. This model will also help model soil and water erosion by predicting ridge dimensions of a soil profile.

Tool parameters such as rake angle, incline angle, and width would affect soil displacement and would impact soil distribution. For designing a new sweep tool, this model can be used to simulate the desired soil distribution for specific application by varying the tool parameters. In real situations, multiple tools are used. The adjacent tools will throw soil back and forth that would further impact the soil profile from the one developed in this study using a single tool. The soil profile resulting from multiple tools would be the objective of future studies.

The inside ridge angle was used as a given parameter in model calculation. This angle was affected by tillage speed as shown in Table 1. The validation in Section 4.1 used measured values, which were different for three tillage speeds. The validation in Section 4.3, however, used the ridge angle of the soil to calculate those parameters of soil profile. Validation results showed that the ridge angle worked well for this model prediction. For a rough estimation, the ridge angle of soil can be used to predicting soil profile. Thus, methods of determining the inside ridge angle ($\varphi_i$) for the model calculation remains unclear. The suggested solution may be to study the impact of tillage speed on the angle ($\varphi_i$) under different soils and soil conditions.

## 6. CONCLUSIONS

Soil profile of a single sweep was mathematically modeled and validated with results from soil bin tests. The main conclusions were as follows:

- The mathematical model of soil profile by the tillage with a single sweep was improved by modifying the process of forming soil ridges. The geometry of a soil ridge changed from an isosceles triangle to a triangle with two different ridge angles.
- Experimental results indicated that ridge spacing increased with increasing tillage speed, but the ridge height decreased slightly. The total width of soil distribution significantly increased with increase in tillage speed. The inside ridge angle was reduced at higher tillage speed, and it was always larger than outside ridge angle.
- The improved model was validated with result from two soil bin tests. Both predicted and measured parameters of a soil profile varied similarly with increasing tillage speed.
- Predicted results were close to those measured values at tillage speeds of 5, 7.5, and 10 km/h. The maximum relative error for all the parameters of this improved model was 16%. Compared to 40% for a soil profile model developed in a previous study.

## 7. ACKNOWLEDGEMENT

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