nologiste, Sept., 1845." The water here works chiefly by pressure. Belanger experimented with the wheel, and reported an efficiency of 0,75 to 0,85 for a velocity of 4 feet per second. The wheel consists of a shrouding of plate iron, 13 inches wide and 5 inches deep, and 7′ — 6″ in diameter, and having six elliptical floats strengthened by ribs.

The curb is made to fit very accurately, and sheet iron fenders, fitting close to the wheel, prevent the water in the lead from escaping into the race. The power with which such a wheel revolves, is, of course, the product of the weight of water, measured by the difference of level in the lead and in the race, by the area of the float.

*Literature.* The literature treating of vertical water wheels is very extensive; but there are few works upon the subject worthy of much attention, as the most of them give very superficial and even erroneous views of the theory of these wheels. Eytelwein, in his "Hydraulik," treats very generally of water wheels. Gerstner, in his "Mechanik," treats very fully of undershot wheels. Langedorf's "Hydraulik" contains little on this subject. D'Aubuisson, in his work "Hydraulique à l'usage des Ingénieurs," treats very fully of overshot wheels. Navier treats water wheels in detail in his "Leçons," and in his edition of "Belidor's Architecture Hydraulique." In Poncelet's "Cours de Mécanique appliquée," the theory of water wheels is briefly, but very clearly, set forth. In the "Treatise on the Manufactures and Machinery of Great Britain," P. Barlow has given details on the *construction* of water wheels, but has not entered into the theory of their effects, &c. Very complete drawings and descriptions of good wheels are given in Armengaud's "Traité pratique de Moteurs hydrauliques et à vapeur." Nicholson's "Practical Mechanic," contains some useful information on this subject. The most complete work hitherto published on vertical water wheels is Redtenbacher's "Theorie und Bau der Wasserräder, Manheim, 1846." Poncelet's and Morin's Memoirs have been already cited.

[The experiments of the Franklin Institute are contained in the Journal of that institution for 1831-2 (vols. 7, 8, & 9), and for 1841. In the last mentioned volume, the discussion of the results is commenced, but has not yet been completed. The committee, as originally constituted, does not appear to have given its attention to the application of mathematical reasoning to the observations made and experiments performed. Subsequent European experiments have consequently, in this respect, occupied the attention of physical inquirers to the exclusion of the American.—AM. ED.]

# CHAPTER V.

## OF HORIZONTAL WATER WHEELS.

**§ 126.** In horizontal water wheels, the water produces its effect either by *impact*, by *pressure*, or by *reaction*, but never directly by its weight. Hence, horizontal water wheels are classified as impact wheels, hydraulic pressure wheels, and reaction wheels. These wheels are now very commonly designated by the generic term *turbines* (Ger. *Kreiselräder*).

The *impact* wheels have plane or hollow pallets, on which the water acts more or less perpendicularly. The *pressure* wheels have curved buckets, along which the water flows, and the *reaction* wheels have as their type a close pipe, from which the water discharges more or less tangentially. Pressure wheels and reaction wheels are generally very similar to each other in construction, the essential difference between them being, that in the former the cells or conduits between two adjacent buckets are not filled up by the water flowing through them, while in reaction wheels the section is quite filled.

According to the different directions in which the water moves in the conduits of pressure and reaction wheels, two systems arise. The relative motion of the water in the conduits is either horizontal, or in a plane inclined to the horizon, and usually vertical.

In the first system, there are to be distinguished those wheels in which the water flows from the *interior* to the *exterior*, and those in which the water takes the opposite course; and in the second system, there are the distinct cases of the water flowing from above downwards, and that in which it flows from below upwards.

Horizontal water wheels in which the water flows from above downwards, are often named *Danaïdes*.

§ 127. *Impact Wheels.*—*Impact turbines*, as shown in Fig. 236, are the simplest, but also the least efficient form of impact wheels. They consist of 16 to 20 rectangular floats $AB$, $A_1B_1$, &c., so set upon the wheel as to incline 50° to 70° to the horizon. The water is laid on to them by a pyramidal trough $EF$, inclined from 40° to 20°, so that the water impinges nearly at right angles to the floats. Such wheels are employed for falls of from 10 to 20 feet, when a

Fig. 236.

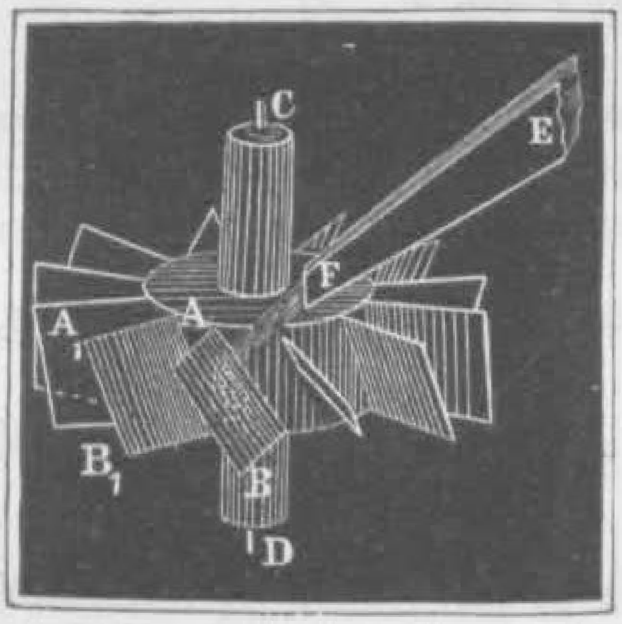
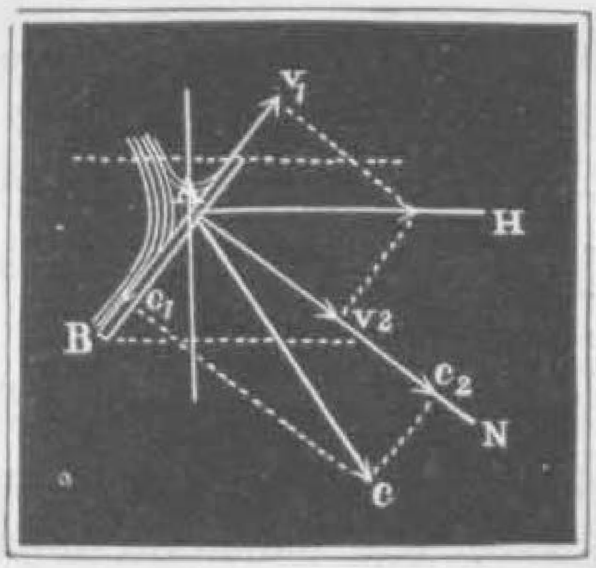



Fig. 237.




great number of revolutions is desired, and when simplicity of construction is a greater desideratum than efficiency. Wheels of this form are met with in all mountainous countries of Europe, and in the north of Africa, applied as mills for grinding corn. They are made from 3 to 5 feet in diameter, the buckets being about 15 inches deep, and 8 to 10 inches long.

The mechanical effect of these wheels is determined according to the theory of the impact of water, as follows. The velocity $Ac = c$, Fig. 237, of the water impinging, and the velocity $Av = v$ of the buckets may be each decomposed into two velocities expressed by the formulas

$c_1 = c\ sin.\ \delta$, $c_2 = c\ cos.\ \delta$, $v_1 = v\ sin.\ \alpha$, and $v_2 = v\ cos.\ \alpha$, $\delta$ being the angle $c\,AN$, by which the direction $Ac$ of the stream of water deviates from the normal $AN$, and $\alpha$ the angle $HAN$ at which the normal is inclined to the horizon, or by which the direction of the wheel's motion deviates from the normal, or the plane of the bucket from the vertical. The component velocity $c_1 = c\ sin.\ \delta$, remains unchanged, as its direction coincides with that of the plane of the bucket; the component $c_2 = c\ cos.\ \delta$, is, on the other hand, changed by impact into $v_2 = v\ cos.\ \alpha$, as the bucket moves away in the direction of the perpendicular with this velocity. The water, therefore, loses by impact a velocity

$$c_2 - v_2 = c\ cos.\ \delta - v\ cos.\ \alpha,$$

and the corresponding loss of effect $= \frac{(c\ cos.\ \delta - v\ cos.\ \alpha)^2}{2g} Q\gamma$. If, now, we deduct from the whole available mechanical effect $\frac{c^2}{2g} Q\gamma$, the above, and further, the effect, $\frac{(c\ cos.\ \delta - v\ cos.\ \alpha)^2}{2g} Q\gamma$, and $\left(\frac{c^2\ sin.\ \delta^2 + v^2\ cos.\ \alpha^2}{2g}\right) Q\gamma$, which the water flowing away with the velocity $w = \sqrt{c_2\ sin.\ \delta^2 + v^2\ cos.\ \alpha^2}$, retains, the mechanical effect communicated by the wheel is

$$L = Pv = [c^2 - (c\ cos.\ \delta - v\ cos.\ \alpha)^2 - (c^2\ sin.\ \delta^2 + v^2\ cos.\ \alpha^2)]\ \frac{Q\gamma}{2g}$$

$$= \frac{(c\ cos.\ \delta - v\ cos.\ \alpha)\ v\ cos.\ \alpha}{g} \cdot Q\gamma.$$

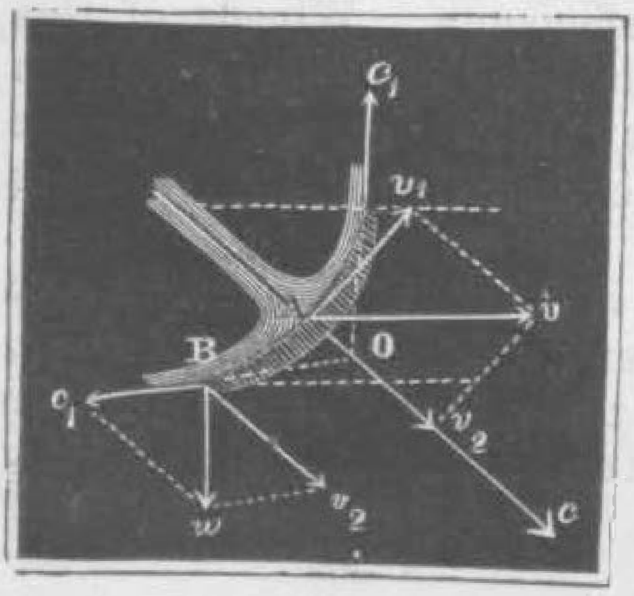
To get the maximum effect, we must make $cos.\ \delta = 1$, or $\delta = 0$, or direct the stream at right angles to the bucket, and besides this, as in other similar cases already treated, we must make $v\ cos.\ \alpha = \frac{1}{2} c$, or $v = \frac{c}{2\ cos.\ \alpha}$. The maximum effect corresponding, is $Pv = \frac{1}{2}\frac{c^2}{2g} Q\gamma = \frac{1}{2} h\, Q\gamma$, or the half of the entire mechanical effect available.

§ 128. The effect of impact wheels is increased by surrounding the buckets with a projecting border or frame, or by forming them like spoons, as shown in Fig. 238. Vol. I. § 385 explains the cause of this increased effect, but we may here determine the amount of this increase. As the bucket moves in the direction of the stream with the velocity $v_2 = v\ cos.\ \alpha$, the relative velocity of the water in reference to the bucket may be put:

$$c_1 = c - v_2 = c - v\ cos.\ \alpha,$$

and if $\beta =$ the angle $c_1\,O\,c$, by which the water is turned aside from its ori-

Fig. 238.



ginal direction, the absolute velocity of the water flowing off:

$$w = \sqrt{c_1^2 + v_2^2 + 2\, c_1\, v_2 \cos.\, \beta}$$
$$= \sqrt{(c - v \cos.\, \alpha)^2 + v^2 \cos.\, \alpha^2 + 2\,(c - v \cos.\, \alpha)\, v \cos.\, \alpha \cos.\, \beta},$$

and hence, the corresponding loss of effect:

$$= [c^2 - 2\,(c - v \cos.\, \alpha)\, v \cos.\, \alpha\, (1 - \cos.\, \beta)]\, \frac{Q\gamma}{2g},$$

and the effect of the wheel:

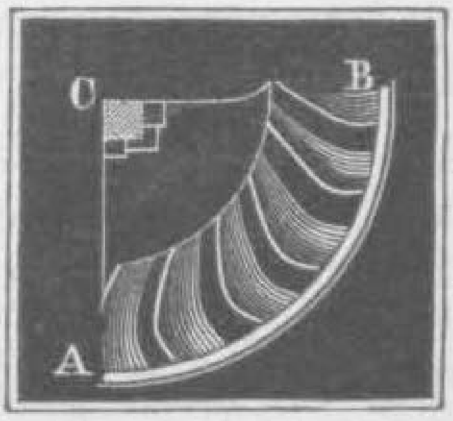
$$L = Pv = \left(\frac{c^2 - w^2}{2g}\right) Q\gamma = (1 - \cos.\, \beta)\, \frac{(c - v \cos.\, \alpha)\, v \cos.\, \alpha}{g}\, .\, Q\gamma.$$

When the buckets are plane, $\beta = 90°$, $\therefore \cos.\, \beta = 0$, and, therefore

$$L = \frac{(c - v \cos.\, \alpha)\, v \cos.\, \alpha}{g}\, Q\gamma,$$

as we have already found, though by an entirely different method of inquiry. In the case of hollow buckets, $\beta$ is greater than 90°, and, therefore, $\cos.\, \beta$ is negative, and hence $1 - \cos.\, \beta$ is greater than 1, consequently the effect is greater than in plane buckets.

Fig. 239.




To this class of wheels belong those termed in France *rouets volants*, upon the effect of which MM. Piobert and Tardy have recorded experiments in a work entitled "Expériences sur les Roues hydrauliques à axe vertical, &c., Paris, 1840." The following are results of experiments on a small wheel of 5 feet diameter, 8 inches high, having 20 curved buckets, Fig. 239, with a fall of 14 feet (measuring from surface of water in lead to bottom of wheel), and with 10 cubic feet of water per second:

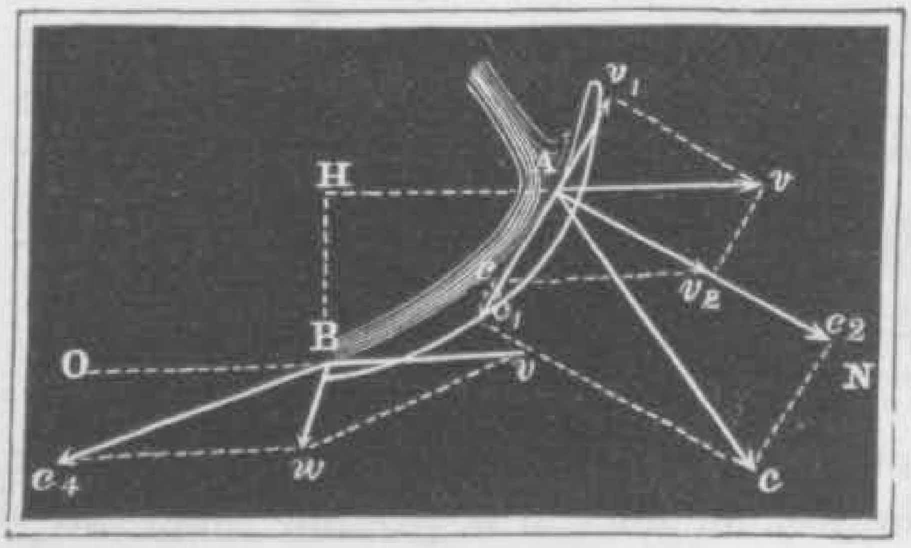
$$\text{For } \frac{v}{c} = 0{,}72,\ \eta = 0{,}16;$$
$$\text{“ } \frac{v}{c} = 0{,}66,\ \eta = 0{,}31;$$
$$\text{“ } \frac{v}{c} = 0{,}56,\ \eta = 0{,}40;$$

and hence, in cases in which the velocity ratio $\frac{v}{c}$ does not much differ from 0,6 : $Pv = 0{,}75\,(c - v \cos.\, \alpha)\, \frac{v \cos.\, \alpha}{g}\, Q\gamma.$

*Example.* What effect may be expected from an impact turbine with hollow buckets (Fig. 239), there being 6 cubic feet of water, and a fall of 16 feet at disposition? If we neglect the depth of the wheel itself, the theoretical velocity of entrance of the water $c = \sqrt{2gh} = 8{,}02 \sqrt{16} = 32{,}08$ feet, and if the inclination of the trough be assumed as 20°, the most advantageous velocity for the wheel $v = \frac{c}{2 \cos.\, \alpha} = \frac{16{,}04}{\cos.\, 20°} = 17$ feet and hence, from the above formula, the effect attainable is $L = Pv = 0{,}75\, .\, \frac{c\, v \cos.\, \alpha - v^2 \cos.\, \alpha^2}{g}\, Q\gamma = \frac{3}{4}\, .\, 031\, .\, (512{,}09 - 15{,}96^2)\, .\, 6\, .\, 62{,}5 = {,}023 \times 257.4\, .\, 375 = 2220$ feet lbs.

§ 129. *Impact and Reaction Wheels.*—If we give the buckets greater length, and form them to such a hollow curve, that the water leaves the wheel in a nearly horizontal direction, the water then not only impinges on the bucket, but exerts a pressure on it, and, therefore, the effect of the wheel is greater than in the impact wheel. The theory of such wheels is merely an extension of that given in § 127. If we conceive a normal erected at the point of entrance $A$, Fig. 240, and if we again put the angle $c\,AN = \delta$, and the angle $v\,AN = \alpha$, we have the lost velocity arising from impact:

Fig. 240.




$c_2 - v_2 = c \cos.\ \delta - v \cos.\ \alpha$, and the loss of effect corresponding $= \dfrac{(c \cos.\ \delta - v \cos.\ \alpha)^2}{2g} Q\gamma.$

The velocity with which the water begins to flow down the buckets is $c_1 + c_3 = c \sin.\ \delta + v \sin.\ \alpha$, and if we put the height $BH$, through which the water descends on the bucket $= h_1$, we have the relative velocity of the water at the bottom $B$ of the bucket:

$$c_4 = \sqrt{(c_1 + c_3)^2 + 2gh_1} = \sqrt{(c \sin.\ \delta + v \sin.\ \alpha)^2 + 2gh_1}.$$

But the water possesses the velocity $v$ in common with the wheel, and, therefore, the absolute velocity of the water flowing from the wheel $w = \sqrt{c_4^2 + v^2 - 2\, c_4\, v \cos.\ \theta}$, where $\theta =$ the angle $c_4BO$, at which the lowest element of the bucket is inclined to the horizon. The loss of effect corresponding to this is:

$$\frac{w^2}{2g} Q\gamma = \left(\frac{c_4^2 + v^2 - 2\, c_4\, v \cos.\ \theta}{2g}\right) Q\gamma.$$

If we deduct these two losses from the whole available effect, we get the useful effect communicated to the wheel

$$L = Pv = [c^2 - (c \cos.\ \delta - v \cos.\ \alpha)^2 - (c_4^2 + v^2 - 2\, c_4\, v \cos.\ \theta)] \frac{Q\gamma}{2g},$$

in which we have to substitute for $c_4$ the value above given.

If the water impinges at right angles $\delta = 0$, and $c_4 = \sqrt{v^2 \sin.\ \alpha^2 + 2gh_1}$, and, therefore,

$$L = [c^2 - (c - v \cos.\ \alpha)^2 - (c_4^2 + v^2 - 2\, c_4\, v \cos.\ \theta)] \frac{Q\gamma}{2g}$$

$$= [2\, c\, v \cos.\ \alpha - (1 + \cos.\ \alpha^2)\, v^2 - v^2 \sin.\ \alpha^2 - 2gh_1 + 2\, v \cos.\ \theta\ .\ \sqrt{v^2 \sin.\ \alpha^2 + 2gh_1}]\frac{Q\gamma}{2g}$$

$$= [(c \cos.\ \alpha - v)\, v - gh_1 + v \cos.\ \theta \sqrt{v^2 \sin.\ \alpha^2 + 2gh_1}]\frac{Q\gamma}{2g}.$$

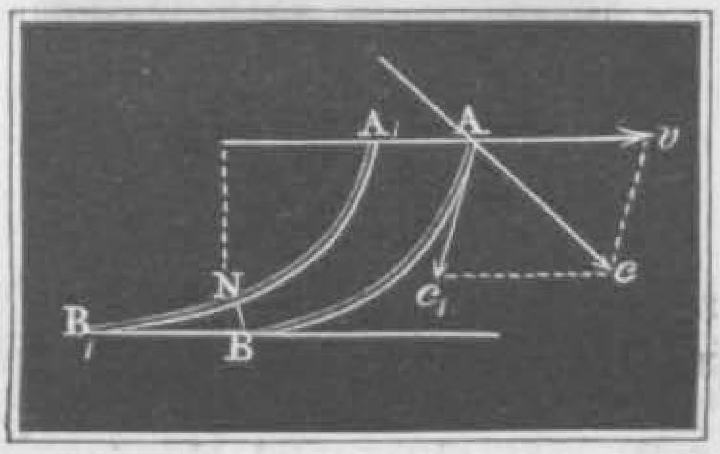
In order that the water may produce its maximum effect, it should fall dead from the wheel, or $w$ should $= 0$. This requires that $\Theta = 0$, and $c_4 = v$, i. e., $v^2 \, sin. \, \alpha^2 + 2gh_1 = v^2, \therefore v = \frac{\sqrt{2gh_1}}{cos. \, \alpha}$.

§ 130. *Pressure Wheels.*—If the water is to be laid on without impact, then $v \, cos. \, \alpha$ must $= c \, cos. \, \delta$, and in order that the water may quit the wheel, deprived of its *vis viva*, we must have: $\Theta = 0$, and $c_4 = v$, i. e. $(c \, sin. \, \delta + v \, sin. \, \alpha)^2 + 2gh_1 = v^2$, or $c^2 \, sin. \, \delta^2 + 2 \, c \, v \, sin. \, \alpha \, sin. \, \delta + 2gh_1 = v^2 \, cos. \, \alpha^2 = c^2 \, cos. \, \delta^2$. If, again, we subtract from both sides: $2 \, c \, v \, cos. \, \alpha \, cos. \, \delta = 2 \, c^2 \, cos. \, \delta^2$, then: $c^2 \, sin. \, \delta^2 - 2 \, c \, v \, (cos. \, \alpha \, cos. \, \delta - sin. \, \alpha \, sin. \, \delta) + 2gh_1 = - c^2 \, cos. \, \delta^2$, or $c^2 + 2gh_1 = 2 \, c \, v \, cos. \, (\alpha + \delta)$, and, therefore,

$$v = \frac{c^2 + 2gh_1}{2 \, c \, cos. \, (\alpha + \delta)} = \frac{g \, (h + h_1)}{c \, cos. \, \phi},$$

in which $h$ is the velocity of the water at entrance, and, therefore, $h + h_1$ the whole fall, $\phi$ is the angle $c \, A \, v$ between the direction of the water and that of the wheel. The theoretical effect is, in the latter case, $= (h + h_1) \, Q\gamma$, and the efficiency $\eta = 1$, because there is no loss from any cause. When, for such a wheel, the best velocity of rotation $v$ has been found, we get the requisite position of the buckets by drawing through the point of entrance $A$, Fig. 241, a line parallel to $v \, c$, completing the parallelogram $A \, v \, c \, c_1$. The side $A \, c_1$, thus given, gives the relative velocity $c_1$, with which water begins to descend along the bucket, in magnitude and direction, and also the direction of the upper element of the bucket.

Fig. 241.




That the water may flow unimpeded through the openings $BB_1$, &c., the foot of the buckets must have a slight inclination to the horizon. If we put the mean radius of the wheel $= a$, and the mean length of the buckets, measured on the radius, $= l$, we may put the section of the orifice of discharge $= \overline{BN} \, . \, l = \overline{BB_1} \, sin. \, \Theta \, . \, l$, and, therefore, the section of the united orifices of the wheel $= 2 \, \pi \, a \, l \, sin. \, \Theta$. If, again, $c_2 =$ the relative velocity with which the water arrives at the bottom of the wheel, or, if

$$c_2 = \sqrt{c^2 + v^2 - 2 \, c \, v \, cos. \, \phi + 2gh_1},$$

we have $2 \, \pi \, a \, l \, sin. \, \Theta = \frac{Q}{c_2}$, and, therefore, for the requisite angle

$$sin. \, \Theta = \frac{Q}{2 \, \pi \, a \, l \, c_2}.$$

*Remark.* According to the theory of the impact of water, or of *hydraulic pressure*, expounded in our first volume, it is not necessary that $v \, cos. \, \alpha = c \, cos. \, \delta$, or, which amounts to the same, that the component $c_1$ of the velocity, should fall in the direction of the bucket. According to Vol. I. § 43, the relative velocity $c_1$ of the water in reference to the bucket $AB$, Fig. 242, is the diagonal of the parallelogram constructed from the absolute velocity of the water $c$ and the velocity of the wheel $v$, taken in the opposite direction; therefore, $c_1 = \sqrt{c^2 + v^2 - 2\,c\,v\cos\phi}$. If, now, the direction, but not the magnitude, of this velocity be changed by the shock on the bucket, we have the relative velocity at discharge, after descent through the height $BH = h_1$, $c_2 = \sqrt{c_1^2 + 2gh_1} = \sqrt{c^2 + v^2 - 2\,c\,v\cos.\,\phi + 2gh_1}$: lastly, that the whole effect may be taken up from the water, we have to makee

$$c_2 = v, \text{ or } c^2 + v^2 - 2\,c\,v\cos.\,\phi + 2gh_1 = v^2, \text{ therefore,}$$

$$v = \frac{c^2 + 2gh^1}{2\,c\cos.\,\phi} = \frac{g\,(h + h_1)}{c\cos.\,\phi}.$$

§ 131. *Borda's Turbine.*—The wheels discussed in the last paragraph, are called *Borda's* turbines, from their having been the suggestion of that distinguished officer and philosopher. Their con-

Fig. 242.

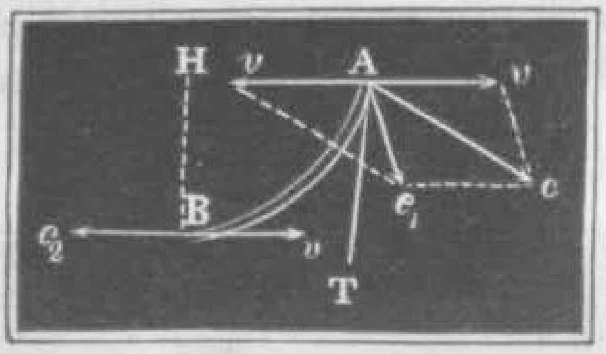
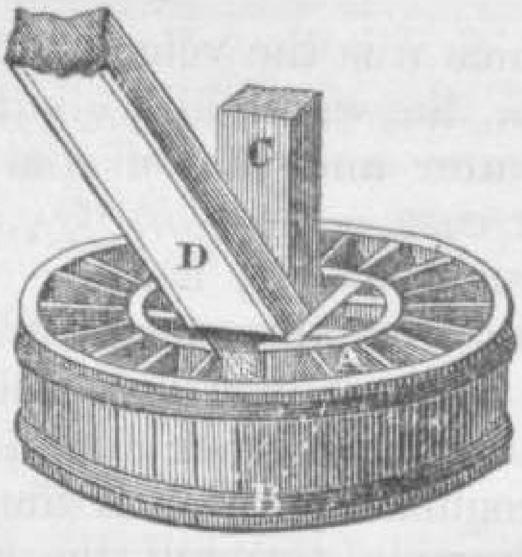



Fig. 243.




struction is shown by Fig. 243, which is a sketch of one driving 6 amalgamation barrels, at the silver mines of Huelgoat in Brittany. The curved buckets are composed of three beech boards put carefully together, and the inner and outer casings are composed of staves, the outer one being bound by two iron hoops. The diameter of the wheel is 5 feet. The buckets 14 inches long or deep, and $16\frac{1}{2}$ inches wide. There are 20 of them. The fall was $16' - 3''$, and the wheel makes 40 revolutions per minute.

There are no good experiments on the efficiency of Borda's turbines. Borda gives 0,75 of the theoretical effect as the useful effect, or $L = 0{,}75\,.\,[h + h_1 - (c\cos.\,\delta - v\cos.\,\alpha)^2 - w^2]\;Q\,\gamma$. Poncelet very justly remarks that it is advisable to make the diameter and the height of the wheels as great as possible, so as to curtail the length of bucket, that is, bringing the outer and inner casings near to each other. By giving height to the wheel, the fall due to the velocity is diminished, and, therefore, the velocity of the water and of the wheel is less. By keeping the *diameter* great, the number of revolutions falls out less, and as for a larger wheel, the capacity remaining the same, the width of the wheel may be less, and then the difference of velocity of the particles of water adjacent to each other will be less.

*Example.* What quantity of water must be supplied to a Borda's turbine, constructed as shown in Fig. 243, which, with a fall of 15 feet, is to drive a pair of millstones requiring 2 horse power? Suppose the wheel to be $1\frac{3}{4}$ feet high, then the theoretical velocity of entrance of the water:

$$c = 8{,}02\sqrt{15 - 1{,}75} = 8{,}02\sqrt{13{,}25} = 29{,}19 \text{ feet.}$$

If the water be laid on at an angle of 30° to the horizon, then the least velocity of rotation is : $v = \frac{g(h + h_1)a}{c \cos. \phi} = \frac{32,2 \times 15}{29,19 \times \cos. 30°} = 19,1$. If the water enters without shock, the velocity with which it begins its descent along the bucket is : $c_1 = \sqrt{c^2 + v^2 - 2\,c\,v \cos. \phi} = \sqrt{c^2 + v^2 - c^2 - 2gh_1} = \sqrt{v^2 - 2gh_1} = 15,88$ feet. For the angle $\psi$, at which the upper element of the buckets must incline to the horizon, we have: $\frac{\sin. \psi}{\sin. \phi} = \frac{c}{c_1} \therefore \sin. \psi = \frac{29,19}{15,88} \sin. 30° = 0,9189 \therefore \psi = 66°, 46'$. If we give the bottom of the bucket an inclination of 25° to the horizon, we get for the absolute velocity of the water flowing away :

$$w = 2\,v \sin. \frac{\theta}{2} = 2 \,.\, 19,1 \sin. 12\tfrac{1}{2}° = 8,2 \text{ ft.},$$

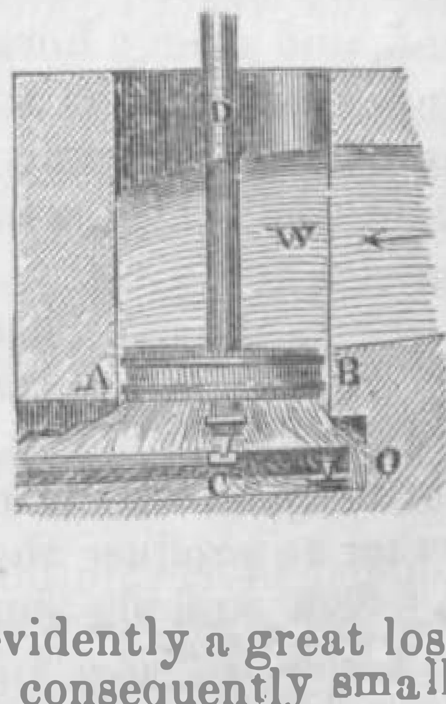
and, hence, the effect:

$$L = \tfrac{1}{2}\left(h + h_1 - \frac{w^2}{2g}\right) Q\gamma = \tfrac{1}{2}\left(15 - \frac{8,2^2}{2g}\right) . 62,5\, Q = 654\, Q.$$

That we may have 2 horse power, or 1100 feet pounds per second, we must have $\frac{1100}{652} = 1,7$ cubic feet of water per second. If the mean radius (measured to the centre of the buckets) of the wheel be 2 feet, and if the water space be 6 inches wide, we get the united areas of section of the orifices of discharge at the bottom of the wheel $= 2\,\pi\,a\,l \sin. \theta = \pi \,.\, 4 \,.\, \tfrac{1}{2} \sin. 25° = 2,65$ square feet, which is quite sufficient to pass 1,7 cubic feet of water per second, with a velocity of 19 feet.

§ 132. *Roues en Cuves.*—To this category of turbines belong those horizontal wheels enclosed in a pit or *well*, frequently met with in the south of France, and called *roues en cuves* (Ger. *Kufenräder*). They are described by Belidor in the "Architecture Hydraulique," by D'Aubuisson in his "Hydraulique," and Piobert and Tardy, in the work already cited, have given the results of experiments instituted on one of these wheels. These wheels are very similar in form to those last described (Fig. 239). They are generally 1 metre in diameter, and have 9 curved buckets. They are made of only two pieces, and are bound together by iron hoops. The axis *CD* (Fig. 244) stands on a pivot, the footstep of which is on a lever *CO*, by which the wheel may be raised and lowered as the millstone may require. The wheel is near the bottom of a well, 2 metres deep, and 1,02 metres in diameter. The water comes into the well by a lead laid tangentially to it, about 13 feet long, the breadth at the outer extremity being 2′ — 6″, and at the entrance to the well about 10 inches. The water flows in with a great velocity, acquires a rotary motion in the wheel chamber, and acts by impact and pressure on the wheel buckets, flowing through it into the tail-race. There is evidently a great loss of water in such wheels, and their efficiency is consequently small. Piobert and Tardy found an efficiency of 0,27 for a *well* wheel at Toulouse, the fall being 10 feet, with $13\frac{1}{2}$ cubic feet of water per second, and the number of revolutions $u = 100$. For $u = 120$, the efficiency $\eta$ was $= 0,22$, and for $u = 133$, $\eta = 0,15$. The wheels of the Basacle mill, at Toulouse, give an efficiency of 0,18.

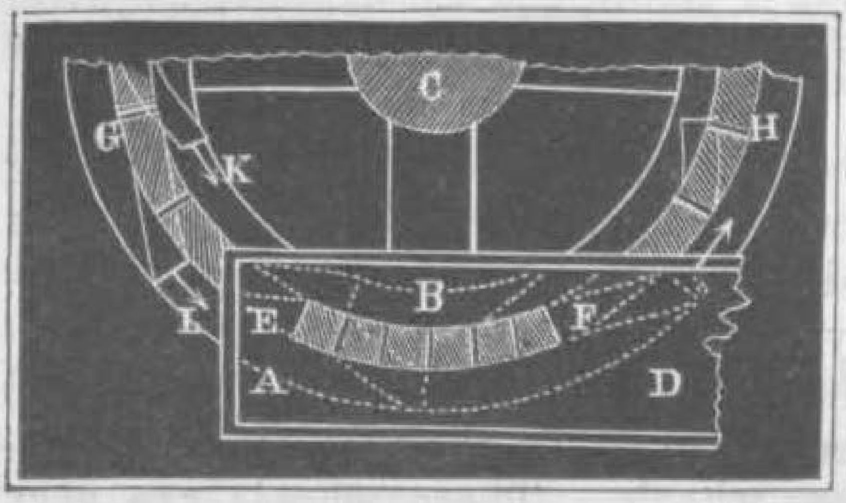
Fig. 244.



D'Aubuisson mentions that wheels of this kind have been erected recently, the wheel being put immediately under the bottom, and made of somewhat greater diameter than the well. The pyramidal trough for laying on the water is much shortened, and by these means the efficiency has been raised to 0,25. These wheels are, therefore, at best, inferior to the impact wheels already treated of.

§ 133. *Burdin's Turbines.*—M. Burdin, a French engineer of mines, proposed what he terms a "turbine à évacuation alternative." They are the best wheels of the category now under examination. They differ from Borda's wheels only in this essential, namely, that the water enters them at various points simultaneously, and that the orifices of discharge are distributed over 3 concentric rings. This latter arrangement is adopted, that the water, discharged with a small absolute velocity, may not hinder the revolution of the wheel.

Fig. 245.




The first wheel of this kind was erected by Burdin for a mill at Pont-Gibaud, and is described in the "Annales des Mines, III. série, t. III." Fig. 245 represents a plan of this wheel. *ABD* is the pen-trough immediately above the wheel, having a series of orifices *EF* in the bottom, through which the water is laid on to the wheel with a slight inclination. The wheel revolving on the axis *c* consists of a series of conduits, the entrances to which make together the annular space *GBH*, which moves accurately under the arc *EF* formed by the trough-openings, so that the water passes unimpeded from the one into the other. The conduits (Fr. *couloirs*) are vertical at the upper end, and nearly horizontal, and tangential at the bottom. The lower ends are brought into *three* distinct rings, so that the third of the number of entrances only discharge in the ring vertically under them; one-third, as *K*, discharge *within*, the others, as *L*, discharge *outside* this ring.

From the experiments made on the turbine erected at Pont-Gibaud by Burdin, it appears that for 3 cubic feet of water per second, and a fall of 10,35 feet, the efficiency was 0,67. The impact turbine formerly in the same position consumed 3 times this quantity of water to produce the same effect. The diameter of the wheel was 4,6 feet, and the depth 15 inches. The number of buckets 36.

§ 134. *Effect of Centrifugal Force.*—In the turbines hitherto under consideration, the water moves nearly, if not exactly, on a cylindrical surface, and, therefore, each element of water retains the same relative position to the axis, or at least does not vary it much. But we have now to consider wheels, in which the water, besides a rotary and vertical motion, possesses a motion inwards or outwards in reference to the axis, and more or less radial. The peculiarity of such turbines is, that their motion depends on the centrifugal force of the water, so that they might be termed centrifugal turbines. Before entering on a discussion of these wheels, it will be well to investigate the effect of the water's centrifugal force, when its motion is in a spiral line round a centre, or when the motion is radial and rotary at the same time. The centrifugal force of a body of the weight $G$, revolving at a distance $y$, with an angular velocity $\omega$, round a given point, is $F = \frac{\omega^2 G y}{g}$ (Vol. I. § 231). If this weight moves also a small distance $\sigma$ radially outwards, or inwards, then this force will have produced, or absorbed, an amount of mechanical effect represented by: $F \sigma = \frac{\omega^2 G y \sigma}{g}$. If, then, we assume that the motion commences in the centre of rotation, and continues radially outwards, so that ultimately the distance of the weight from the axis $= r$, we may ascertain the mechanical effect produced by the centrifugal force by substituting in the last formula $\sigma = \frac{r}{n}$ $y$, introducing successively, however, $\frac{r}{n}, \frac{2r}{n}, \frac{3r}{n} \ldots \frac{nr}{n}$, and uniting the mechanical effects resulting by summation. Hence the mechanical effect in question is:

$$L = \frac{\omega^2 G r}{n g}\left(\frac{r}{n} + \frac{2r}{n} + \frac{3r}{n} + \cdots + \frac{nr}{n}\right)$$

$$= \frac{\omega^2 G r^2}{n^2 g}(1 + 2 + 3 + \ldots + n) = \frac{\omega^2 G r^2}{n^2 g} \cdot \frac{n(n+1)}{2}$$

or, as we must assume $n$ infinite:

$$L = \frac{\omega^2 G r^2}{n^2 g} \cdot \frac{n^2}{2} = \frac{\omega^2 r^2}{2g} \cdot G = \frac{v^2}{2g} G,$$

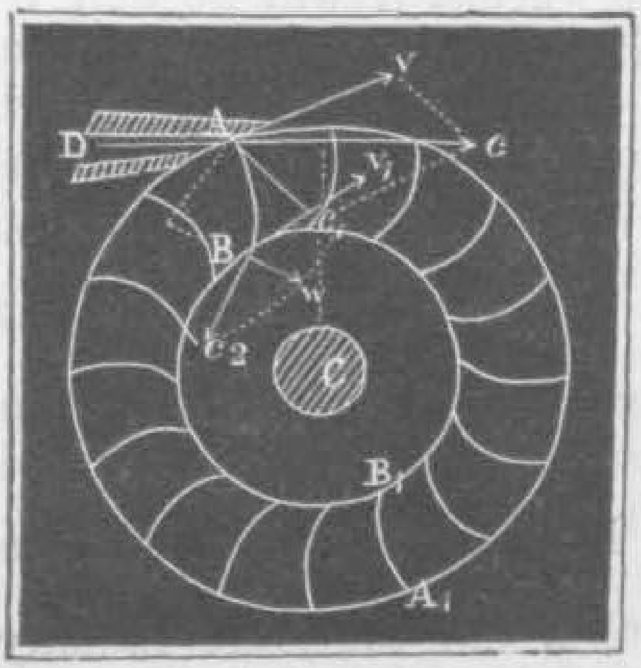
when $v$ is the velocity of rotation $\omega r$ of the body at the extreme point of its motion. As this mechanical effect is produced by the centrifugal force when the motion is from within *outwards*, it must be consumed when the motion is from without *inwards*. If the body does not come to the centre at the end of its motion, but remains at a distance $r_1$ from it, then there remains an amount of effect $\frac{\omega^2 r_1^2}{2g} G$, and the body consumes, therefore, only the effect

$$L = \frac{\omega^2 r^2}{2g} G - \frac{\omega^2 r_1^2}{2g} G = (r^2 - r_1^2)\frac{\omega^2}{2g} G = \left(\frac{v^2 - v_1^2}{2g}\right) G,$$

if $v_1$ represent the velocity of rotation at the distance $r_1$ or end of the motion, as $v$ represents it at the distance $r$ or commencement of the motion. If the motion is from within outwards, then the effect produced by centrifugal force is $L = \left(\frac{v^2 - v_1^2}{2g}\right) G$.

§ 135. *Poncelet's Turbine.*—One of the most simple horizontal wheels, in which centrifugal force influences the working, is Poncelet's turbine, shown in Fig. 246, in plan. This turbine has curved buckets between shroudings, and is, in fact, one of Poncelet's undershot wheels, laid on its side. The water is laid on by a trough $AD$ nearly tangentially, and runs along the curved bucket to discharge itself in the interior. That the effect of the water on the wheel may be a maximum, it is necessary that the water should enter without shock and discharge into the interior deprived of its *vis viva*. The direction of the end of the bucket $A$, insuring no shock, is determined exactly as for Poncelet's undershot wheel,

Fig. 246.




by constructing a triangle with the velocity $v$ of the wheel and that $c$ of the water entering and drawing $Ac_1$ parallel to the side $vc$. The relative velocity $Ac_1$ with which the water enters the wheel is: $c_1 = \sqrt{c^2 + v^2 - 2\, c\, v \cos. \delta}$, $\delta$ being the angle $c\,A\,v$ by which the direction of the stream of water deviates from the tangent to the circumference of the wheel. This velocity is, however, diminished by centrifugal force during the motion of the water on the bucket, and, therefore, the relative velocity $Bc_2 = c_2$ with which the water comes to the inside of the wheel, is less than the above velocity $c_1$. According to the result of the investigation in the last paragraph, the water loses an amount of effect represented by $\left(\frac{v^2 - v_1^2}{2g}\right) Q\gamma$, or $\frac{v^2 - v_1^2}{2g}$ in pressure or velocity height, $v$ being the velocity of rotation at the commencement, and $v_1$ that at end of the motion. If, therefore, $\frac{c_1^2}{2g}$ be the height due to the velocity at the entrance $A$, and $\frac{c_2^2}{2g}$ that at the exit $B$, we have

$\frac{c_2^2}{2g} = \frac{c_1^2}{2g} - \left(\frac{v^2}{2g} - \frac{v_1^2}{2g}\right)$, and, therefore, $c_2^2 = c_1^2 - v^2 + v_1^2$, or as $c_1^2 = c^2 + v^2 - 2\ c\ v\ \cos.\ \delta$, $c_2^2 = c^2 + v_1^2 - 2\ c\ v\ \cos.\ \delta$, and $c_2 = \sqrt{c^2 + v_1^2 - 2\, c\, v \cos. \delta}$, it being constantly borne in mind that $v$ is the velocity of rotation at the outer periphery, and $v_1$ that at the inner. In order to rob the water of all its *vis viva*, the end $B$ of the bucket should be laid tangentially to the inner periphery of the wheel, and also $c_2$ should be made

$$= v_1, \text{ or } c^2 + v_1^2 - 2\, c\, v \cos.\ \delta = v_1^2, \text{ i. e., } v \cos.\ \delta = \frac{c}{2}.$$

For the sake of an unimpeded discharge of the water to the interior, the angle $\delta_1$ at which the inner end of the bucket cuts the wheel, must be made 15° to 30°, and, hence, the absolute velocity of the water discharged $w = \sqrt{c_2^2 + v_1^2 - 2\, c_2\, v_1 \cos.\ \delta_1}$, or, if we assume

$v \cos.\delta = \frac{c}{2}$, or, $v_1 = c_2$, $w = 2\, v_1 \sin. \frac{\delta_1}{2}$, and the loss of mechanical effect corresponding is:

$$\frac{w^2}{2g} Q\gamma = \left(2\, v_1 \sin. \frac{\delta_1}{2}\right)^2 \frac{Q\gamma}{2g},$$

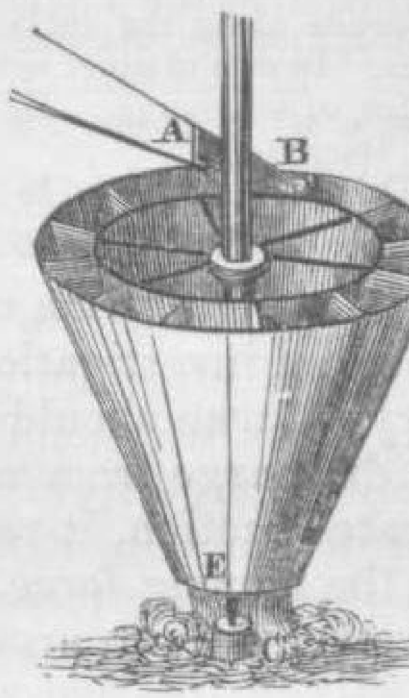
lastly, the remaining useful effect of the wheel:

$$L = \left[c^2 - \left(2\, v_1 \sin. \frac{\delta_1}{2}\right)^2\right] \frac{Q\gamma}{2g}.$$

According to Poncelet, these wheels should give an efficiency of 0,65 to 0,75.

§ 136. *Danaïdes.*—We shall next treat of horizontal wheels which have more or less the form of an inverted cone, and which are termed in France *roues à poires*, or *Danaïdes.* Belidor describes them in the "Architecture hydraulique." Fig. 247 represents the general arrangement of these wheels. They consist essentially of a vertical axis, with a double conical casing attached. The space between the casing is intersected by division plates, forming conduits running from top to bottom. The water is laid on by a trough *A* at top, and flows off through the bottom of the cone at *E*, near to the axis, after having passed through the conduits above mentioned. In the simplest form of wheels, the division plates are plane surfaces, running vertically; in other cases they are spiral or *screw-formed.* Belidor describes the wheel without the outer casing, but the wheel is placed in a conical vessel fitting pretty closely to the blades, or division plates.

Fig. 247.




In these wheels, gravity and centrifugal force act simultaneously on the water. If the water enters the wheel with the relative velocity $c_1$ above, at the point $B$, the velocity of rotation of which is $v$, and flows, in the wheel, through a height $h_1$, the velocity at the bottom of the wheel near the axis will be $c_2$, determined by the formula $c_2^2 = c_1^2 + 2gh_1 - v^2$. In order that this may be 0, we must have $v^2 = c_1^2 + 2gh_1$. Further, that the water may enter the wheel without shock, the horizontal component of its velocity must equal the velocity of rotation, that is, $c \cos. \delta = v$, $\delta$ being the inclination of the stream of water to the horizon.

The relative velocity of entrance is $c_1 = c \sin. \delta$, and, therefore, the above equation of condition becomes,

$$c^2 \cos. \delta^2 = c^2 \sin. \delta^2 + 2gh_1, \text{ i. e., } c^2 \cos. 2\,\delta = 2gh_1.$$

The fall necessary for the velocity is, therefore, $h_2 = \frac{c^2}{2g} = \frac{h_1}{\cos. 2\,\delta}$.

If, now, the whole fall $h_1 + \frac{h_1}{\cos. 2\,\delta} = h$, then the depth of the wheel $h_1 = \frac{h \cos. 2\delta}{1 + \cos. 2\delta}$, and the height due to the velocity:

$h_2 = \frac{h}{1 + \cos. 2\delta}$.

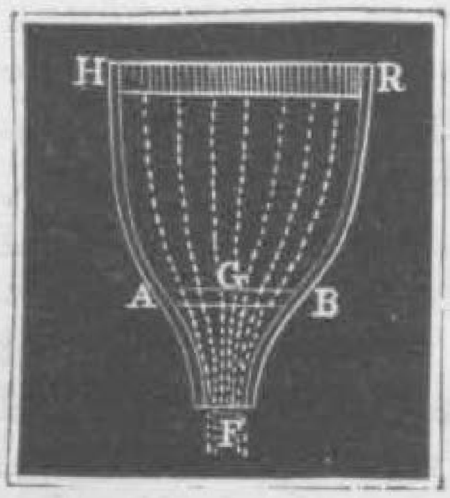
By this arrangement there is no loss of mechanical effect, but as the axis has a certain sectional area, and the water too requires a certain area of orifice for discharge, and thus the water can only be brought to within a certain distance of the axis, its *vis viva* cannot be entirely taken up, so that the efficiency is not nearly 1,0.

*Remark.* The wheel just described is known as Burdin's Danaïde. The older Danaïde of Manouri d'Ectot was differently constructed, though in principle it was the same. This wheel consisted of a sheet iron cylinder, with an orifice in the bottom for the discharge of the water, and through which the axis passed. In this hollow cylinder, there is placed a closed cylinder in such a position as to leave an annular space between it and that first mentioned, and also a space between the bottoms of the two. This latter is divided by plates and buckets placed vertically and radially into a series of compartments. The water is laid on tangentially into the space between the two cylinders, descends along the surface to the bottom, inducing a rotary motion of the whole apparatus. In this manner it flowed gradually to the bottom, and from thence reached the orifice of discharge. See "Dictionnaire des Sciences mathématiques par Montferrier, art. Danaïde."

This form of Danaïde has been recently perfected by Mr. James Thomson, of Glasgow, so that the efficiency of a model has been proved to be 0,85.

§ 137. *Reaction of Water.*—Before proceeding with the description and investigation of the theory of *reaction wheels*, it is necessary that we should illustrate the nature of the reaction of water in its discharge from vessels. As a solid body endowed with an accelerated motion, it reacts in the opposite direction with a force equal to the moving force, so it is in the case of water when it issues from a vessel with an accelerated motion from the orifice. This acceleration always takes place when the area of the orifice is less than the area of the vessel, or the velocity of discharge greater than the velocity of the water through the vessel. On these grounds the vertical pressure of the water in the vessel $HRF$, Fig. 248, from which the water flows downwards at $F$, is *less* than the weight of the mass of water in the vessel. The decrease of this force, or *the reaction of the water* flowing away, may be determined as follows. If the horizontal layer $AB$ of the water flowing out, has a variable section $G$, a variable thickness $x$, and a variable acceleration $p$, its weight is $G x \gamma$ and its masse $= \frac{G x \gamma}{g}$, and, therefore, its reaction $K = \frac{G x \gamma}{g} \cdot p$. If, now, $w$ represent the variable velocity of the layer of water, and $_x$ its increase in passing through the elementary distance $x$, we have (according to Vol. I. § 19) $p x = w_x$, and, therefore, $K = \frac{G \gamma}{g} w_x$.

Fig. 248.



If $F$ be the area of the orifice, and $v$ the velocity of discharge, then $Gw = Fv$, and, therefore, $K = \frac{F\gamma}{g} v\, x$. To obtain the reaction of all the layers of water, we must substitute in the last expression for $x$, the increments of velocity $x_1, x_2, x_3, \ldots x_n$ of all the layers of water, and sum the results. The reaction of the whole mass is thus $P = \frac{Fv}{g} \gamma (x_1 + x_2 + \ldots + x_n)$. If $c$ be the velocity of entrance of the water, the sum of all the increments of velocity $= v - c$, and, therefore, the reaction required:

$$P = \frac{F v \gamma}{g}(v - c) = \frac{(v - c)}{g} Fv \,.\, \gamma = \frac{v - c}{g} \,.\, Q\gamma,$$

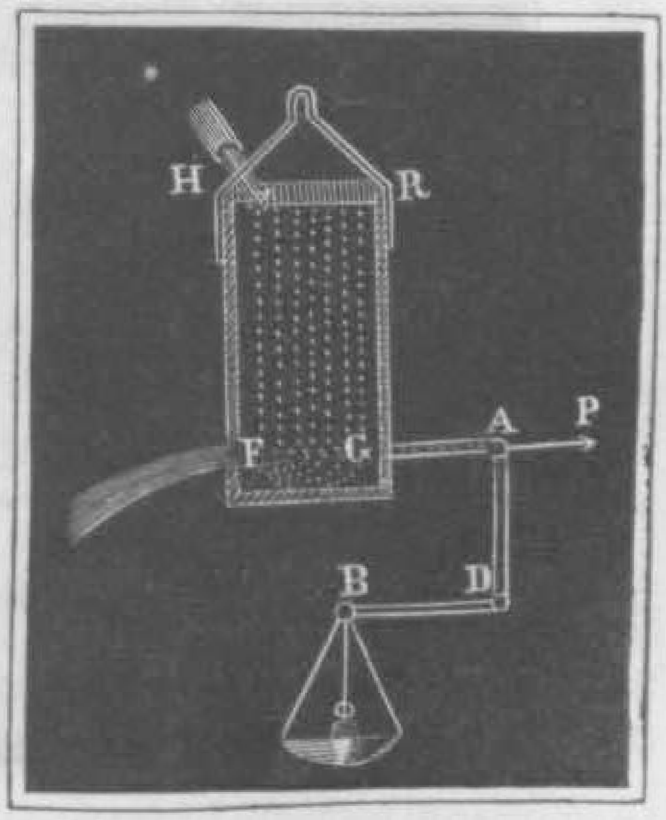
$Q$ being the quantity discharged per second. If, however, the orifice $F$ be very small compared with the surface $HR$, then $c$ may be neglected, compared with $v$, and

$$P = \frac{v^2}{g} F\gamma = 2 \,.\, \frac{v^2}{2g} F\gamma = 2h \,.\, F\gamma.$$

So that the reaction is as great as the vertical impact of the water on a plane surface (Vol. I. § 385), that is, *equal to the weight of a column of water, the basis of which is the area $F$ of the orifice, or of the stream, and whose height is double the height due to the velocity* ($2h$) *of the water discharged.*

If the water flow out by the side of the vessel, as shown in Fig. 249, the direction of the reaction is then horizontal, and the amount is in like manner $= \frac{c^2}{g} F\gamma$. If the water vein be contracted, and if $a$ be the co-efficient of contraction, then, instead of $F$, we must put $Fa$, or $P = \frac{c^2}{g} \,.\, a\, F\gamma$.

Fig. 249.




*Remark.* Mr. Peter Ewart, of Manchester, made experiments to test this result, ("Memoirs of the Manchester Phil. Soc.," Vol. II.) The vessel $HRF$ (Fig. 249) was hung on a horizontal axis $C$, and the reaction measured by a bent lever balance $ADB$, upon which the vessel acted by means of a rod $GA$, bearing on the point directly opposite to the orifice $F$. In the discharge through an orifice in the *thin plate*, it was found that

$$P = 1{,}14 \frac{v^2}{2g} F\gamma.$$

If we take the section of the streamn $F_1 = 0{,}64 \,.\, F$, and the effective velocity of discharge $v_1 = 0{,}960$ (Vol. I. § 315), we have, according to the theoretical formula:

$$P = 2 \,.\, \frac{v_1^2}{2g}, F_1\gamma = 2 \,.\, 0{,}96^2 \,.\, 0{,}64 \,.\, \frac{v^2}{2g} F\gamma$$
$$= 1{,}18 \frac{v^2}{2g} F\gamma,$$

nearly the same as the experimental result. When the orifice was provided with a mouth-piece formed like the *vena contracta*, it was found that:

$P = 1,73 \cdot \frac{v^2}{2g} F\gamma$, the co-efficient of discharge being 0,94. As in this case $F_1 = F$, and $v_1 = 0,94\, v$, the theoretical result is:

$$P = 2 \cdot 0,94^2 \frac{v^2}{2g} F\gamma = 1,77 \cdot \frac{v^2}{2g} F\gamma,$$

or a very close agreement with the experimental result.

§ 138. *Reaction Wheels.*—If a vessel, as *HRF*, Fig. 250, be placed on a wheeled carriage, the reaction moves the carriage in the opposite direction from that in which the discharge takes place, and if a vessel *AF*, Fig. 251, be connected with a vertical axis *C*, it will

Fig. 250.

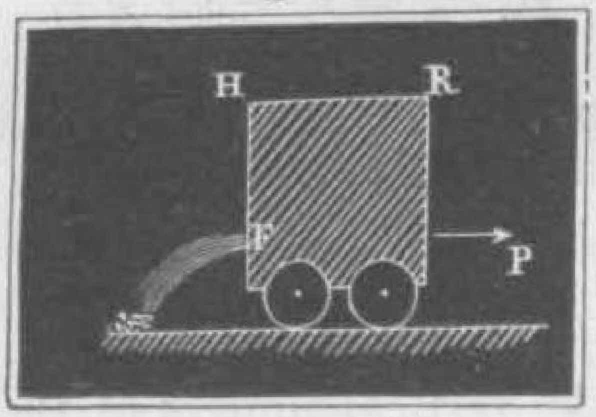
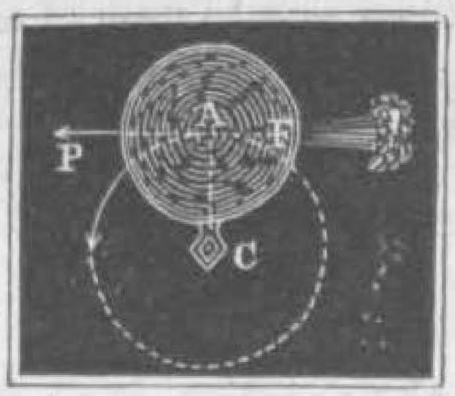



Fig. 251.




cause it to revolve in the direction opposite to that in which discharge takes place. If a constant supply of water be maintained, a continuous rotary motion results. This contrivance is the *reaction* wheel (Fr. *roue à réaction;* Ger. *Reactionsrad*), commonly called *Barker's* mill in Britain, and *Segner's* water wheel in Germany. The simplest form of this wheel is shown in Fig. 252. It consists of a pipe *BC*, firmly connected with a vertical axle *AX*, of two pipes *CF* and *CG* at right angles to the first, having orifices in the sides at *F* and *G*. The water discharged from these orifices is continually supplied by a trough leading into the top of the upright

Fig. 252.

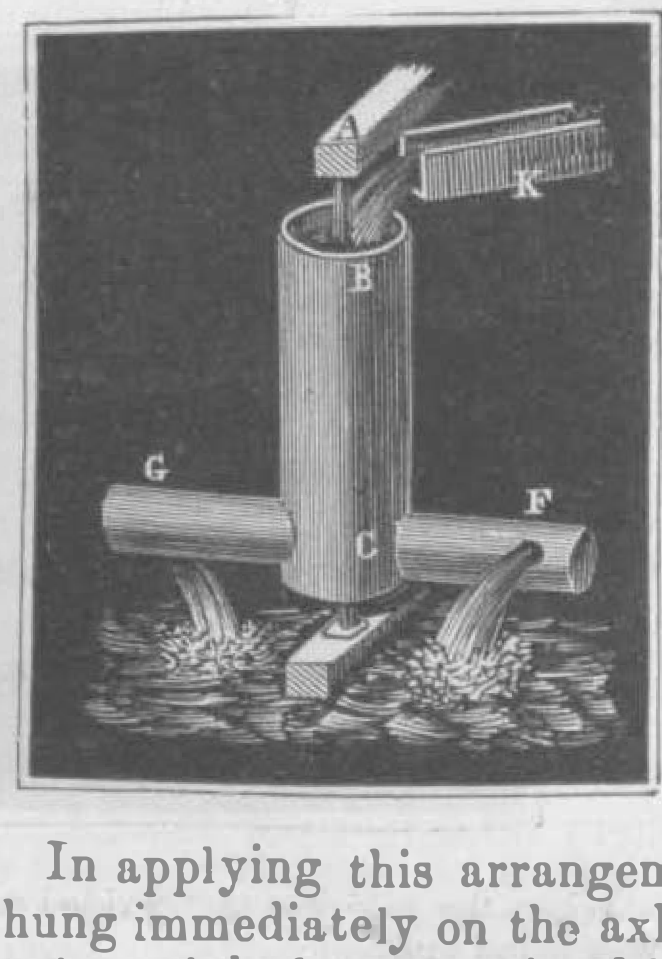
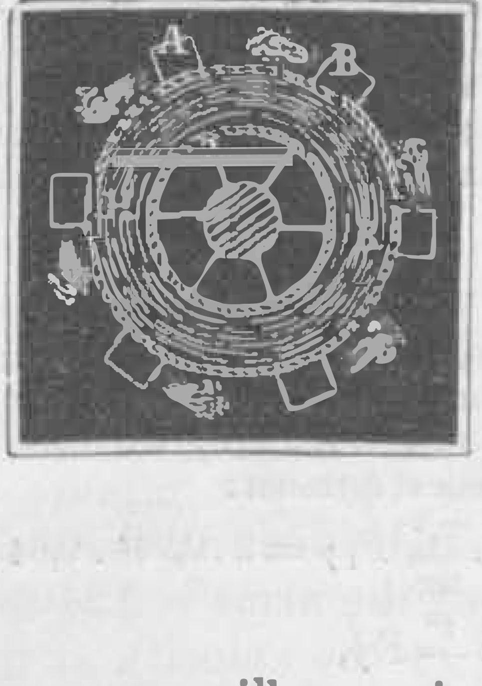



Fig. 253.

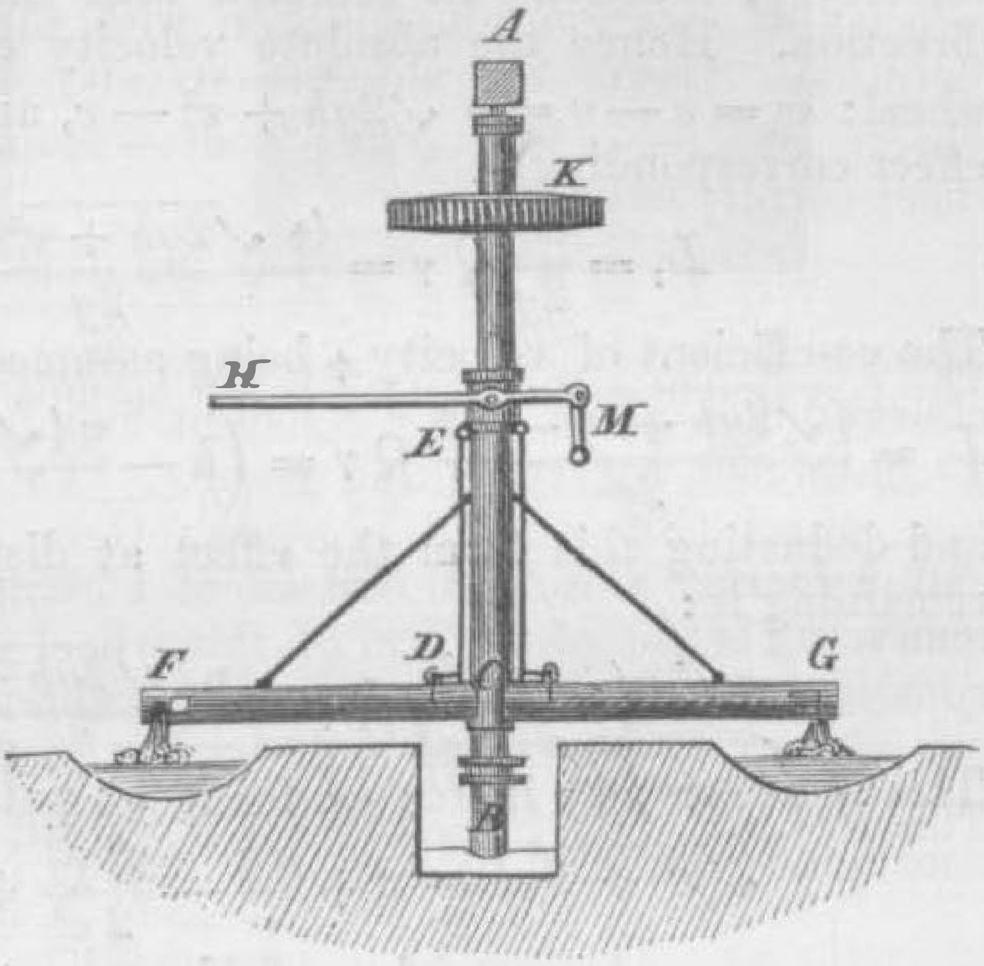



pipe. In applying this arrangement, the upper millstone is generally hung immediately on the axle *AX*; but for other applications the motion might be transmitted by any suitable gear.

Reaction wheels are also made with more discharge pipes or conduits than two, as shown in Fig. 253. The vessel $HR$ is made either cylindrical or conical. In order to bring in the water at the top without shock, the great Euler adapted a cylindrical end to the pentrough, immediately above the wheel, putting a series of inclined guide-buckets into it, analogous to the arrangement introduced by Burdin for his turbines (Vol. II. § 133).

There is a simple reaction wheel erected by M. Althans, of Vallender, in the neighborhood of Ehrenbreitstein, for driving two pair of grindstones, which we have seen and admired. The arrangement of this machine is shown in the accompanying sketch, Fig. 254. The water is laid on by a pipe $B$ descending beneath the wheel, and turning vertically upwards. The upright axle $AC$, with its two arms $CF$ and $CG$, is hollow, and fits on to the end $B$ of the supply pipe. There is a stuffing box at $B$ allowing of the free rotary motion of the wheel, and at the same time preventing loss of water at the joint. The rectangular orifices $F$ and $G$ are opened or shut by means of vanes or slide valves moved by rods attached to a collar $E$ on the axle, movable by means of the lever $HM$. The water supplied by the 9 inch pipe $B$ flows through the arms of the wheel, and through the apertures $F$ and $G$. This arrangement has the advantage of supporting the whole, or great part of the weight of the machine upon the water, so that there is little or no friction on the base. If $G$ be the weight of the machine, $h$ the head of water, $2\,r$ the diameter of the pipe at $B$, then $\pi r^2 h \gamma = G$, and, therefore, in order to support the machine, the radius of the pipe should be $r = \sqrt{\frac{G}{\pi h \gamma}}$. The quantity of water expended by this machine is 18 cubic feet per minute, the fall is 94 feet, and, therefore, the mechanical effect at disposition is 1755 feet lbs. per second. The length of the arms is $12\frac{1}{2}$ feet, and the number of revolutions 30 per minute, or the velocity at the periphery 39,3 feet per second.

Fig. 254.




*Remark* 1. The first account of a reaction wheel, as an invention of Barker, is given in Desagulier's "Course of Experimental Philosophy, vol. ii. London, 1745." Euler treats in detail of the theory and best construction of these wheels in the "Memoirs of the Berlin Academy, 1750—1754."

*Remark* 2. The efficiency of reaction wheels is reputed as extremely small. Nordwall makes it only half of that of an overshot wheel, and Schitko's experiments on such a wheel gave the efficiency only 0,15.

§ 139. *Theory of the Reaction Wheel.*—The effects of reaction wheels may be determined theoretically as follows. If $h$ be the fall, or the depth of the centre of the orifices below the surface in the feed-pipes, we have the height measuring the pressure of water on the orifices $h_1 = h + \frac{v^2}{2g}$, and, therefore, the theoretical velocity of discharge $c = \phi \sqrt{2gh + v^2}$. This is not, however, the absolute velocity of the water at efflux from the wheel, for it partakes of the velocity of rotation $v$ in common with the wheel, in the opposite direction. Hence the absolute velocity of the water leaving the wheel: $w = c - v = \phi \sqrt{2gh + v^2} - v$, and the loss of mechanical effect corresponding:

$$L_1 = \frac{w^2}{2g} Q \gamma = \frac{(\phi \sqrt{2gh + v^2} - v)^2}{2g} Q \gamma.$$

The co-efficient of velocity $\phi$ being assumed $= 1$, then

$$L_1 = \frac{(\sqrt{2gh + v^2} - v)^2}{2g} Q \gamma = \left(h - \frac{v(\sqrt{2gh + v^2} - v)}{g}\right) Q \gamma,$$

and deducting this from the effect at disposition, the useful effect remaining is:

$$L = \left(h - \frac{w^2}{2g}\right) Q \gamma = \frac{v(\sqrt{2gh + v^2} - v)}{g} Q \gamma.$$

This increases as $v$ increases, for if we put:

$$\sqrt{v^2 + 2gh} = v + \frac{gh}{v} - \frac{g^2 h^2}{2 v^3} + \ldots \text{ we have:}$$

$$L = v \left(\frac{gh}{v} - \frac{g^2 h^2}{2 v^3} + \ldots\right) \cdot \frac{Q \gamma}{g},$$

and for $v = \infty$, $L = Q h \gamma$, the whole effect available.

This circumstance of the maximum effect depending on the wheels acquiring an infinite velocity, is very unfavorable; because, as the velocity increases, the prejudicial resistances increase, and even when the wheel runs without any useful resistance, the velocity it acquires is far from being infinite, proving the absorption of effect at these great velocities.

The question of course is, as to whether the effect for mean velocities of rotation be very much less than the maximum effect, or $Q h \gamma$. If we load the machine to such an extent that the height due to velocity, corresponding to the velocity of rotation, is equal tothe fall, or $\frac{v^2}{2g} = h$, or $v = \sqrt{2gh}$, then, according to the above formula:

$$L = \frac{\sqrt{2gh}\,(\sqrt{4gh} - \sqrt{2gh})}{g} Q\gamma = 2(\sqrt{2} - 1)\, Q h \gamma = 0{,}828\, Q h \gamma,$$

but if $\frac{v^2}{2g} = 2h$, then:

$$L = \frac{\sqrt{4gh}(\sqrt{6gh} - \sqrt{4gh})}{g} Q\gamma = 4(\sqrt{1{,}5} - 1) Qh\gamma = 0{,}899\, Qh\gamma.$$

and, lastly, if $\frac{v^2}{2g} = 4h$, then:

$$L = \frac{\sqrt{8gh}(\sqrt{10gh} - \sqrt{8gh})}{g} Q\gamma = 8(\sqrt{1{,}25} - 1) Qh\gamma = 0{,}944\, Qh\gamma.$$

It thus appears that, in the first case, we lose 17, in the second 10, and in the third only 6 per cent. of the available effect, and, therefore, for moderate falls, and when a velocity of rotation exceeding the velocity due to the height of fall may be adopted, there is a great effect to be expected from these wheels. Considering the great simplicity of these wheels, the balance must often be much in their favor when compared with other wheels.

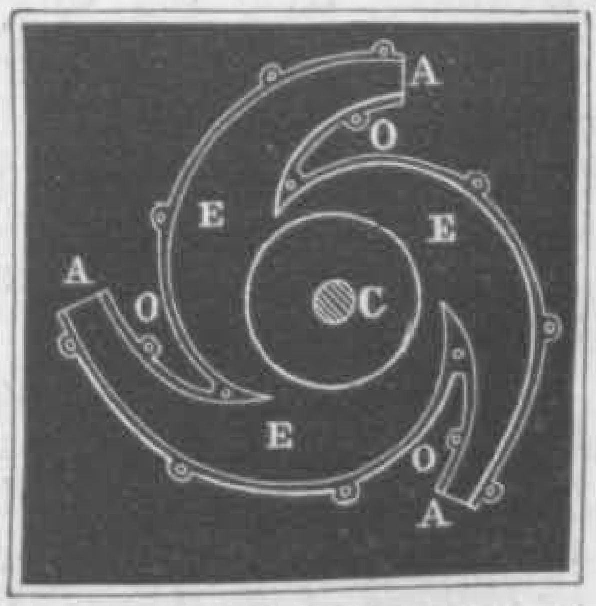
*Remark.* The *force* of rotation or of reaction is:

$$P = \frac{L}{v} = \frac{\sqrt{2gh + v^2} - v}{g} Q\gamma,$$

and for $v = 0$, $P = \frac{\sqrt{2gh}}{g} Q\gamma = \frac{c}{g} Q\gamma = 2 \cdot \frac{c^2}{2g} F\gamma$, as we showed, Vol. II. § 146, although by a different method.

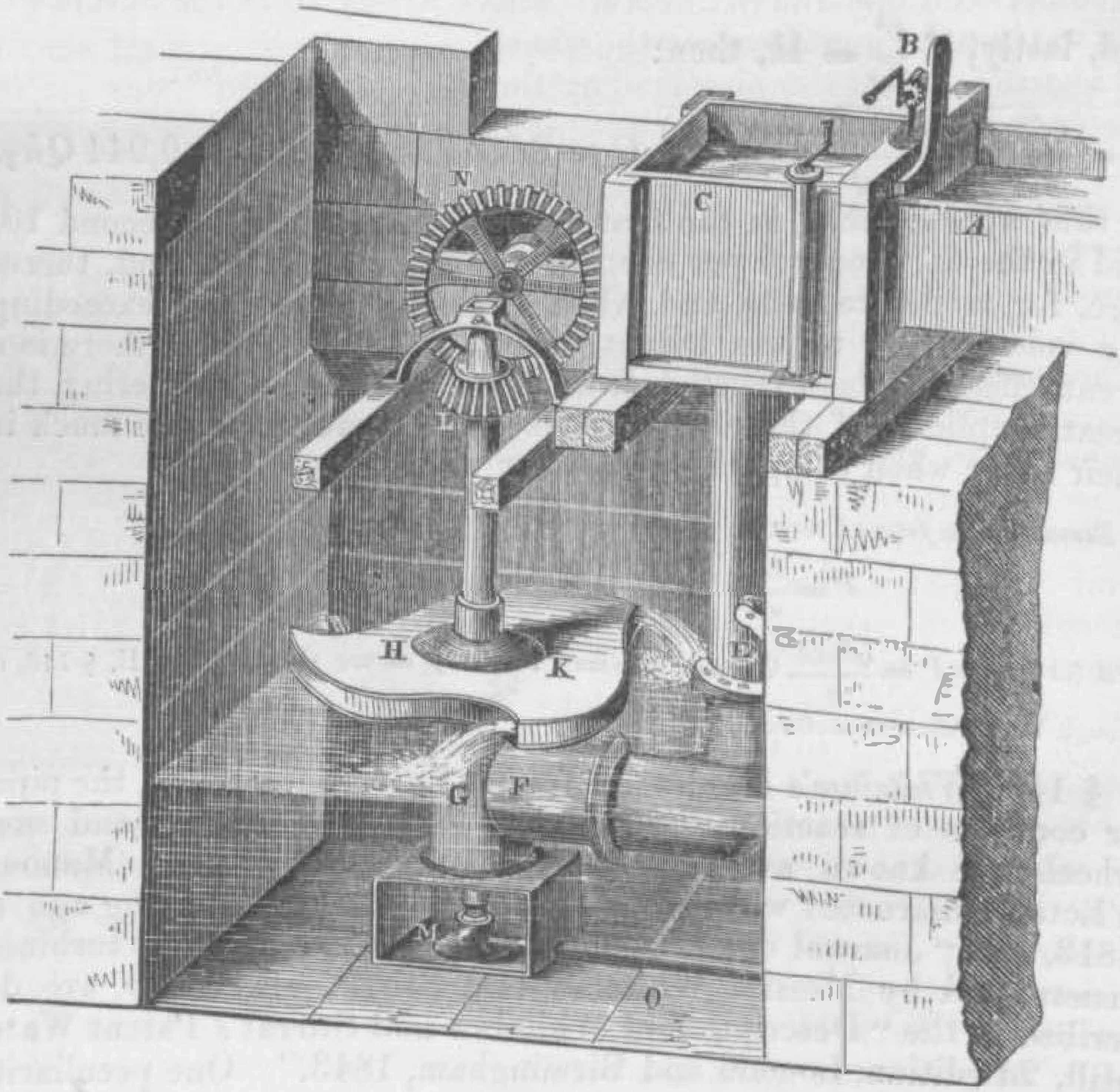
§ 140. *Whitelaw's Turbine.*—Within the last few years, the pipes or conduits of reaction wheels have been made curved, and such wheels are known as *Whitelaw's*, or *Scottish* turbines. Manouri d'Ectot constructed wheels on nearly the same plan as long ago as 1813, (*see* "Journal des Mines, t. xxxiii.") The Scottish turbines, constructed by Messrs. Whitelaw and Stirrat, of Paisley, are described in the "Description of Whitelaw and Stirrat's Patent Water Mill, 2d edition, London and Birmingham, 1843." One peculiarity of Whitelaw's wheel consists in the introduction of a movable piece at the *outer orifice* of the conduits by which its section is regulated. Fig. 255 is a horizontal section of one of Whitelaw's turbines. There are, in this case, three arms. The water enters at $E$, and is discharged at $A$. $OA$ is a valve movable round $O$, by which the orifice of discharge is regulated. The adjustment of these regulators has been made self-acting by a peculiar arrangement, but is better regulated by the band, by an apparatus analogous to that shown in Fig. 254.

Fig. 255.




The general arrangement of Whitelaw's turbines is clearly shown by Fig. 256. $A$ is the lead for the water. $B$ the sluice. $C$ a reservoir immediately above the pressure pipe. $E$ is a valve for regulating the expenditure of water. At $F$ the water enters the cylinder $G$, and goes from thence into the wheel $HK$ placed above it, and fixed on the vertical axis $LM$. The reaction of the water

Fig. 256.




streaming from the three orifices, drives the wheel round in the opposite direction, and this motion is transmitted by the bevelled gear $LN$. The wheel, the axle, and the pressure pipe are of cast iron. The footstep for the pivot at $M$ is of brass. Oil is introduced by a pipe $O$ from the wheel room.

We shall hereafter recur to the theory and geometrical construction of this wheel.

§ 141. *Combe's Reaction Wheels.*—As being analogous to Whitelaw's, we may next consider Combe's reaction wheel. The water flows from below upwards into these wheels, and the wheel differs essentially from Whitelaw's in having so great a number of conduits or orifices of discharge, that it may be said to discharge at every point of the circumference, as the plan of the wheel in Fig. 257 shows. $AA$ is a plate connected with the axis, and forming the upper shrouding or cover of the wheel. $BB$ is the under shrouding, and npon it, between this and the upper plate, the buckets $EE$ are fastened. $DD$ is a cylinder surrounding the lower part of the axis, through which the water is laid on to the wheel, entering the wheel by all the aper-tures between the buckets on the inside, and streaming through the conduits formed by the buckets to be discharged at every point of the outer circumference. Another essential difference between this and Whitelaw's wheel arises from the absence of the water-tight joint between the wheel *B* and the end of the pipe leading the water to the wheel, and which is quite necessary to Whitelaw's wheels. The reason of this difference is, that the pressure of water in a reservoir or vessel, from which the water is running, is different at different points. The pressure is greatest where the water is nearly still, and least where the velocity is greatest (Vol. I. § 307). The velocity of the water depends, however, upon the section of the reservoir or vessel, and is inversely proportional to the section, and thus, by varying the section, the pressure may be made to vary at will, or it may be made only equal to that of the atmosphere. If we bore a hole in the side of a vessel at the point where the pressure of the water flowing past is only equal to the atmospheric pressure, there will be no discharge, nor any indraft. Thus, that no water may escape, and no air be drawn in through the space necessarily left between *B* and *D*, it is only necessary to give the section certain dimensions at the point of passage.

Fig. 257.

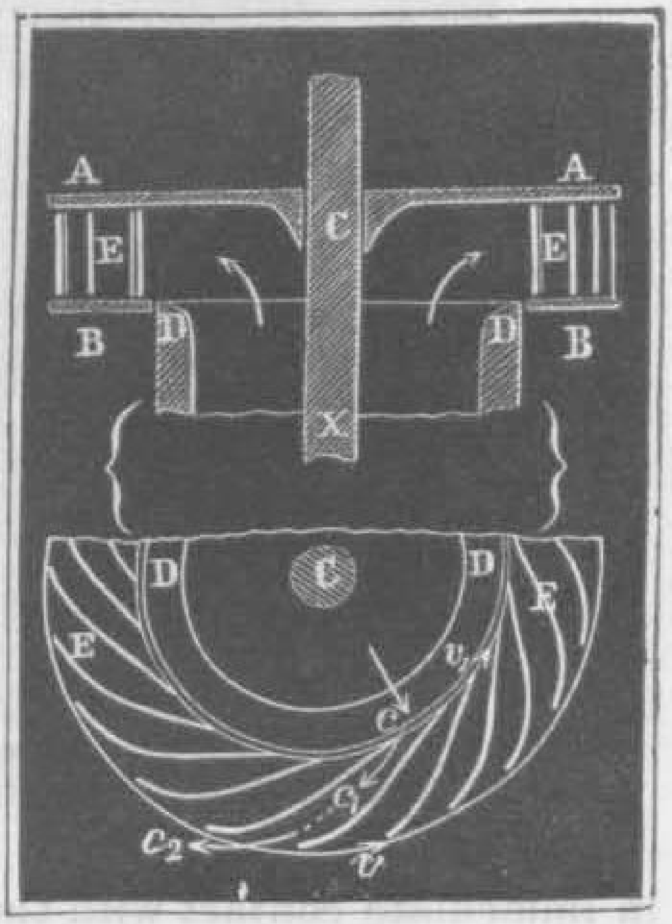
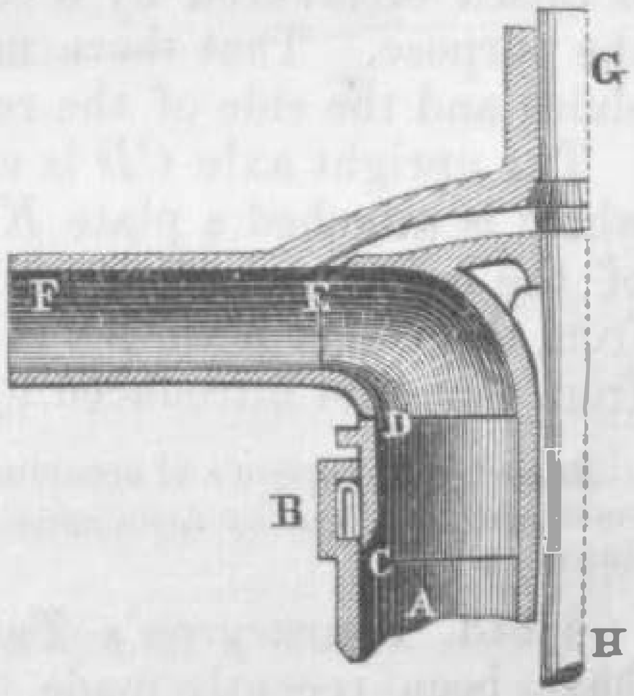



Fig. 258.

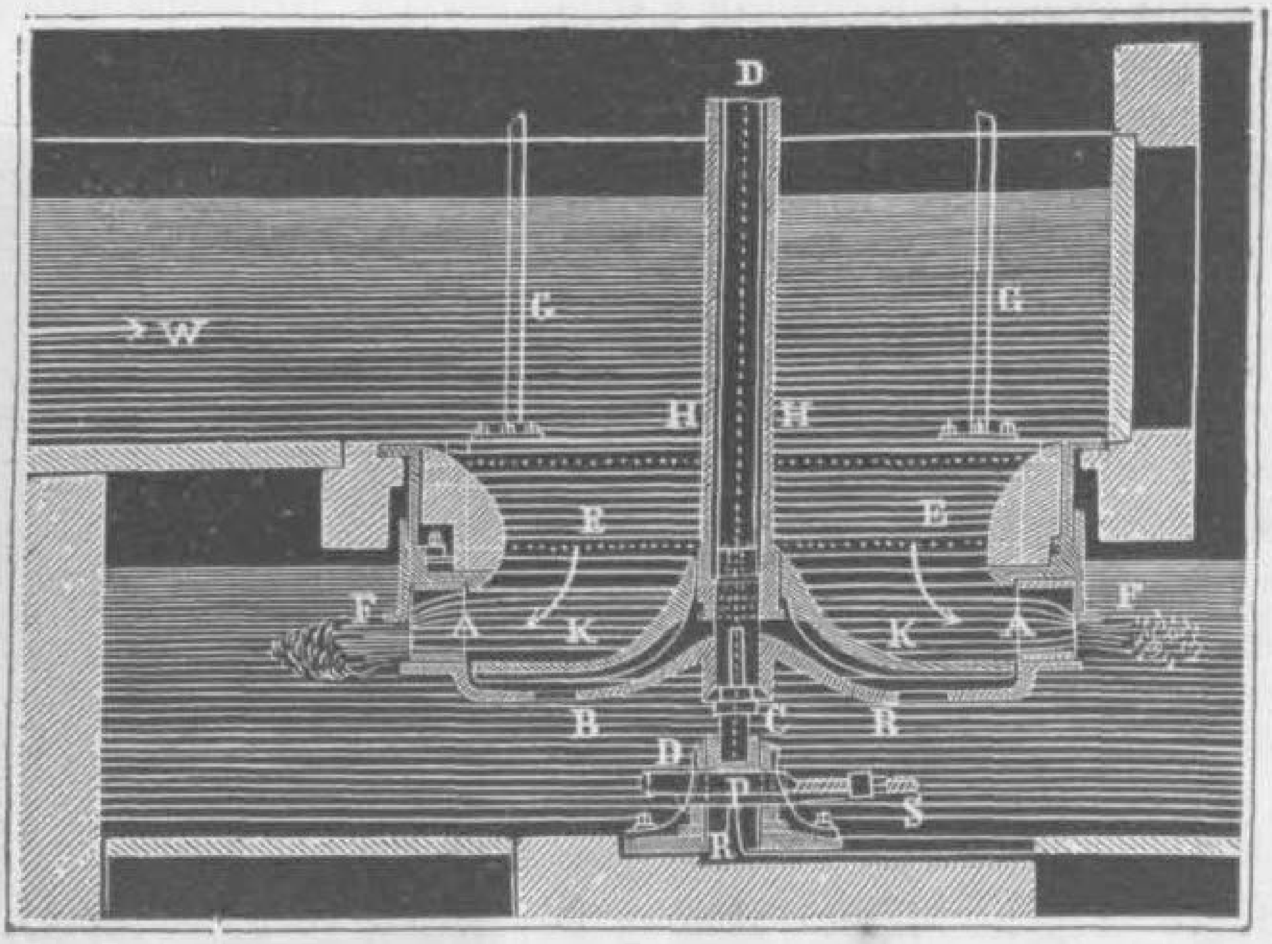



*Remark.* Combe's wheels are sometimes provided with guide-buckets for laying the water on to the wheel in a definite direction.

Redtenbacher forms the water-tight joint between the main pipe *AB* and the wheel *DEF*, by means of a movable brass ring *CD*, which is pressed so tightly up against the lower ring-surface *D* of the wheel, that the water does not escape. The ring *CD* must slide in a well-constructed, water-tight collar.

§ 142. *Cadiat's Turbine.*—The next wheel we shall describe is Cadiat's turbine. These have no guide-curves, like Whitelaw's and Combe's, but as in Fourneyron's turbine, the water is brought in from above. The peculiarity of this wheel is the introduction of a cylindrical sluice, which shuts the wheel *on the outside.* Fig. 259 is a vertical section of this wheel. *AA* is the wheel, *BB* being a saucer-formed plate connecting the wheel with the axle *CD*. The pivot *C* of this axle rests in a footstep, which we shall hereafter have occasion to allude to more particularly. *EE* is the reservoir with circular section communicating with the lead *W*, and coming

Fig. 259.




in immediate contact with the upper plate or shrouding of the wheel. That the water coming into the wheel may not be unnecessarily disturbed or contracted, the reservoir gradually widens both upwards and downwards, as the figure shows. The discharge of the water is regulated by the cylindrical sluice *FF* on the outside. This sluice is raised or lowered by 3 or 4 rods, connected with mechanism for the purpose. That there may be no escape of water between the sluice and the side of the reservoir, the joint is made with leather.

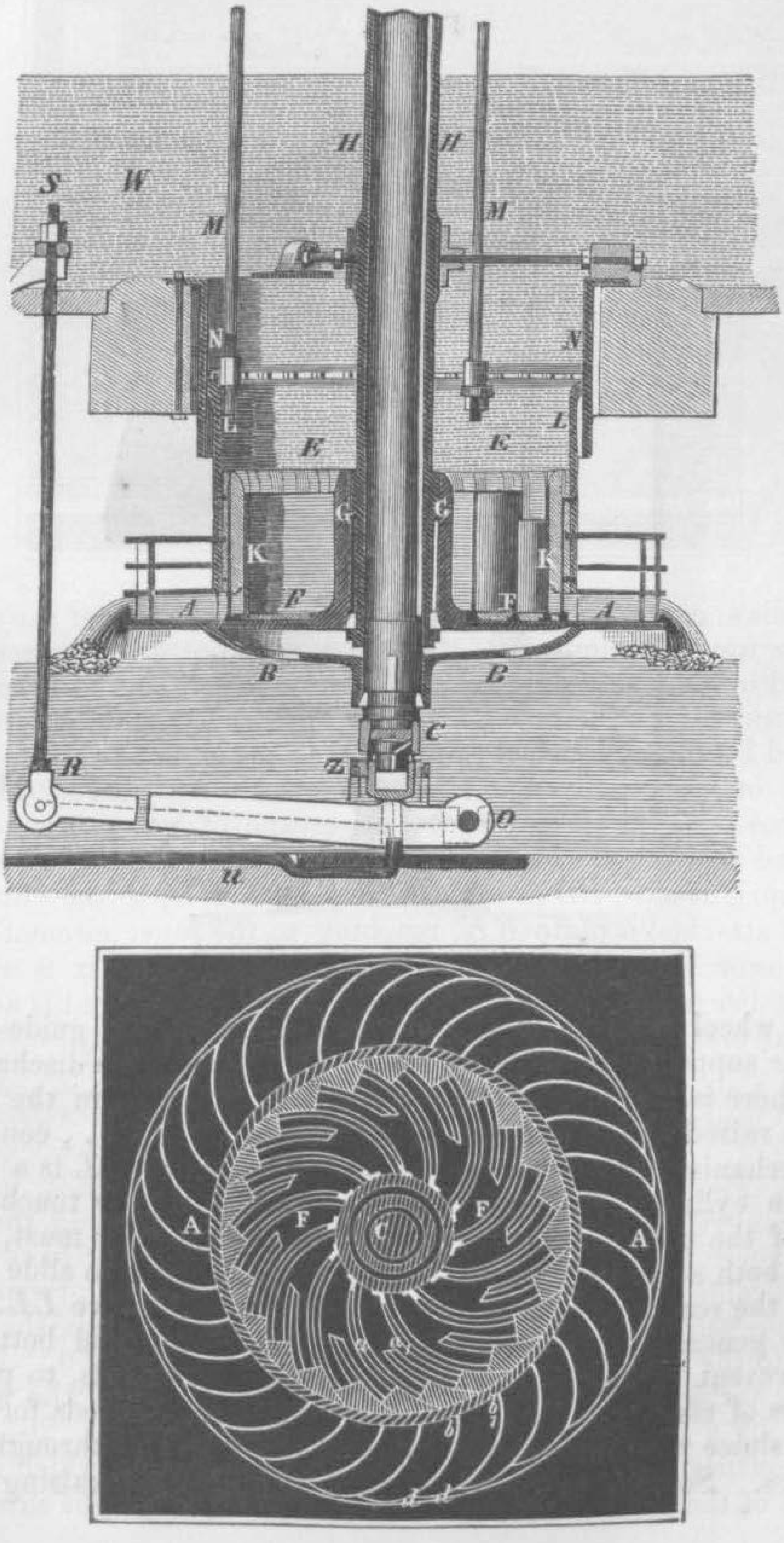
The upright axle *CD* is enclosed in a pipe *HH*, to the bottom of which is attached a plate *KK*, reaching to the inner circumference of the lower shrouding of the wheel, so that the water is shut off from the disc or plate *BB* of the wheel. This arrangement is adopted from that first introduced by Fourneyron in his turbine.

*Remark.* A complete and accurate description of one of Cadiat's turbines, as originally constructed, is given by Armengaud, sen., in the second volume of his "Publication Industrielle."

§ 143. *Fourneyron's Turbine.*—Fourneyron's turbines, as they have been recently made, may be considered as among the most perfect horizontal wheels. They work either in or out of back-water, are applicable to high and to low falls—are either high pressure and or low pressure turbines. In the low pressure turbine, the water flows into the reservoir, open above, as shown in Fig. 260. In high pressure turbines, the reservoir is shut in at top, and the water is laid on by a pipe at one side, as represented in Fig. 261. The wheel consists essentially of two shroudings, between which are the buckets of the connecting plate or arms, and the upright axle, as in

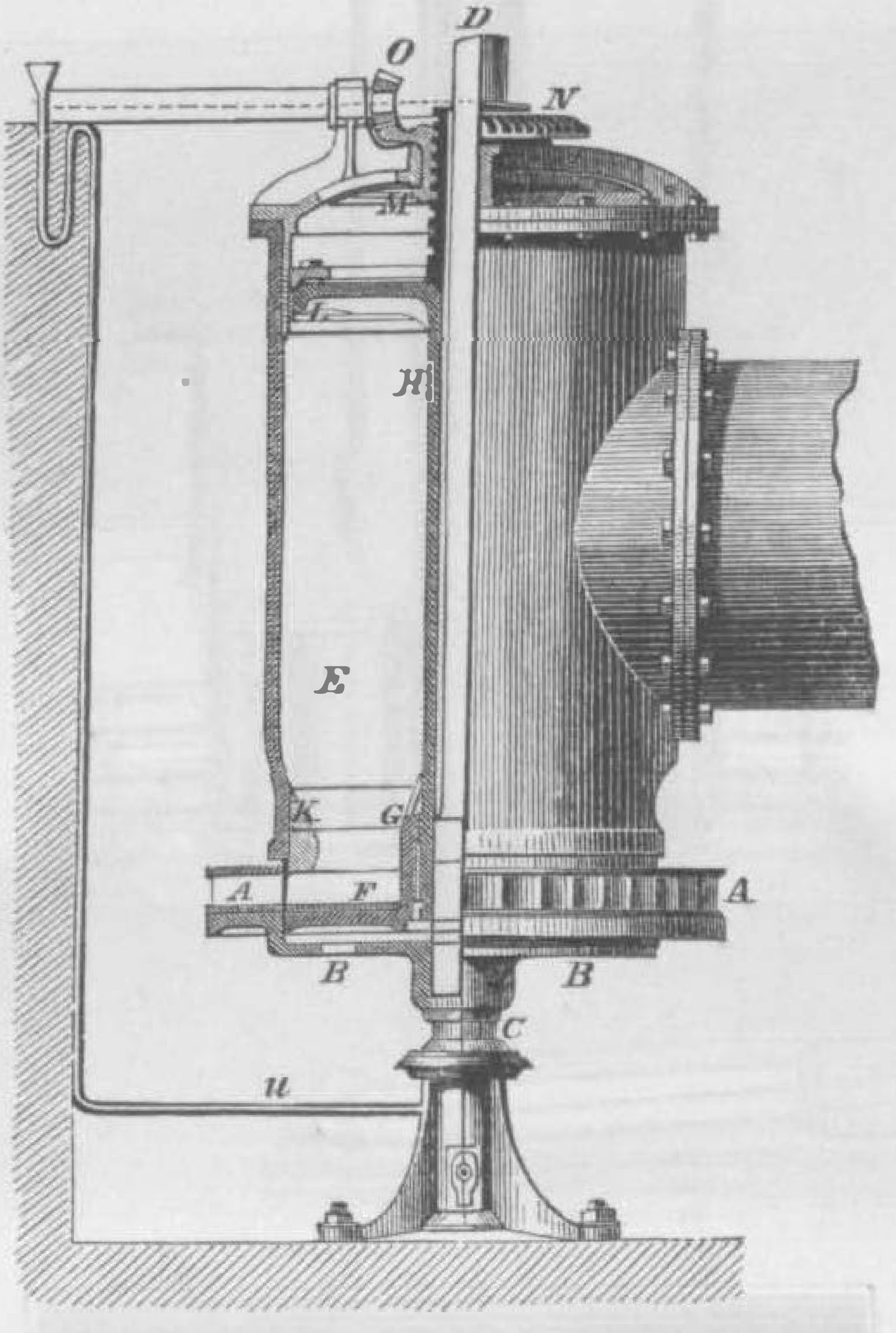
Cadiat's turbine. The water from the lead *W* flows into the reservoir *EE*. In order that the water may not rest directly on the wheel disc *BB*, which would greatly increase the pivot friction, a pipe encloses the upright axle, to which there is attached a disc *FF*,

Fig. 260.



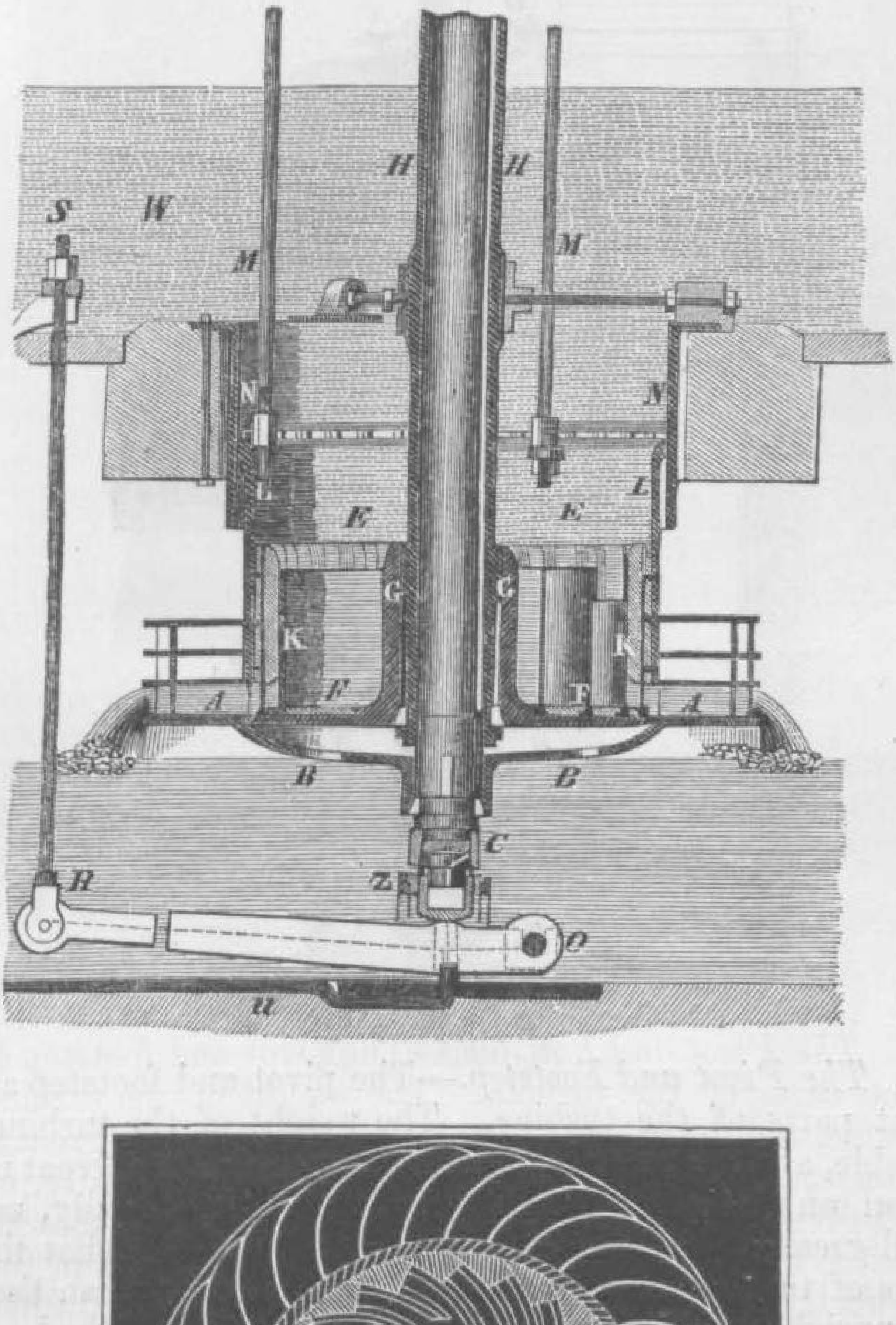
upon which the mass of water presses as it flows to the wheel. On this disc the so-called *guide-curves*, *ab*, $a_1b_1$, &c. (Fig. 260 or 262), are fastened. These give the water a certain direction of motion on to the wheel, which surrounds them, and through it along the buckets, to be discharged at the outer circumference. The reaction is such

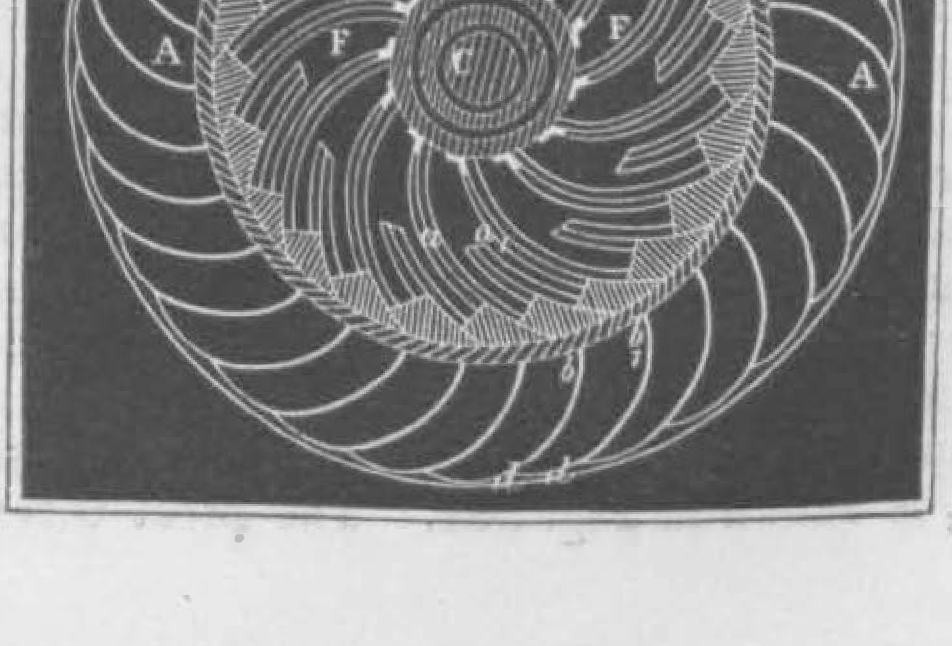
Fig. 261.




that the wheel revolves in the opposite direction, the guide-curves and their support remaining at rest. To regulate the discharge of water, there is a cylindrical sluice *KLLK*, Fig. 262, in the inside, which is raised and depressed by three rods *M*, *M* . . e, connected with mechanism suited to the object. The sluice *KL* is a hollow cast iron cylinder, the outer surface of which nearly touches the inside of the upper shrouding of the wheel, and they must, therefore, be both accurately turned. The sluice isemade to slide water-tight in the reservoir by a leather or other fitting above *LL*. The sluice is generally lined with wood, rounded at top and bottom, so as to prevent contraction as much as possible, that is, to prevent all losses of *vis viva*. In high pressure turbines, the rods for working the sluice pass through the cover of the reservoir through stuffing boxes. Sometimes the regulation is effected by raising or depressing the bottom plate Fig. 263. For this purpose, the top of the pipe $GH$ is screwed, and the female screw $M$ is attached to a conical wheel, moved by gear, as $O$. The female screw is fixed so

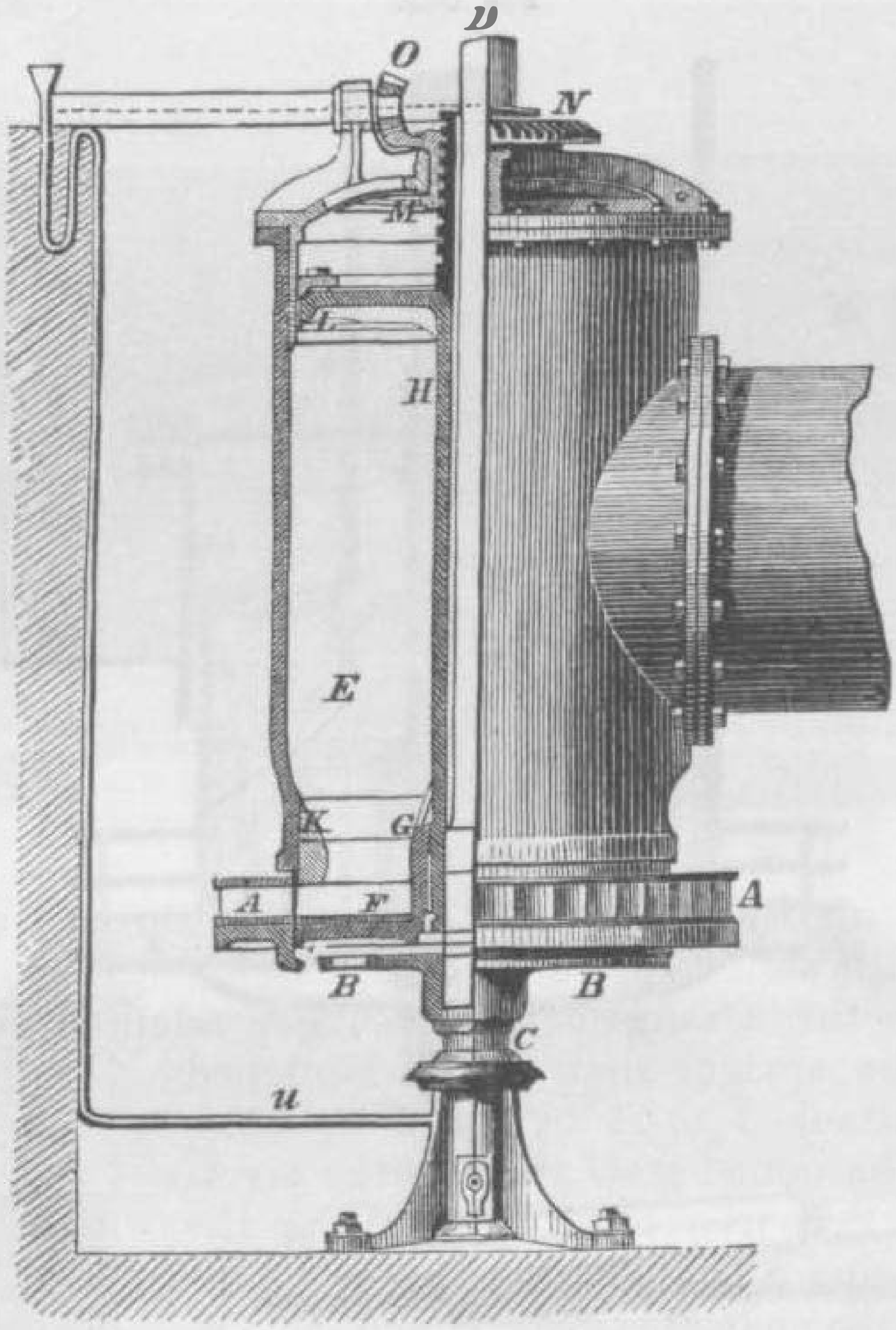
Fig. 262.



that its motion raises or lowers the pipe *GH*. There is also attached to *GH* a plate or piston *HL*, having a water-tight packing.

Fig. 263.



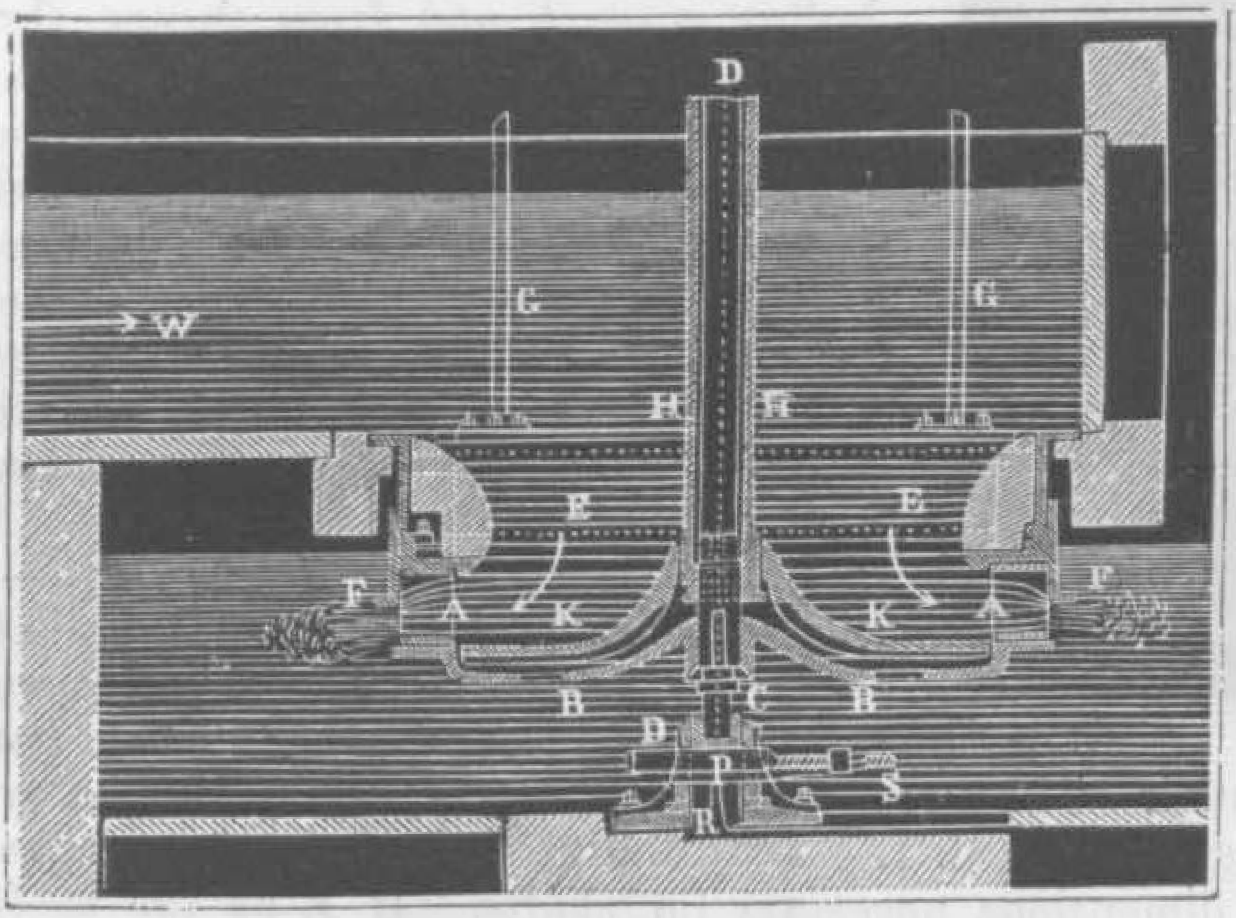

§ 144. *The Pivot and Footstep.*—The pivot and footstep are very important parts of the turbine. The weight of the turbine, often considerable, and the velocity of rotation, give rise to a great moment of friction on the pivot, which would wear very rapidly, unless it were well greased. It has been frequently observed that the pivot and brass of turbine axles wear much more rapidly than the pivots of other upright axles. This is attributable partly to the heating of the pivot from great velocity of rotation, but chiefly to the difficulty of lubricating the bearing-points which are under water. In order to meet this evil, turbine makers have endeavored to diminish the weight as much as possible, to increase the rubbing surface, to prevent the contact of the water with the rubbing surfaces, and also, to keep up a continuous supply of olive oil or nut oil, between the surfaces in contact.

At the upper end of the axle, there must of course be a collar or other support, in which it can revolve.

A very simple footstep, applicable chiefly when the weight is inconsiderable, is shown in Fig. 264. The pivot *C* rests in a *brass*

*D*, which is supported on a block, movable up or down, as may be required, by means of the *folding key PS*. The oil is supplied by

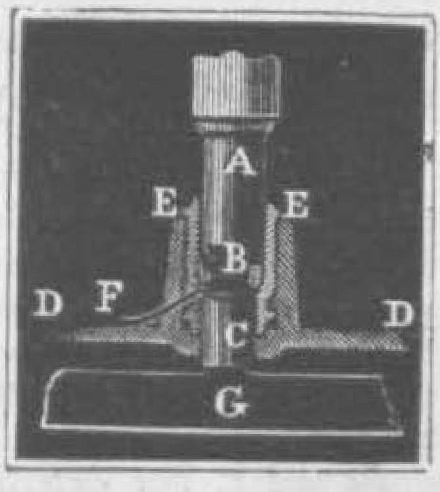
Fig. 264.




a pipe *R* passing by the side of the key, and through the bottom of the block and brass.

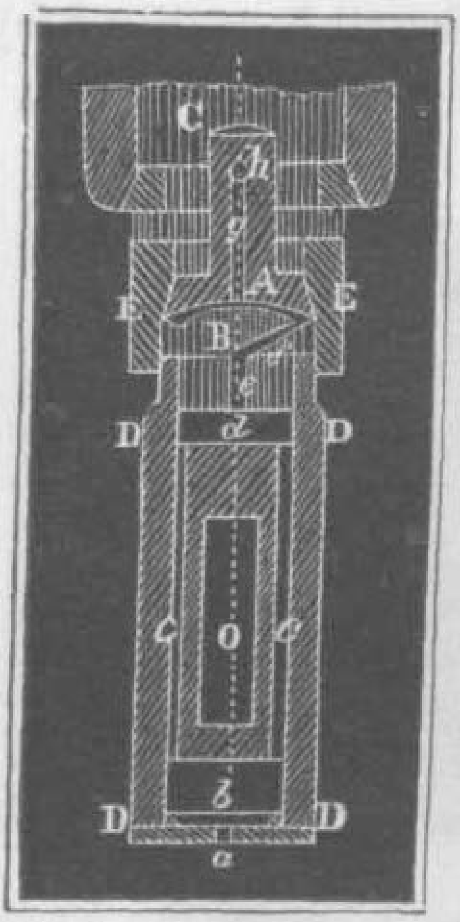
Fig. 265 is the arrangement of footstep adopted by Cadiat. *A* the foot of the upright shaft. *B* a hardened steel pivot attached to *A* by screw or welding. *C* is a hardened steel *step* for the pivot. *DEED* is a cast iron block or case for the step. *EE* being a brass casing. *F* is a pipe taking oil to the space between *B* and *E*. *G* is a lever for raising or depressing the turbine.

Fig. 265.




Fourneyron has very much complicated the construction of the pivot and footstep, to attain permanence. The general arrangement is shown in Fig. 262, and its details are shown in Fig. 266. Fig. 262 shows that the footstep *Z* is in a block which rests on a lever *OR*, turning round *O*, when elevated or depressed by the rod *RS*. *U* is a pipe for bringing oil to the footstep, the head of which should be as high as possible. That the circulation of the oil may be active, it should, at all events, be considerably above the surface of the water in the lead in low pressure turbines, and there should be a means of *forcing* in a supply for high pressure turbines. The parts *A* and *B* immediately in contact with each other are of hard steel. The upper part *A* is fixed in the shaft *G*, and the lower part *B* fits into a hollow piece *DD*, and is movable upwards and downwards by a lever supporting the whole, and passing through *O* (*OR*, Fig. 262). The surface of *A* is hollowed,

Fig. 266.




and the head of $B$ rounded, and both are surrounded by a collar $EE$, which keeps the oil between the rubbing surfaces. The oil, brought down in a pipe, enters at $a$ into the hollow space $b$, and from thence, through $c$, passes into the space $d$. Out of this it flows through three channels $e\,f$, on the periphery of the steel bearing, rising perpendicularly from the bottom and running inclined to the top, where three radial furrows serve to distribute it sufficiently. Lastly, there goes from the centre of $A$, a hole $gh$ into the axis, through which the oil escapes outwards, so that a circulation is maintained.

§ 145. *Strength or Dimensions.*—In designing a turbine for a certain fall of water, there is, besides the chief dimensions of the wheel itself, the strength of certain parts to be calculated. The strength of pipes, &c., is to be calculated by the formulas given in Vol. I. § 283; and at Vol. II. § 84, we have treated of the dimensions necessary for shafts. If $L$ be the useful effect of a turbine in horse's power, and $u$ the number of revolutions per minute, we have for the requisite diameter of the upright shaft

$$d = 6{,}12 \sqrt[3]{\frac{L}{u}} \text{ inches.}$$

The strength of the bottom plate, &c., is easily determined by reference to the theory of the strength of materials; but the nature of the casting fixes the dimensions, so that there can very rarely occur any necessity for calculation.

*Remark* 1. In the erection of turbines, not only the weight of the parts of the machine, but the water pressure, has to be considered. The latter has especially to be considered in high pressure turbines. It must not, for example, be lost sight of, that the water presses against the reservoir with a force equal to the weight of a column of water having the section of the pressure pipes as base, and the *head of water* as height; and that the knee piece on the pressure pipes tends to move with the same force, but in the opposite direction.

*Remark* 2. For determining the dimensions of the pivot, the rule which limits the pressure on every square inch of the brass to 1500 lbs., and that for a steel pivot on a steel bearing to 7000 lbs., might be used; but we know from what has been said above in reference to the wear of these points, that it is preferable to make the pivot only very little less than the diameter of the shaft. The above numbers refer to shafts having a moderate velocity of rotation, and as the wear increases as the weight, and as the velocity of rotation besides, it is evident that turbines, revolving with great speed, should have proportionally large pivots and bearings.

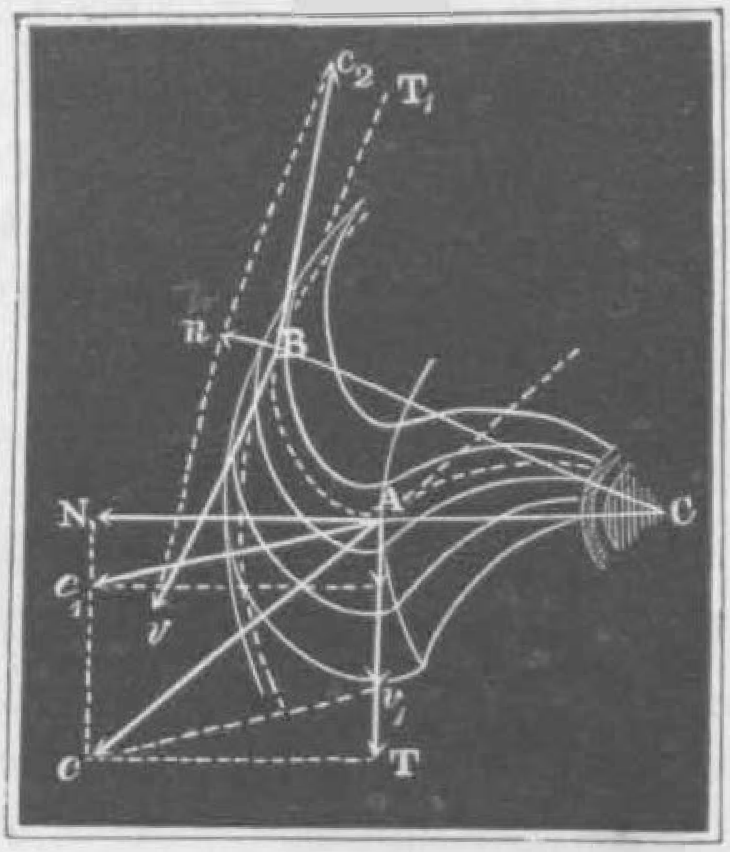
§ 146. *Theory of Reaction Turbines.*—For the investigation of the mechanical proportions and effects of Fourneyron's turbines, we shall use the following notation:

$r_1 = CA$, Fig. 267, the radius of the inside of the wheel, or approximately, that of the periphery of the bottom plate.

$r = CB$, the radius of the outside of the wheel.

$v_1 =$ the velocity of the interior periphery.

Fig. 267.




$v =$ the velocity of the exterior periphery.

$c =$ the velocity with which the water flows from the reservoir or guide-curves.

$c_1 =$ the velocity with which the water enters the wheel.

$c_2 =$ the velocity with which the water leaves the wheel.

$\alpha =$ the angle $cAT$ which the direction of the water leaving the reservoir makes with the inner circumference of the wheel.

$\beta =$ the angle $c_1AT$ made by the water entering the wheel buckets with the inner periphery of the wheel.

$\delta =$ the angle $c_2AT$ made by the water stream leaving the wheel with the outer periphery.

$F =$ the area of the orifices of discharge from the guide-curves.

$F_1 =$ the sum of the areas of the orifices by which the water enters the wheel.

$F_2 =$ the sum of the areas of the orifices at the outside of the buckets.

$h =$ the entire fall of water.

$h_1 =$ the height from surface of lead to centre of wheel, or centre of orifice of discharge from reservoir.

$h_2 = h_1 - h$ the depth of the entrance orifices to the wheel, below the orifices of discharge, or, if the wheel works under water, below the surface of the tail-race; and, lastly:

$x =$ the height, measuring the pressure of the water at the point where it passes from the reservoir or guide-curves into the wheel.

In the first place, for the velocity $c$ due to the difference of pressures $h_1 - x$, we have $\frac{c^2}{2g} = h_1 - x$, or, more accurately, if the water, in flowing from the guide-curves, loses an amount of hydraulic pressure $= \zeta \cdot \frac{c^2}{2g}$, then $(1 + \zeta)\frac{c^2}{2g} = h_1 - x$, therefore,

$$c = \sqrt{\frac{2g\,(h_1 - x)}{1 + \zeta}}, \text{ and, inversely, } x = h_1 - (1 + \zeta)\frac{c^2}{2g}.$$

In order that the water may enter the wheel without shock, it is necessary that the velocity of discharge should resolve itself into two components, the one of which must coincide in magnitude and direction with the velocity of the inner circumference of the wheel, and the other must coincide in direction with the stream entering the wheel conduits or channels. This being taken for granted: if the velocity with which the water begins to flow through the channels $Ac_1 = c_1$, we have it from the formula:

$$c_1^2 = c^2 + v_1^2 - 2\,c\,v_1 \cos.\,\alpha.$$

The velocity of discharge $c_2$ may be deduced from the pressure height $x$ and $h_2$ at entrance and exit, from the height $\frac{c_1^2}{2g}$, being that corresponding to the velocity of entrance, and from the increase of pressure height corresponding to centrifugal force of the water int the wheel, $\frac{v^2 - v_1^2}{2g}$ (Vol. II. § 143):

$\frac{c_2^2}{2g} = x - h_2 + \frac{c_1^2}{2g} + \frac{v^2 - v_1^2}{2g}$, or, substituting the above values of $x$ and $c_1$:

$$\frac{c_2^2}{2g} = h_1 - h_2 - (1 + \zeta)\frac{c^2}{2g} + \frac{c^2}{2g} + \frac{v^2}{2g} - \frac{2\,c\,v_1\,cos.\,\alpha}{2g};$$

or as $h_1 - h_2 = h$, the whole fall:

$$c_2^2 = 2gh + v^2 - 2\,c\,v_1\,cos.\,\alpha - \zeta\,.\,c^2.$$

If we further assume that, by friction and curvilinear motion in the wheel channels, there is a loss of hydraulic head $= \frac{x\,c_2^2}{2g}$, then, more accurately: $(1 + x)\,c_2^2 = 2gh + v^2 - 2\,c\,v_1\,cos.\,\alpha - \zeta\,.\,c^2$.

The quantity of water $Q = Fc = F_1 c_1 = F_2 c_2 \therefore c = \frac{F_2 c_2}{F}$, and $v_1 = \frac{r_1}{r}v$, and, therefore, we have for the velocity of the water leaving the wheel:

$$\left[1 + \zeta\left(\frac{F_2}{F}\right)^2 + x\right]c_2^2 + 2\frac{F_2}{F}\,.\,\frac{r_1}{r}\,c_2\,v\,cos.\,\alpha - v^2 = 2gh.$$

§ 147. *Best Velocity.*—In order to get the maximum effect from the water, the absolute velocity of the water leaving the wheel must be the least possible. But this velocity is the diagonal $Bw$ of a parallelogram constructed from the velocity of discharge $c_2$ and the velocity of rotation $v$:

$$w = \sqrt{c_2^2 + v^2 - 2\,c_2\,v\,cos.\,\delta} = \sqrt{(c_2 - v)^2 + 4\,c_2\,v\left(sin.\,\frac{\delta}{2}\right)^2};$$

and $\delta$ is to be made as small as possible, and $c_2 = v$. But in order that there may be free passage for the necessary quantity of water, it is not possible to make $\delta = 0$, but this has to be made 10° to 20°; whenever, therefore, we make $c_2 = v$, there remains the absolute velocity:

$$w = \sqrt{4\,c_2\,v\left(sin.\,\frac{\delta}{2}\right)^2} = 2\,v\,sin.\,\frac{\delta}{2},$$

and the loss of effect corresponding is:

$$\frac{w^2}{2g}\,Q\,\gamma = \frac{\left(2\,v\,sin.\,\frac{\delta}{2}\right)^2}{2g}\,Q\,\gamma.$$

We now perceive that the maximum effect is not got when $v = c_2$, but when $v$ is something less than $c_2$; but it is also manifest that for $v = c_2$, and for a small value of $\delta$, the deficiency below the maximum effect can only be very small. As, besides, in assuming $v = c_2$ we get very simple relations, we shall do so in the sequel, and introduce this into the last equation of the preceding paragraph. We then have:

$$\left[1 + \zeta\left(\frac{F_2}{F}\right)^2 + \varkappa\right] v^2 + 2\frac{F^2}{F} \cdot \frac{r_1}{r} v^2 \cos. \alpha - v^2 = 2gh, \text{ or,}$$

$$\left[2\frac{F_2}{F} \cdot \frac{r_1}{r} \cos. \alpha + \zeta\left(\frac{F^2}{F}\right)^2 + \varkappa\right] v^2 = 2gh;$$ and, therefore, the velocity of the wheel corresponding to the maximum effect required, is:

$$v = \sqrt{\frac{2gh}{2\frac{F_2}{F} \cdot \frac{r_1}{r} \cos. \alpha + \zeta\left(\frac{F_2}{F}\right)^2 + \varkappa}}.$$

Instead of the section ratio $\frac{F_2}{F}$, we may introduce the angle $\beta$. The unimpeded entrance of the water into the wheel requires that $c$ should not be altered in entering into it, or that the *radial component* of $c$, $AN = c \sin. \alpha$, should be equal to the radial component of $c_1$, i. e. $c_1 \sin. \beta$, and also to the tangential component $c \cos. \alpha$ of $c$ the tangential velocity $AT = c_1 \cos. \beta + v_1$ of the water already within the wheel. According to this:

$$\frac{c_1}{c} = \frac{\sin. \alpha}{\sin. \beta},\ c \cos. \alpha - c_1 \cos. \beta = v_1, \text{ and } \frac{c}{v_1} = \frac{\sin. \beta}{\sin. (\beta - \alpha)}.$$

Besides this, as $Fc = F_2 c_2 = F_2 v = \frac{r}{r_1} F_2 v_1$; we have

$$\frac{F_2}{F} = \frac{r_1}{r} \cdot \frac{c}{v_1} = \frac{r_1}{r} \cdot \frac{\sin. \beta}{\sin.c(\beta - \alpha)},$$

and the velocity of the outer periphery of the wheel:

$$v = \sqrt{\frac{2gh}{2\left(\frac{r_1}{r}\right)^2 \frac{\sin. \beta \cos. \alpha}{\sin. (\beta - \alpha)} + \zeta\left(\frac{r_1 \sin. \beta}{r \sin. (\beta - \alpha)}\right)^2 + \varkappa}}.$$

and hence the velocity of the inner periphery:

$$v_1 = \frac{r_1}{r} v = \sqrt{\frac{2gh}{2\frac{\sin. \beta \cos. \alpha}{\sin. (\beta - \alpha)} + \zeta\left(\frac{\sin. \beta}{r \sin. (\beta - \alpha)}\right)^2 + \varkappa\left(\frac{r}{r_1}\right)^2}}.$$

Neglecting the prejudicial resistances, we should have:

$$v_1 = \sqrt{\frac{gh \sin. (\beta - \alpha)}{\sin. \beta \cos. \alpha}} = \sqrt{gh\,(1 - \text{tang. } \alpha \text{ cotang. } \beta)}.$$

§ 148. *Pressure of the Water.*—With the aid of this formula for $v$, we can determine the pressure exerted on that part of the reservoir where the water passes from it on to the wheel; we have:

$$x = h_1 - (1 + \zeta)\frac{c^2}{2g} = h_1 - (1 + \zeta)\frac{v_1^2}{2g}\left(\frac{\sin. \beta}{\sin. (\beta - \alpha)}\right)^2$$

$$= h_1 - \frac{(1+\zeta)\, h\, sin.\, \beta^2}{2\, sin.\, \beta\, cos.\, \alpha\, sin.\, (\beta - \alpha) + \zeta\, sin.\, \beta^2 + \varkappa \left(\frac{r}{r_1}\right)^2 [sin.\, (\beta - \alpha)]^2}$$

$$= h_2 - \frac{(1+\zeta)\, h}{1 + cos.\, 2\, \alpha - cotg.\, \beta\, sin.\, 2\, \alpha + \zeta + \varkappa \left(\frac{r}{r_1}\right)^2 \left(\frac{sin.\, (\beta - \alpha)}{sin.\, \beta}\right)^2}.$$

Neglecting prejudicial resistances, we have:

$$x = h_1 - \frac{h}{1 + cos.\, 2\, \alpha - cotg.\, \beta\, sin.\, 2\, \alpha}.$$

If the turbine work free of back water, we have, in the case of the turbines of Fourneyron, Cadiat, and Whitelaw, $h_1 = h$, and, therefore,

$$x = \frac{cos.\, 2\, \alpha - cotg.\, \beta\, sin.\, 2\, \alpha}{1 + cos.\, 2\, \alpha - cotg.\, \beta\, sin.\, 2\, \alpha}\, h\,;$$

if, however, the turbine works in back water, then $h_1 = h + h_2$, and, therefore,

$$x = \frac{cos.\, 2\, \alpha - cotg.\, \beta\, sin.\, 2\, \alpha}{1 + cos.\, 2\, \alpha + cotg.\, \beta\, sin.\, 2\, \alpha} \cdot h + h_2.$$

If, in the first case, the pressure is to be $= 0$, *i. e.*, equal to the atmospheric pressure, then $x = 0$; but if in the second case, it must be equal to the pressure of the back water against the orifices of the wheel, then $x = h_2$, but in both cases we should have $cos.\, 2\, \alpha - cotg.\, \beta\, sin.\, 2\, \alpha = 0$, i. e., $tang.\, \beta = tang.\, 2\, \alpha$, or $\beta = 2\, \alpha$.

*If, therefore, the angle of the water's entrance $\beta$ be twice the angle of exit $\alpha$, the pressure at the point where the water passes from the reservoir to the wheel, is equal to the external pressure of the atmosphere, or of the back water.*

On the other hand, it is easy to perceive that this internal pressure is greater than the external pressure, if $\beta > 2\, \alpha$, and it is less than this when $\beta < 2\, \alpha$. These relations are somewhat different when the prejudicial resistances are taken into account, as it is very proper to do. The equation between the external and internal pressure then stands thus:

$$1 + cos.\, 2\, \alpha - cotg.\, \beta\, sin.\, 2\, \alpha + \zeta + \varkappa \left(\frac{r}{r_1}\right)^2 \left(\frac{sin.\, \beta - \alpha)}{sin.\, \beta}\right)^2 = 1 + \zeta,$$

or $cotg.\, \beta\, sin.\, 2\, \alpha = cos.\, 2\, \alpha + \varkappa \left(\frac{r}{r_1}\right)^2 (cos.\, \alpha - cotg.\, \beta\, sin.\, \alpha)^2.$

If, in the last member, we introduce:

$$cotg.\, \beta = cotg.\, 2\, \alpha = \frac{cos.\, 2\, \alpha}{sin.\, 2\, \alpha},$$

that is:

$$cotg.\, \beta\, sin.\, 2\, \alpha = cos.\, 2\, \alpha + \varkappa \left(\frac{r}{r_1}\right)^2 \left(\frac{sin.\, \alpha}{sin.\, 2\, \alpha}\right)^2$$

$$= cos.\, 2\, \alpha + \varkappa \left(\frac{r}{r_1}\right)^2 \cdot \frac{1}{4\, (cos.\, \alpha)^2},$$

it follows that:

$$tang.\,\beta = \frac{sin.\,2\,\alpha}{cos.\,2\,\alpha + x\left(\frac{r}{r_1}\right)^2 \cdot \frac{1}{4\,(cos.\,\alpha)^2}},$$

consequently, $\beta$ is somewhat smaller than $2\,\alpha$.

If we neglect again $\zeta$ and $x$, we have, by introducing the value $\beta = 2\,\alpha$.

$$v_1 = \sqrt{gh\,(1 - tang.\,\alpha\; cotg.\,2\,\alpha)} = \sqrt{\frac{gh\,(1 + tang.\,\alpha^2)}{2}} = \frac{\sqrt{\frac{1}{2}\,gh}}{cos.\,\alpha},$$

and $c = \sqrt{2gh}$. If the internal pressure be greater than the external, then $v_1 > \frac{\sqrt{\frac{1}{2}gh}}{cos.\,\alpha}$, and $c < \sqrt{2gh}$, and if it be less than this, then $v_1 < \frac{\sqrt{\frac{1}{2}gh}}{cos.\,\alpha}$, and $c > \sqrt{2gh}$.

§ 149. The discussion as to pressures in the last paragraph is of great importance in the question of the construction of turbines; for the point of transit from the reservoir to the wheel cannot be made water-tight, and, therefore, water may escape, or water and air get in, by the annular aperture. That neither of these circumstances may occur, turbines must be so constructed, that the internal pressure of the water passing the slit may equal the external pressure of the atmosphere, or of the back water, if the wheel be submerged; we must, in short, have $\beta = 2\,\alpha$, or, better still, satisfy the equation:

$$tang.\,\beta = \frac{sin.\,2\,\alpha}{cos.\,2\,\alpha + x\left(\frac{r}{r_1}\right)^2 \cdot \frac{1}{(2\,cos.\,\alpha)^2}}.$$

Turbines are constructed so that, in the normal state of the sluice being fully opened, the above equation is satisfied, or, so that a slight excess of pressure $x$ exists, at the risk of losing some water through the space left between the bottom plate and the wheel, thus providing for variations of velocity of discharge, from variations in the area of the section $F$ of the orifices by the different positions of the turbine-sluice, regulating the quantity discharged.

§ 150. *Choice of the Angles $\alpha$ and $\beta$.*—If we do not take the internal pressure into consideration, the angles $\alpha$ and $\beta$ may have very arbitrary values. The formula

$$v_1 = \sqrt{gh\,(1 - tang.\,\alpha\; cotg.\,\beta)} = \sqrt{gh\left(1 - \frac{tang.\,\alpha}{tang.\,\beta}\right)}$$

gives an impossible value for $v_1$, when $\frac{tang.\,\alpha}{tang.\,\beta} > 1$, that is, when $\alpha < 90°$, and $\beta < \alpha$, or when $\alpha > 90°$, and $\beta > \alpha$. These values of $\alpha$ and $\beta$ are, therefore, not to be admitted. If $\alpha = \beta$, then $v_1 = 0$, and hence we see that the best velocity of rotation becomes so much the less, the nearer the angles $\alpha$ and $\beta$ approach equality. The formulas $c = \frac{v_1\,sin.\,\beta}{sin.\,(\beta - \alpha)}$, and $F_2 = \frac{r_1}{r} \cdot \frac{sin.\,\beta}{sin.\,\beta - \alpha)}\,F$, given negative values, involving impossible conditions, when $\beta \ngtr \alpha$. It is, therefore, necessary, in the construction of turbines, that $\beta > \alpha$, and that $\alpha < 90°$.

Between these limits, we may choose various values for $\alpha$ and $\beta$, although they do not all lead to an equally convenient or advantageous construction. Fourneyron makes $\beta = 90°$, and $\alpha = 30°$ to $33°$, some constructors make $\beta$ less, and others greater than $90°$. When $\beta$ is made $90°$, or less, the curvature of the buckets is greater than when $\beta$ is made more obtuse. Great curvature involves greater resistance in the efflux, and hence it is advisable to make $\beta$ rather obtuse than acute, that is, to make $\beta = 100°$ to $110°$. The angle $\alpha$ must then be $50°$ to $55°$, if the internal pressure is to balance the external. In order, however, that the channels formed by the guide-curves may not diverge too much, and that the equilibrium of pressure may not be disturbed by depressions of the sluice, this angle is made from $30°$ to $40°$, and if the turbine revolves free of back water, then $\alpha$ should not be more than $25°$ to $30°$; $\alpha$ is, however, never made very acute, because with the angle $\alpha$, the area of the orifices of discharge varies, and, therefore, the quantity of water discharged would diminish, or the diameter of the wheel must be greater for a given quantity of water expended. On the other hand, we have to bear in mind, that the losses of effect increase as $v^2$, and that, therefore, *cæteris paribus*, a turbine will yield a greater effect —will be more efficient—when revolving slowly, than when it has a great velocity of rotation. According to this, the construction should be so arranged that $\alpha$ and $\beta$ do not differ widely from one another, from which would follow an internal pressure less than the external. If $a$ be the height of a column of water balancing the atmospheric pressure, the absolute pressure of the water as it passes over the space between the wheel and bottom plate is $a + x$, and if this pressure $= 0$, then the water flows with a maximum velocity $c = \sqrt{2g\,(h_1 - x)} = \sqrt{2g\,(h_1 + a)}$ from the reservoir. If $a + x$ were negative, or $x < -a$, there would arise a vacuum at the point of passage of the water into the wheel, for the water would flow even faster through the wheel than it flowed on to it from the reservoir, and so air would rush in from the exterior, greatly disturbing the flow of the water. If, therefore, we introduce into the formula, instead of

$$x = h - \frac{h}{1 + cos.\, 2\,\alpha - cotg.\, \beta\, sin.\, 2\,\alpha}.$$

$x = -a$, we then have:

$$1 + cos.\, 2\,\alpha - cotg.\, \beta\, sin.\, 2\,\alpha = \frac{h}{h + a}, \text{ hence:}$$

$$tang.\, \beta = \frac{sin.\, 2\,\alpha}{1 + cos.\, 2\,\alpha - \frac{h}{h + a}} = \frac{(h + a)\, sin.\, 2\,\alpha}{(h + a)\, cos.\, 2\,\alpha + a},$$

and, therefore, the corresponding best velocity of rotation:

$$v_1 = \sqrt{gh\left(1 - tang.\,\alpha\,.\,\frac{(h+a)\,cos.\,2\,\alpha + a}{(h+a)\,sin.\,2\,\alpha}\right)} = \frac{h}{cos.\,\alpha}\sqrt{\frac{g}{2(h+a)}}.$$

§ 151. *Turbines without Guide-Curves.* — For turbines without guide-curves we may make $\alpha = 90^\circ$, because the water flows by the shortest way, or radially, out of the reservoir. In this light we have to consider the turbines of Combe, Cadiat, and Whitelaw. If we introduce into the formula for the best velocity $\alpha = 90^\circ$, we get:

$$v_1 = \sqrt{\frac{2gh}{\frac{2\,sin.\,\beta\,cos.\,90^\circ}{cos.\,\beta} + \zeta\left(\frac{sin.\,\beta}{cos.\,\beta}\right)^2 + x\left(\frac{r}{r_1}\right)^2}}.$$

$$= \sqrt{\frac{2gh}{\zeta\,(tang.\,\beta)^2 + x\left(\frac{r}{r_1}\right)^2}};$$ neglecting prejudicial resistances,

however, $v_1 = \sqrt{\frac{2gh}{0}} = \infty$. But, for two reasons, the wheel cannot acquire an infinite velocity. There is a limit, in the first place, when the mechanical effect at disposition is absorbed in overcoming prejudicial resistances, that is, when

$$Q\,h\,\gamma = \left(\frac{w^2}{2g} + \zeta\,\frac{c^2}{2g} + x\,.\,\frac{c_2^{\,2}}{2g}\right)Q\gamma,\ i.\ e.,$$

$$h = \left[\left(2\,sin.\,\frac{\delta}{2}\right)^2 + \zeta\left(\frac{r_1}{r}\,tang.\,\beta\right)^2 + x\right]\frac{v^2}{2g},\ \text{or},$$

$$v = \sqrt{\frac{2gh}{\left(2\,sin.\,\frac{\delta}{2}\right)^2 + \zeta\left(\frac{r_1}{r}\,tang.\,\beta\right)^2 + x}},$$

and in the second place when

$$x = -a,\ \text{i. e.}\ h - \frac{c^2}{2g} = -a,\ \text{or}\ \frac{c^2}{2g} = a + h,\ \text{or},$$

$$\frac{1}{2g}\left(\frac{r_1}{r}\,.\,\frac{v\,sin.\,\beta}{sin.\,(\beta - 90^\circ)}\right)^2 = a + h,\ \text{that is, when}$$

$v = \frac{r}{r_1}\,cotg.\,\beta\,\sqrt{2g\,(a + h)}$, because then the full discharge by *full flow* ceases, and the circumstances are quite changed, seeing that the water cannot flow from the reservoir in quantities sufficient to supply the discharge of the wheel channels, when their section is filled.

If we introduce into the above formula:

$$v_1 = \sqrt{\frac{2gh}{\zeta\,(tang.\,\beta)^2 + x\left(\frac{r}{r_1}\right)^2}},$$

the experimental co-efficients $\zeta$ and $x$, it is still far from giving us $v = \infty$. Now for the most accurate construction of the guide-curve apparatus, the co-efficient of velocity $\phi$ is not greater than

0,93, and, therefore, the co-efficient of resistance $\zeta$ corresponding $= \frac{1}{\phi^2} - 1 =$ not less than $\frac{1}{0{,}93^2} - 1 = 0{,}16$, or about 16 per cent. In the case of turbines without this apparatus, this resistance does not exist; but still there remains always a certain loss for the entrance into the wheel channels, which, in Combe's and Cadiat's wheel, does not probably exceed 5 per cent. though in Whitelaw's it may be taken as 10 per cent. at least, for the channels are too wide to admit of the supposition that the whole stream of water has the definite direction ($\beta$). The co-efficient $\varkappa$ corresponding to the resistance from the curvature and friction in the channels may be set at from 0,5 to 0,15, as we shall see in the sequel, and hence, for turbines without guide-curves, we have, putting $\varkappa = 0{,}1$, the best velocity

$$v_1 = \sqrt{\frac{2gh}{0{,}05\,(tang.\ \beta)^2 + 0{,}1\left(\frac{r}{r_1}\right)^2}},$$

and for Whitelaw's reaction wheelc

$$v_2 = \sqrt{\frac{2gh}{0{,}1\,(tang.\ \beta)^2 + 0{,}1\left(\frac{r}{r_1}\right)^2}}.$$

If, again, we put $\beta = 60°$, and $\frac{r}{r_1} = \frac{4}{3}$, we have, in the first case:

$$v_1 = \sqrt{\frac{2gh}{0{,}148 + 0{,}178}} = 1{,}75\sqrt{2gh},$$

and in the second case:

$$v_1 = \sqrt{\frac{2gh}{0{,}296 + 0{,}178}} = 1{,}45\sqrt{2gh}.$$

For other reasons, we shall hereafter learn that the most advantageous velocity of rotation is not even equal to the velocity due to the height $h$.

In order that the water may enter the wheel with the least shock possible, in the case of wheels without guide-curves, the equation $\frac{F_2}{F} = \frac{r_1}{r}\ tang.\ \beta$ must be satisfied. But as $F_2$ is determined by the relative position of the sluice, it follows that the maximum effect is got for a certain position of the sluice.

§ 152. *Influence of the Position of the Sluice.*—In one point of view, turbines are inferior to overshot and breast wheels. When in these wheels there is only a small quantity of water available, or it is only required to produce a portion of the power of the *fall*, and for this purpose we partially close the sluice, we know that the efficiency of such wheels is rather increased than diminished from the cells being proportionally less filled. In the turbine the contrary is the case, for as the sluice is lowered, the water enters the wheel under circumstances involving greater loss of effect. This is a circumstance so much the more unfavorable, inasmuch as it is generally requisite to economize power the more, as the water supply fails. The loss, however, by lowering the sluice, is never very great, as the following investigation proves.

If we decompose the velocities $c$ and $c_1$ into their radial and tangential components $c\ sin.\ \alpha$, $c\ cos.\ \alpha$, $c_1\ sin.\ \beta$ and $c_1\ cos.\ \beta$, and subtract the two from each other, there remain the relative velocitiese

$$c\ sin.\ \alpha - c_1\ sin.\ \beta, \text{ and } c\ cos.\ \alpha - c_1\ cos.\ \beta;$$

as, however, the water has the velocity $v_1$ in common with the wheel, the latter relative velocity is in fact $= c\ cos.\ \alpha - c_1\ cos.\ \beta - v_1$. According to a known law, the loss of pressure height corresponding to a sudden cessation of this velocity is:

$$y = \frac{1}{2g}\left[(c\ sin.\ \alpha - c_1\ sin.\ \beta)^2 + (c\ cos.\ \alpha - c_1\ cos.\ \beta - v_1)^2\right],$$

or, in mechanical effect:

$$Y = y\ Q\gamma = \left[(c\ sin.\ \alpha - c_1\ sin.\ \beta)^2 + (c\ cos.\ \alpha - c_1\ cos.\ \beta - v_1)^2\right]\frac{Q\gamma}{2g}.$$

If we introduce into this formula $c_2 = v$ and $v_1 = \frac{r_1}{r}\ v$, further $c = \frac{F_2}{F} v$ and $c_1 = \frac{F_2}{F_1} v$, we have, as the loss of mechanical effect:

$$Y = \left[\left(\frac{F_2\ sin.\ \alpha}{F} - \frac{F_2\ sin.\ \beta}{F_1}\right)^2 + \left(\frac{F_2\ cos.\ \alpha}{F} - \frac{F_2\ cos.\ \beta}{F_1} - \frac{r_1}{r}\right)^2\right]\frac{v^2}{2g}\ Q\gamma e$$

From this we may judge as to the loss of effect in turbines that do not fulfil the conditions expressed in the equation:

$$F_1\ sin.\ \alpha = F\ sin.\ \beta \text{ and } F_1\ cos.\ \alpha = F\ cos.\ \beta + \frac{FF_1}{F_2}\cdot\frac{r_1}{r}.$$

However, even if these conditions be fulfilled in the normal state of the turbines' working, *i. e.*, when the sluice is fully drawn, they cannot be so when the sluice is depressed, and $F$ becomes $F_x$. The loss of mechanical effect then, even when the effect is a maximum, viz.: $c_2 = v$, is:

$$Y = \left[\left(\frac{F_2\ sin.\ \alpha}{F_x} - \frac{F_2\ sin.\ \beta}{F_1}\right)^2 + \left(\frac{F_2\ cos.\ \alpha}{F_x} - \frac{F_2\ cos.\ \alpha}{F_1} - \frac{r_1}{r}\right)^2\right]\frac{v^2}{2g}\ Q\gamma,$$

or substituting $F\ sin.\ \beta = F_1\ sin.\ \alpha$ and $F\ cos.\ \beta + \frac{FF_1}{F_2}\cdot\frac{r_1}{r} = F_1\ cos.\ \alpha$,

$$Y = \left[\left(\frac{1}{F_x} - \frac{1}{F}\right)^2 (F_2\ sin.\ \alpha)^2 + \left(\frac{1}{F_x} - \frac{1}{F}\right)^2 (F_2\ cos.\ \alpha)^2\right]\frac{v^2}{2g}\ Q\gamma$$

$$= \left(\frac{F_2}{F_x} - \frac{F_2}{F}\right)^2 \frac{v^2}{2g}\ Q\gamma.$$

If, as an example, we put $\frac{v_1^2}{2g} = \frac{1}{2} h$, which is allowable in Fourneyron's turbines, we have:

$$Y = \left(\frac{F_2}{F_x} - \frac{F_2}{F}\right)^2 \cdot \left(\frac{r}{r_1}\right)^2 \cdot \tfrac{1}{2}\ Q h\gamma;$$

or, for the sluice half open, in which case

$$F_x = \tfrac{1}{2} F,\ Y = \tfrac{1}{2}\left(\frac{F_2 r}{F r_1}\right)^2 Q\, h\, \gamma.$$

We see from this, that this loss may be diminished by making the ratios $\frac{F_2}{F}$ and $\frac{r}{r_1}$ small, that is to say, by making the orifice of discharge of the wheel and the external radius small, but keeping the orifices and radius of the reservoir large. As:

$$\frac{F_2}{F} = \frac{r_1 \sin.\ \beta}{r \sin.\ (\beta - \alpha)},$$

we have in the last case

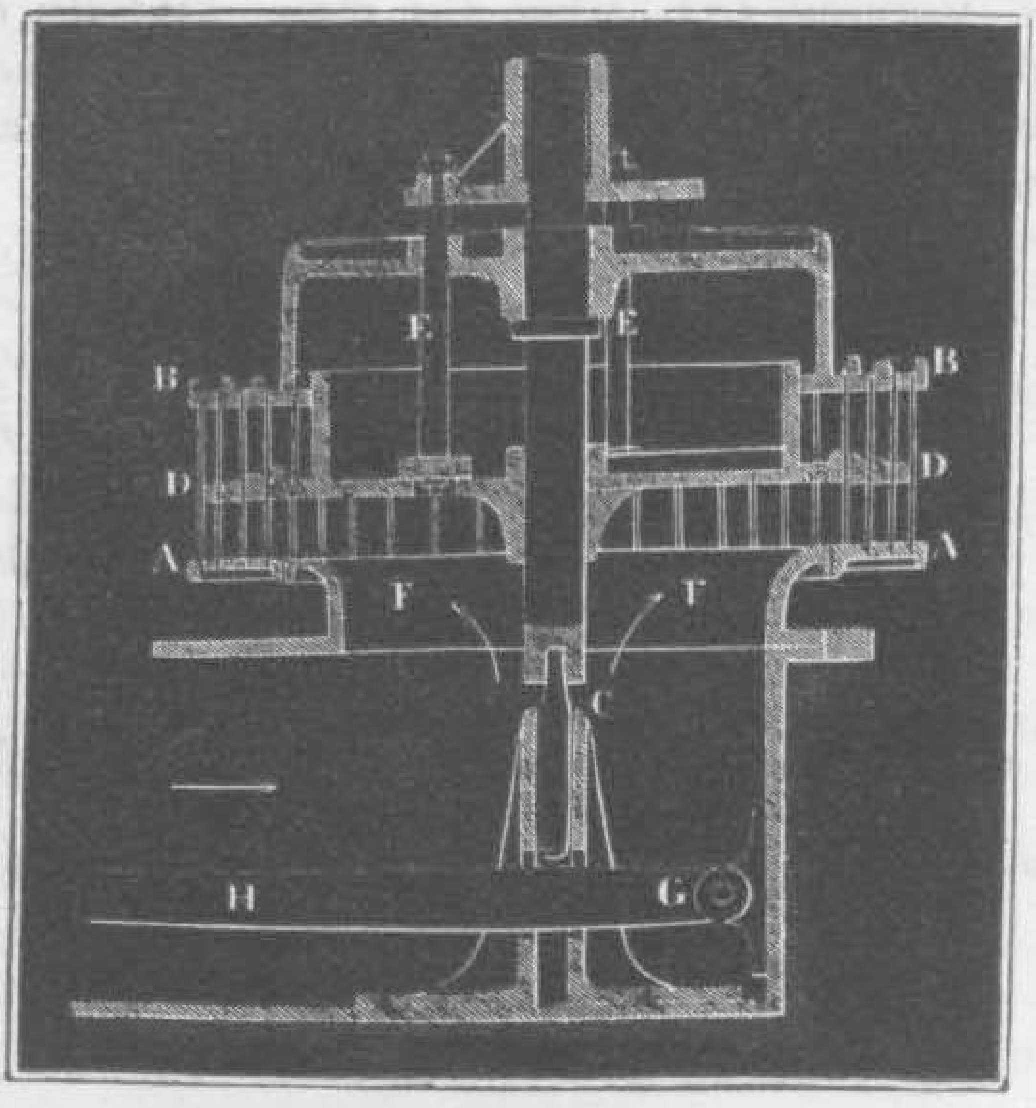
$$Y = \tfrac{1}{2}\left(\frac{\sin.\ \beta}{\sin.\ (\beta - \alpha)}\right)^2 Q\, h\, \gamma,$$

and, therefore, for $\beta = 90°$, and $\alpha = 40°$, $Y = 0{,}57\ Q\, h\, \gamma$, or there is in this case a loss of 57 per cent. of the effect.

Generally, when the sluice is much depressed, when $F_x < \frac{1}{2} F$, the *full discharge* ceases, that is, the water no longer fills up the wheel channels, the wheel becomes a pressure turbine only.

§ 153. *Sluice Adjustment.*—To avoid, or at least to diminish the loss of mechanical effect which results from lowering the sluice, and in order to retain the *full flow* of water through the wheel, many devices have been recently introduced by Fourneyron and others. Fourneyron divides the whole depth of the wheel into stages, by introducing horizontal annular division plates, dividing the total depth into two or three separate spaces, so that, when the sluice is lowered, one or two of the spaces are completely shut off, and the water flows through the other subdivision. This arrangement does

Fig 268.



not entirely fulfil its object; but the apparatus shown in Fig. 268, invented by Combes, does. This contrivance consists in a plate or disc $DD$, between the two shroudings of the wheel, which, by means of rods $EE$, can be raised or depressed by means of a simple mechanism, so that the water flowing through the wheel always fills up the channels open to it. This apparatus fulfils the required conditions, but it is difficult, and very costly in construction.

The turbines of Callon, and also those of Gentilhomme, are likewise constructed so that the water may fill up the channels, how small soever the quantity supplied.

Fig. 269 represents a part of Callon's wheel in elevation and section. This shows that the guide-curves are covered in on top, and in the inside by sluices $E, E \ldots$, each of which closes two apertures. To regulate the discharge of water by this arrangement, it is only necessary to keep a certain number of apertures closed. Although this arrangement certainly provides against impact on the wheel, yet it is imperfect, inasmuch as the water can work little, if at all, by reaction, as it does not run through the wheel channels in an unbroken stream. In this alternate filling and emptying of the wheel-channels, the velocities $c$, $c_1$ and $c_2$ undergo continual variations unless $x = 0$, that is $\beta = 2\alpha$. If, for example, the wheel-channels not being filled, $c = \sqrt{2gh_1}$, we should have $c = \sqrt{2g(h - x)}$, when the water stream filled up the channels. Thus for each filling and emptying, or while the wheel passes from one open aperture to another, the velocity $c$ continually oscillates between the limits $\sqrt{2gh}$, and $\sqrt{2g(h - x)}$. As the maximum effect can only be obtained for determinate values of $v$ and $c_2 = \frac{Fc}{F_2}$, it is quite evident that, as $c_2$ varies, we fail in this.

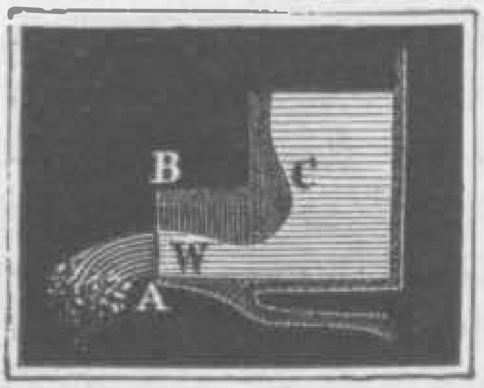
Fig. 269.


For Gentilhomme's turbine, the same object is attained by circular sectors, which are so placed by means of mechanism, that they close a part of the guide-curve apparatus; an arrangement evidently even more imperfect than Callon's. Hänel, a German engineer, describes an arrangement of sluice for effecting the objects now in question, very similar to that of Combe's. (*See* "Deutsche Gewerbzeitung, 1846.)

§ 154. *Pressure Turbines.*—This is the place to compare the reaction turbines hitherto under discussion with the *impact* and pressure turbines, into which they always become converted when the sluice $C$, Fig. 270, closes the greater part of the depth of the wheel

*AB*. As the water *W* only partially fills the section of the wheel channels, the remainder is filled with air, unless the wheel works free of back water, and, therefore, the pressure on the outside of the wheel is that of the atmosphere; the velocity is $c = \sqrt{2gh}$, and is independent of the motion of the wheel. But for the velocity of discharge we have $c_2^2 = 2gh + v^2 - 2\,c\,v_1 \cos.\,\alpha$, and for the maximum effect $c_2 = v$, and then for these wheels, the expressionc

Fig. 270.




$2\,c\,v_1 \cos.\,\alpha = 2gh$, or substituting $c = \sqrt{2gh}$, $v_1 = \frac{\sqrt{2gh}}{2 \cos.\,\alpha}$, obtains.

In the case of reaction wheels, we found:

$$v_1 = \sqrt{gh\,(1 - tang.\,\alpha\, cotg.\,\beta)};$$

and hence we perceive that the conditions of maximum effect are identical in both cases, if $\frac{1}{2 \cos.\,\alpha^2} = 1 - tang.\,\alpha\; cotg.\,\beta$, or if $tang.\,\beta = tang.\,2\,\alpha$, that is $\beta = 2\,\alpha$; which result we have already ascertained in the form of the condition that $x = 0$. There is, therefore, an essential difference in the turbines of the two classes, in as far as the velocity for the maximum of effect does not in the one depend upon $\beta$, whilst it does in the other; and it is only when $\beta = 2\,\alpha$ that this velocity is the same for both. While, therefore, for reaction wheels the velocity $v_1$ may be made to vary within wide limits by selection of the angle $\beta$, we have no such joice in the case of impact turbines.

In reference to the effect of both wheels, we adduce the following facts. When, in a reaction wheel, the sluice is gradually lowered, the efficiency diminishes, and when it is so far lowered that the water does not fill up the wheel channels, the turbine then passing into a pressure-turbine, the efficiency suddenly rises, because the loss of mechanical effect by the sudden change of velocity ceases. By lowering the sluice still further, the further loss of effect is inconsiderable. According to this, the pressure turbine seems to be a better wheel than the reaction turbine; but from other circumstances the advantages do not preponderate, and can only be accorded when the supply of water is liable to much fluctuation.

As the water entering the wheel finds a much greater sectional area than at its velocity it can fill, it gets into an irregular oscillatory motion, and not only does not discharge with the velocity $c_2$ above calculated, but also loses a part of its *vis viva* absorbed in creating the eddying irregular motion alluded to (and which no doubt is *consumed in raising the temperature* of the water, Tr.). Numerous experiments have proved this, and these may be repeated with any turbine, if it be made to revolve with the best velocity, first as a reaction wheel, and then as a pressure wheel. Turbines always give a greater effect for an open sluice and full discharge than when the sluice is lowered and the water does not fill the section of the wheel's channels.

When turbines work under water, the flow is always *full* through them, and these wheels are, therefore, always *reaction* wheels. A greater efficiency is naturally to be expected from these when the sluice is fully opened, than from the turbines revolving free of back water; on the other hand, we may safely assume that, when the sluice is lowered so that only $\frac{2}{3}$ or less of the depth of wheel is open, the efficiency of the reaction wheel will be less than that of the pressure turbine. From this we can easily understand the great advantage of introducing the horizontal dividing plates.

*Remark.* All the older turbines of Fourneyron were pressure turbines; but, as experience pointed out the greater efficiency of the reaction turbines, almost all turbines are now reduced to this principle.

§ 155. *Mechanical Effect of Turbines.*—We can now calculate the mechanical effect of turbines. The effect, which is not taken from the water if it flow from the wheel with an absolute velocity:

$$w = \sqrt{c_2^2 + v^2 - 2\, c_2\, v\, cos.\ \delta}, \text{ or if } c_2 = v,\ w = 2\, v\, sin.\ \frac{\delta}{2},$$

$$\text{is, } L_1 = \frac{w^2}{2g} Q \gamma = \frac{4\, v^2 \left(sin.\ \frac{\delta}{2}\right)^2}{2g} Q \gamma.$$

The effect which the water loses in the guide-curve apparatus, or in getting into the wheel, is:

$$L_2 = \zeta \frac{c^2}{2g} Q \gamma = \zeta \left(\frac{F_2}{F}\right)^2 \cdot \frac{v^2}{2g} Q \gamma,$$

in which $\zeta = 0{,}10$ to $0{,}20$, according as the wheel is provided with guide-curves or not.

The third loss of effect is $= \varkappa \cdot \frac{c_2^{\,2}}{2g} Q \gamma$, and consists of the friction and curve resistance. The resistance in passing round the curve may be found by the rules in Vol. I. § 334. The corresponding loss of head $= \zeta_1 \frac{\phi}{\pi} \cdot \frac{1}{2g} \cdot \left(\frac{Q}{F}\right)^2$ in which $\zeta$ is a co-efficient dependent on the ratio $\frac{d}{2\,a}$ of the half of the mean width of the channel to the mean radius, $\phi$ the central angle, $F$ the sum of the mean section of the channels. If $n$ be the number of channels or of buckets, and if $e$ be the mean height of a channel, then $F = n\, d\, e$, and, therefore, the height of head lost by the resistance in the curves:

$$y = \zeta_1 \frac{\phi}{\pi} \cdot \frac{1}{2g} \cdot \left(\frac{Q}{n\, d\, e}\right)^2, \text{ or if we put:}$$

$$\zeta_1 = 0{,}124 + 3{,}104 \left(\frac{d}{2a}\right)^{\frac{7}{2}} \text{ (as in Vol. I. § 334), then:}$$

$$y = \left[0{,}124 + 3{,}104 \left(\frac{d}{2a}\right)^{\frac{7}{2}}\right) \cdot \frac{\phi}{\pi} \cdot \frac{1}{2g} \left(\frac{Q}{n\, d\, e}\right)^2\Big]$$

and the loss of mechanical effect corresponding, is:

$$L_3 = \left[0{,}124 + 3{,}104\left(\frac{d}{2a}\right)^{\frac{7}{2}}\right] \cdot \frac{\phi}{\pi} \cdot \frac{1}{2g}\frac{Q^3\gamma}{(n\,d\,e)^2},$$

or putting $Q^2 = (F_2\,v)^2$,

$$L_3 = \left[0{,}124 + 3{,}104\left(\frac{d}{2a}\right)^{\frac{7}{2}}\right]\frac{\phi}{\pi} \cdot \left(\frac{F_2}{n\,d\,e}\right)^2 \cdot \frac{v^2}{2g} \cdot Q\,\gamma.$$

The wheel buckets consist usually of two parts of different curvatures, and, hence, $L_3$ would be made up of two items. It is evident from the above, that this source of resistance increases the wider the channels are, and the less the radius of curvature $a$. Hence it is advisable to make $\beta$ *obtuse*, so as to diminish the curvature of the buckets, which iscalso advisable in respect of a *full flow* through them.

The resistance from friction is to be calculated according to Vol. I. § 330. If $\zeta_2$ be the co-efficient of friction, $p$ the mean periphery, and $l$ the length of the wheel channel, the height due to the resistance from friction:

$$z = \zeta_2 \cdot \frac{p\,l}{de} \cdot \frac{1}{2g} \cdot \left(\frac{Q}{n\,d\,e}\right)^2 = \zeta_2 \cdot \frac{p\,l}{de} \cdot \left(\frac{F_2}{n\,d\,e}\right)^2 \cdot \frac{v^2}{2g},$$

and the loss of effect corresponding:

$$L_4 = \zeta_2 \cdot \frac{p\,l}{de} \cdot \left(\frac{F_2}{n\,d\,e}\right)^2 \cdot \frac{v^2}{2g}\,Q\,\gamma;$$

or, if $p = 2\,(d + e)$, and $\zeta_2 = 0{,}0144 + 0{,}0169\sqrt{\frac{n\,d\,e}{Q}}$ be introduced:

$$L_4 = \left(0{,}0144 + 0{,}0169\sqrt{\frac{n\,d\,e}{Q}}\right) \cdot \frac{2\,(d+e)\,l}{d\,e} \cdot \left(\frac{F_2}{n\,d\,e}\right)^2 \cdot \frac{v^2}{2g}\,Q\,\gamma.$$

If, lastly, $G$ be the weight of the turbine in revolution, and $r_2$ the radius of the pivot, the loss of effect by the friction, then, is:

$L_5 = \frac{2}{3} f\,G \cdot \frac{r_2}{r}\,v$ (Vol. I. § 171).

If, now, we deduct these five losses of effect from the power at disposition, there remains of useful effect:

$$Pv = \left(h - \left[\left(2\,\sin.\frac{\delta}{2}\right)^2 + \zeta\left(\frac{F_2}{F}\right)^2 + \zeta_1 \cdot \frac{\phi}{\pi}\left(\frac{F_2}{n\,d\,e}\right)^2 \right.\right.$$
$$\left.\left. + 2\,\zeta_2\,l\left(\frac{1}{d} + \frac{1}{e}\right)\left(\frac{F_2}{n\,d\,e}\right)^2\right]\frac{v^2}{2g}\right)Q\,\gamma - \frac{2}{3}\frac{r_2}{r}\,v\,f\,G.$$

In order to have this mechanical effect great, it is necessary to make the velocity of rotation $v$, the area of the orifice $F_2$, the orifice angle $\delta$ small. In how far this is possible we have above shown.

It is only in the case of turbines working under water that the height $h$ is to be measured from water surface to water surface. For turbines working in air, $h$ is to be measured from the upper surface to the centre of the wheel. In the latter case, the freeing the wheel of back water involves a loss of head, measured by the distance from the centre of the wheel to the surface of the race, whilst for wheels working under water there is a loss from the *resistance of the medium.*

*Remark.* For high pressure turbines there is an additional source of loss in the resistance of the flow of water through the pressure pipe.

§ 156. *Construction of Guide-Curve Turbines.*—Let us now endeavor to deduce the *rules* necessary for planning a wheel consistently with the above principles. We may of course assume the quantity of water discharged $Q$, and the fall $h$ to be given; and if, instead of $Q$, the useful effect $L$ were given, we might then at least derive $Q$ from $L$, and the efficiency $\eta$ (about 0,75) by the formula: $Q = \frac{L}{\eta h \gamma}$. The remaining quantities $r, r_1$ $\alpha$, $\beta$, $\delta$, $v$, $n$, $e$, &c., are determined, partly by discretion, partly by experience, and partly by theory.

The angle $\alpha$ is generally assumed. For wheels without guide-curves it is taken as 90°, but for wheels with guide-curves it must be made from 25° to 40°; the former for high falls, the latter for small falls, in order that, in the former case, the orifices may not be too large, and in the latter not too small, or, in order that, in the former case, the wheels may not be too small in diameter, and in the latter not too great. The angle $\beta$ is, in a certain degree, fixed by the value of $\alpha$. That the water may enter the wheel without pressure on the free space, we must have $\beta = 2\alpha$; but as this pressure diminishes as the sluice is depressed, in order to prevent *negative* pressure, $\beta$ is made greater than $2\alpha$, and probably $\beta = 2\alpha + 30°$ to $2\alpha + 50°$ are good limits; the former in high falls, the latter in low falls.

The ratio $\nu = \frac{r}{r_1}$ of the internal and external radii of the wheel falls within the limits of 1,25 and 1,5. For reasons easily understood, the smaller ratio is to be chosen for large values of $\beta$, and for wheels of considerable diameter, and *vice versâ.*

In order further to determine the radius of the wheel, and of the reservoir, we shall, as is the case in the best turbines hitherto made, require fulfillment of the condition that the velocity of the water in the reservoir shall not exceed 3 feet per second. If we adopt this velocity as ground work of our calculation, and leave out of the question the section of the upright pipe encasing the axle, and that of the sluice, then $Q = 3\pi r_1^2$, and, therefore, inversely, the radius of the reservoir, or the internal radius of the wheel:

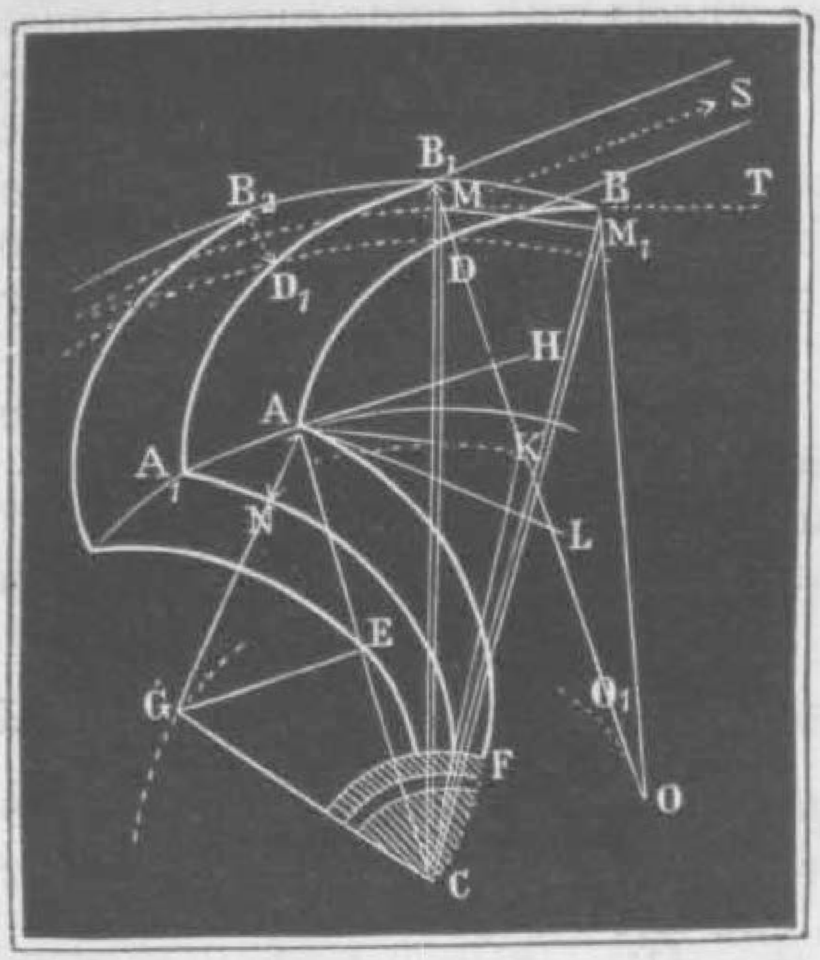
$r_1 = \sqrt{\frac{Q}{3\pi}} = 0,326\sqrt{Q}$, when $r_1$ is in feet, and $Q$ in cubic feet.

From this radius we get the external radius $r = \nu r_1$. The velocity at the inner periphery of the wheel is determined by the formula:

$$v_1 = \sqrt{\frac{2gh}{\frac{2\sin.\beta\cos.\alpha}{\sin.(\beta-\alpha)} + \zeta\left(\frac{\sin.\beta}{\sin.(\beta-\alpha)}\right)^2 + \varkappa\left(\frac{r}{r_1}\right)^2}}$$

into which we must, in the first place, introduce an approximate value of $x$. From this, however, we get the velocity of discharge: $c = \frac{v_1 \, sin. \, \beta}{sin. (\beta - \alpha)}$, and the section $F = \frac{Q}{c} = \frac{Q \, sin. (\beta - \alpha)}{v_1 \, sin. \, \beta}$; further, the velocity of entrance $c_1 = \frac{c \, sin. \, \alpha}{sin. \, \beta} = \frac{v_1 \, sin. \, \alpha}{sin. (\beta - \alpha)}$, and the section $F_1 = \frac{Q}{c_1} = \frac{Q \, sin. (\beta - \alpha)}{v_1 \, sin. \, \alpha}$. Lastly, the velocity of the external periphery of the wheel, and of the exit from it: $v = c_2 = \frac{r}{r_1} v_1$, and the contents of the united orifices of discharge from the wheel: $F_2 = \frac{Q}{c_2} = \frac{r_1}{r} \cdot \frac{Q}{v_1} = \frac{r_1}{r} \cdot \frac{Fc}{v_1}$. Besides this, we ascertain the number of revolutions of the wheel per min. to be: $u = \frac{30 \, v}{\pi \, r} = 9{,}55 \, \frac{v}{r}$.

Fig. 271.




In order to find the height of the wheel, or of the orifice $e$, we pursue the following method. If $n_1$ be the number of guide-curves, and $d_1$ the least distance $AN$ (Fig. 271), between any two of the guide-curves at the entrance on to the wheel, then $n_1 d_1 e = F$. If, further, $d_1$ and $e$ be in a determinate proportion to each other: $\psi_1 = \frac{e}{d_1}$, then $n_1 \psi_1 d_1^2 = F$; and if $s$ be the thickness of one of the guide-curves, we may put with tolerable accuracy:

$$d_1 = AA_1 \, sin. \, \alpha = \frac{2 \pi r_1 \, sin. \, \alpha}{n_1} - s_1,$$

and, hence,

$$n_1 \psi_1 \left( \frac{2 \pi r_1 \, sin. \, \alpha}{n_1} - s_1 \right)^2 = F,$$

so that, by inversion, the number of buckets required:

$$n_1 = \frac{\psi_1 (2 \pi r_1 \, sin. \, \alpha - n_1 s_1)^2}{F},$$

or, approximately: $= \frac{\psi_1 (2 \pi r_1 \, sin. \, \alpha)^2}{F}$, for which a whole number would be taken. If $n_1$ be once fixed, then:

$$d_1 = \frac{2 \pi r_1 \, sin. \, \alpha}{n_1} - s_1 = \frac{F}{\psi_1 (2 \pi r_1 \, sin. \, \alpha - n_1 s_1)},$$

and the height of wheel: $e = \frac{F}{2 \pi r_1 \, sin. \, \alpha - n_1 s_1}$.

§ 157. We have still to deduce rules by which to calculate the number of wheel-buckets, and the dimensions of the orifices of the wheel. The orifices of discharge, the united area of which is $F_2 = \frac{Q}{c_2}$, is *not* the outer periphery of the wheel, but the section $B_1D$, $B_2D_1$, &c., through the outer end of the buckets $B_1$, $B_2$, &c. (Fig. 271). Again, for $r$ in the above formulas we are not to understand the radius of the outer periphery, but the distance $CM$ of the centre of the orifice $B_1D$ from the axis of rotation, and, in like manner, $v$ is not the velocity of rotation of $B$, but of $M$. If, now, $\delta$ be the angle $SMT$, which the axis of the stream flowing through $B_1D$ makes with the tangent $MT$, or the normal to the radius $CM = r$; and, further, if $n$ be the number of wheel-buckets, $s$ their thickness, $d$ the width $B_1D$ of the orifices of discharge, and $\psi$ the ratio $\frac{e}{d}$, we may put: $n\, d\, e = n\, \psi\, d^2 = \frac{n\, e^2}{\psi} = F_2$, therefore, inversely, the number of wheel-buckets $n = \frac{\psi F_2}{e^2}$. Again, as

$$2\,\pi\, r\, sin.\, \delta - n\, s = n\, d = \frac{n\, e}{\psi} = \frac{F_2}{e},$$

we have for the angle of discharge $sin.\, \delta = \frac{F_2\,(e + \psi\, s)}{2\,\pi\, r\, e^2}$.

This angle $\delta$ should not in any case be more than 20°, and, therefore, if it comes out more than this, by the latter formula, some one or more of the elements composing it must be changed. Thus, for example, for this purpose $F_2 = \frac{r_1}{r} \cdot \frac{Q}{v_1}$ may be made less, *i. e.*, $v_1$, or which amounts to the same, the difference between $\alpha$ and $\beta$ may be made greater. Some engineers have endeavored to keep $\delta$ small, by making the wheel deeper at the outside than in the inside, giving two values to $e$ (Fig. 269). This, however, has the disadvantage, that *full* discharge is thereby interfered with—at least in wheels revolving free of back water, and that the water follows the wheel shroudings when these diverge from each other to any considerable extent. As to the ratio $\psi = \frac{e}{d}$, its influence one and $\delta$ is but trifling. It is within the limits 2 and 5 in wheels giving good results. The small value refers to small wheels, and *vice versâ;* for, otherwise, the channels fall out too wide, and the *full flow* is liable to be lost.

§ 158. *Construction of the Bucket.*—The buckets are generally circular arcs. For the guide-curves one arc is sufficient; but for the wheel-curves, or buckets, two arcs, tangential to each other, are usually required. How to fix the radius of these arcs, and how to combine them, may be explained as follows: With $CM = r$, Fig. 272, describe a circle, draw the tangent $MT$, and upon it set off the angle of discharge $SMT = \delta$, as determined above. Draw $MO$ at right angles to $MS$, and set off on each side of $M$, $MD = MB_1 = \frac{1}{2}\, d$.

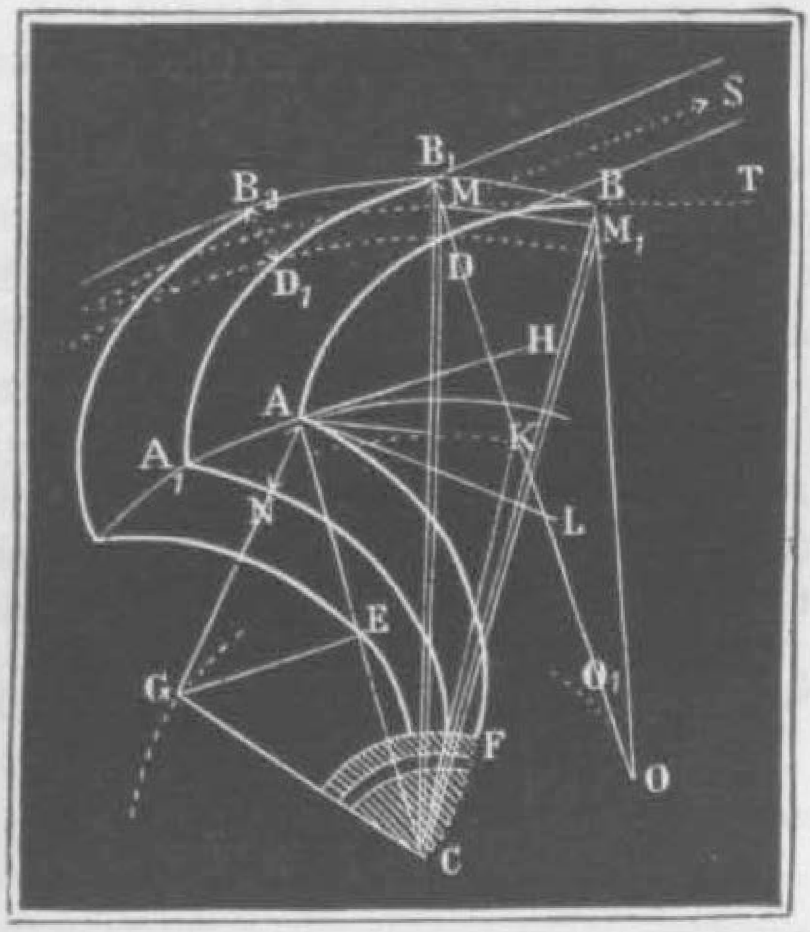
$\sin \delta = \frac{s}{ab}$

$ab = \frac{s}{r \sin \delta}$



Now draw the radius $CB_1$, and from $C$ lay off the angle $B_1CBc = \phi^0$, as determined by the formula $\phi = \frac{2\pi}{n} - \frac{s}{r \sin. \delta}$. Also, from $C$, as centre, draw circles through $B_1$ and $D$. The first of these circles gives the external periphery of the wheel, and the points $B$, $B_1$, &c., are the outer ends of the buckets. If we draw $BO$, so that $BOD = BCB_1 = \phi$, we have in $O$ the centre, and in $BO = DO$, the radius of the arc $DB$ forming the outer portion of the bucket. If we make $B_1O_1 = DO$, we have the centre $O_1$ of the outer piece $B_1D_1$ of the next following bucket, &c. &c. The radius $OB = OD = a$ of the arc $BD$ may be also determined by solution of the triangle $MOM_1$. We

Fig. 272.




have: $\frac{MO}{MM_1} = \frac{\sin. MM_1O}{\sin. MOM_1}$, but the cord $MM_1 = 2\, r \sin. \frac{\phi}{2}$, $MOM_1 = \phi$, and $MM_1O = 90 + \delta - \frac{\phi}{2}$, therefore,

$$OM = \frac{2\, r \sin. \frac{\phi}{2} \cos. \left(\delta - \frac{\phi}{2}\right)}{\sin. \phi} = \frac{r \cos. \left(\delta - \frac{\phi}{2}\right)}{\cos. \frac{\phi}{2}},$$

and the radius $a$ required

$$= \frac{r \cos. \left(\delta - \frac{\phi}{2}\right)}{\cos. \frac{\phi}{2}} - \tfrac{1}{2}\, d.$$

By this method of construction, the end $B_1$ of the bucket is quite parallel to the element $D$ opposite, and, therefore, the stream flows out without contraction. If this parallelism be not effected, it is always disadvantageous; if the tangents to $B_1$ and $D$ diverge outwards, there is danger of losing the *full flow*, and if they converge, there arises a partial contraction, and the stream then strikes upon the outer surface of $BD$ (Vol. I. § 319).

The inner piece $DA$ of a wheel-bucket may generally be formed of one arc of a circle. The radius $KD = KA = a_1$ of this circle is found as follows: In the triangle $CMK$, $CM = r$, $MK = a_1 + \frac{d}{2}$, and $\angle CMK = SMT = \delta$,

$$\therefore \overline{CK}^2 = r^2 + \left(a_1 + \frac{d}{2}\right)^2 - 2\, r \left(a_1 + \frac{d}{2}\right) cos.\ \delta.$$

In the triangle $CAK$, on the other hand, $CA = r_1$, $AK = a_1$, and $CAK = 180° - \beta$, therefore, $\overline{CK}^2 = r_1^2 + a_1^2 + 2\, r_1\, a_1\, cos.\ \beta$. By equating the two expressions, we have:

$$r^2 + a_1\, d + \frac{d^2}{4} - 2\, r\, a_1\, cos.\ \delta - r\, d\, cos.\ \delta = r_1^2 + 2\, r\, a_1 cos.\ \beta,$$

and hence the radius required:

$$a_1 = \frac{r^2 - r_1^2 - r\, d\, cos.\ \delta + \frac{d^2}{4}}{2\,(r\, cos.\ \delta + r_1\, cos.\ \beta) - d}.$$

As to the arc to be adopted as the curvature of the guide-curves, we get its radius and centre by drawing $AL$ at the known angle $\alpha$ to the tangent $AH$ of the inner circumference of the wheel. Raise a perpendicular $AG$ to it, and cut this in $G$ by another normal, raised from the middle point $E$ of the radius $CA$. This point $G$ is the centre of the guide-curve $AF$, which may be drawn either quite up to the case pipe of the axle, or to within any convenient distance of it. The radius $GA = GC = a_2$ of this bucket is:

$$a_2 = \frac{r_1}{2\ cos.\ \alpha}.$$

The centres of the arcs forming the outer arcs are in circles described with the radii $CO$, $CK$, and $CG$.

*Example.* It is required to determine all the proportions and lines of construction of a Fourneyron's turbine for a fall of 5 feet, with 30 cubic feet of water per second. We shall take $\alpha = 30°$, and $\beta = 110°$, and adopt the ratio $\frac{r}{r_1} = \nu = 1,35$. This being assumed, we have, from the rules above given, the internal radius $r_1 = 0,326\sqrt{Q} = 1,785$ feet, for which we take 1,8 feet. Hence $r_1 = CM$ (Fig. 272), the external radius $= 1,8 \times 1,35 = 2,43$ feet, for which we shall put 2,45. The width of the shrouding, therefore, measured to the centre of the orifice of discharge, $= 2,45 - 1,8 = 0,65$ feet. Neglecting prejudicial resistance, the best velocity of the wheel is:

$$v_1 = \sqrt{gh\,(1 - tang.\ \alpha\ cotg.\ \beta)} = \sqrt{5n\ 31,25\,(1 + tang.\ 30°\ cotg.\ 70°)}$$
$$= \sqrt{156,25\ .\ 1,21014} = 13,75 \text{ feet},$$

but, taking these resistances into account, if $\zeta = 0,18$ and $\varkappa = 0,06$:

$$v_1 = \sqrt{\frac{2gh}{\frac{2\ sin.\ \beta\ cos.\ \alpha}{sin.\ (\beta - \alpha)} + 0,18\left(\frac{sin.\ \beta}{(sin.\ \beta - \alpha)}\right)^2 + 0,06\left(\frac{r}{r_1}\right)^2}}$$

$$= \sqrt{\frac{2\ .\ 31,25\ .\ 5}{\frac{2\ sin.\ 110°\ cos.\ 30°}{sin.\ 80°} + 0\ 18\left(\frac{sin.\ 110°}{sin.\ 80°}\right)^2 + 0,06\ .\ 1,4^2}}$$

$$= \sqrt{\frac{312,5}{1,6527 + 0,1639 + 0,1176}} = \sqrt{\frac{312,5}{1,9342}} = 12,71 \text{ feet},$$

and then $v = c_0 = \nu\, v_1 = 1,35\ .\ 12,71 = 17,15$ feet. The velocity of discharge

$$c = \frac{v_1\ sin.\ \beta}{sin.\ (\beta - \alpha)} = \frac{12,71\ sin.\ 70°}{sin.\ 80°} = 12,13 \text{ feet}.$$

The number of revolutions per minute is $u = 9,55\ .\ \frac{v_1}{r_1} = \frac{9,55\ .\ 12,71}{18} = 67,4.$

From this we have the areas of the orifices of discharge:

$F = \frac{Q}{c} = \frac{30}{12,15} = 2,473$ square feet, and $F_2 = \frac{Q}{c_2} = \frac{Q}{v} = \frac{30}{17,15} = 1,748$ square feet. Assuming the thickness of the buckets to be $2\frac{1}{2}$ lines $= 0,017$ feet, and supposing that the ratio of the depth to the width of the orifices $\psi_1 = \frac{e}{d_1} = 3$, we have as the requisite number of buckets:

$$n_1 = \frac{\psi_1 (2 \pi r_1 \sin. \alpha - n_1 s_1)^2}{F} = \frac{3 . (5,655 - 0,017 n_1)^2}{2,473} = 30,$$

and, hence, we have as the height of the wheel, or of the orifices:

$$e = \frac{F}{2 \pi r_1 \sin. \alpha - n_1 s_1} = \frac{2,473}{5,655 - 0,51} = \frac{2,473}{5,145} = 0,4808 \text{ feet.}$$

Supposing $\psi = 5$, the number of buckets would be:

$n = \frac{\psi F_2}{e^2} = \frac{5 . 1,748}{0,4808^2} = \frac{8,74}{0,2311} = 37,8$, for which we may adopt 36. From this we get the required angle of discharge:

$$\sin. \delta = \frac{F_2 (e + \psi s)}{2 \pi r e^2} = \frac{1,748 \, (0,4808 + 0,017 \, . \, 5)}{2 \pi \, . \, 2,45 \, . \, 0,4808^2}$$

$$= \frac{1,748 \, . \, 0,5658}{4,9 \, \pi \, . \, 0,4808^2} = 0,2780, \text{ consequently } \delta = 16^\circ, 8', \text{ and}$$

$$d = \frac{F_2}{n e} = \frac{1,748}{36 \, . \, 0,4808} = 0,1010 \text{ feet.}$$

If the turbine is to be clear of the back water, it must be raised a certain height above the surface of the tail-race; and as the half height of the wheel $\frac{e}{2} = 0,2404$ feet, this distance may be estimated at 0,5 feet. If this excess of fall does not exist, then the calculations must be based on a fall of $4\frac{1}{2}$ feet, instead of on that of 5 feet. In order to judge of the loss of water, we have to find the amount of $x$, the excess of pressure of the water passing under the sluice. We have:

$x = h - (1 + \zeta) \frac{c^2}{2g} = 5 - 1,18 \, . \, 0,016 \, . \, 12,13^2 = 5 - 2,778 = 2,222$ feet, and the velocity corresponding $= 8,906 \sqrt{2,222} = 11,78$ feet. If, therefore, the space between the wheel and the bottom plate be $\frac{1}{24}$ inch, its area is: $2 \, . \, 1,8 \, . \, \pi \, . \, \frac{1}{288} = 0,0393$ square feet, and, therefore, the quantity of water escaping: $Q_1 = 11,78 \, . \, 0,0393 = 0,46$ cubic feet. To diminish this loss, which will be the less the lower the sluice, the *fitting up* of the wheel must be very accurately done, so that the space between the wheel and bottom plate may be as small as possible; or, by increasing $c$ and making $\beta$ less, $x$ must be reduced as low as possible.

The dividing angle of the wheel is $\frac{360^\circ}{36} = 10^\circ$; but the thickness of the buckets takes up an angle $= \frac{s}{r \sin. \delta} = \frac{0,017}{2,45 \sin. 16^\circ, 8'} = 0,02497$, or an angle $= 1^\circ, 26'$, hence $\phi = 8^\circ, 34'$, the angle of curvature of one part of the bucket. The radius corresponding is:

$$a = \frac{r \cos. (\delta - \frac{1}{2} \phi)}{\cos. \frac{1}{2} \phi} - \frac{1}{2} d = \frac{2.45 \cos. 11^\circ, 51'}{\cos. 4^\circ, 17'} - 0,0505 = 2,3541 \text{ feet.}$$

The radius of the second part of the wheel-bucket:

$$a_1 = \frac{r^2 - r_1^2 - r d \cos. \delta + \frac{1}{4} d^2}{2 (r \cos \delta + r_1 \cos. \beta) - d} = \frac{2,785 - 0,2377}{3,476 - 0,101} = \frac{2,5473}{3,375} = 0,7558 \text{ feet.}$$

The corresponding angle of curvature is: $\phi_1 = 180^\circ - \beta - \delta + \sigma - \tau$, in which $\tang. \sigma = \frac{a_1 \sin. \beta}{r_1 - a_1 \cos. \beta}$, and $\tang. \tau = \frac{a_1 \sin. \delta}{r - a_1 \cos. \delta}$; expressed numerically:

$$\phi_1 = 70^\circ - 16^\circ, 8' + 24^\circ, 42' - 6^\circ, 56' = 71^\circ, 38'.$$

These investigations afford us the necessary elements for the construction of a turbine for the fall in question, and we have now only to calculate the useful effect that such a machine will yield. The absolute velocity of the water discharged is $w = 2 \, c_2 \sin. \frac{1}{2} \delta = 2 \, . \, 17,15 \sin. 8^\circ, 4' = 4,813$ feet, and, hence, the loss of fall corresponding $= \frac{w^2}{2g} = 0,016 \, . \, 4,813^2 = 0,371$ feet. Again, the loss of fall occasioned by the resistance in the guide-curves $= 0,18 \frac{c^2}{2g} = 0,18 . 0,16 . 12,13^2 = 0,423$ feet. The loss of fall arising from the hydraulic resistances, may be estimated as follows: From an accurate drawing of the wheel, and the results of the calculations given above, it will be found that each wheel channel consists of two parts, of which the one is 0,11 feet wide, and 0,2 long; the radius 2,35 feet, the central angle $4\frac{1}{2}^{\circ}$, and the other is 0,21 feet wide, and 0,95 feet long, the radius of curvature 0,755 feet, and central angle of $71^{\circ}, 38'$. From this we deduce the co-efficient of resistance for the smaller part $=$

$$\zeta_1 = 0,124 + 3,104 \left(\frac{d}{2\,a}\right)^{\frac{7}{2}} = 0,124 + 3,104 \left(\frac{0,11}{4,71}\right)^{\frac{7}{2}} = 0,124$$

and for the larger: $\zeta_1 = 0,124 + 3,104 . \left(\frac{0,21}{1,51}\right)^{\frac{7}{2}} = 0,127$. Again, the angle ratio for the first part is $\frac{\phi}{\pi} = \frac{4,5}{180} = 0,025$, and for the second $\frac{71.63}{180} = 0,398$. Again, the section of the first is $\frac{F_1}{n\,d\,e} = \frac{1,748}{36 . 0,11 . 0,4808} = 0,918$, and for the second part $= \frac{1,748}{36 . 0,21 . 0,4808} = 0,481$, and hence we have for the co-efficient of the whole resistance arising from curvature $=$

$\kappa_1 = 0,124 . 0,025 . 0,918^2 + 0.127 . 0.398 . 0,481^2 = 0,0026 + 0,0117 = 0,0143.$

Further, the co-efficient of friction in the first part:

$$\zeta_2 = 0,0144 + 0,0169 \sqrt{\frac{n\,d\,e}{Q}} = 0,0144 + 0,0169 \sqrt{\frac{36 . 0,11 . 0,4808}{30}}$$

$= 0,0187$, and for the second $= 0,0144 + 0,0169 \sqrt{\frac{36 . 0,21 . 0,4808}{30}} = 0,0203$. The ratio $\left(\frac{d+e}{2\,d\,e}\right) l$ for the first part $= \frac{0.5908 . 0,2}{2 . 0,11 . 0,4808} = 1,117$, and for the second $= \frac{0,6908 . 0,95}{2 . 0,21 . 0,4808} = 3,250$, and from this we have the co efficient of friction for the whole channel:

$\kappa_2 = 0,0187 . 1,117 . 0,918^2 + 0,0203 . 3,250 . 0,481^2 = 0,0176 + 0,0152 = 0,0328,$

and hence, lastly, the co-efficient for all the resistances in a wheel-channel is:

$\kappa = \kappa_1 + \kappa_2 = 0,0143 + 0,0328 = 0,0471$, and the loss of fall corresponding to this $= \kappa . \frac{v^2}{2g} = 0,0471 . 0,016 . 17,15^2 = 0,222$ feet. The three losses of fall just estimated $= 0,371 + 0.424 + 0,222 = 1,017$ feet, and thus there remains of the total effect at disposition, $Q\,h\,\gamma = 30 . 5 . 66 = 9900$ feet lbs., only

$Pv = 30 . (5 - 1,017)\ 66 = 7886$ lbs. as *useful* effect. There is, however, some portion of this consumed by the friction of the pivot. If the weight of the wheel, &c., be 2000 lbs., and supposing the radius of the pivot $= 1\frac{1}{2}$ inch, and the co-efficient of friction 0,075, the mechanical effect consumed by the friction of the pivot $= \frac{r_2}{r_1} f\,G v_1 = \frac{1}{8 . 1,8} . 0,075 . 2000 . 12,71 = 132$ feet lbs. (Suppose the co efficient of friction 0,12, which is more likely, then the friction $= 210$ feet lbs.) The useful effect available, directly at the axle of the wheel, is then:

$L = 7886 - 210 = 7676$ feet lbs. $= 13,9$ horse power. If we assume 0,5 feet lost besides, by keeping the wheel free of the water in the race, then the efficiency:

$$\eta = \frac{L}{Q\,h\,\gamma} = \frac{7676}{30 . 5,5 . 66} = 0,705.*$$

§ 159. *Turbines without Guide-Curves.*—The proportions of turbines without guide-curves are only partly deducible in the manner of turbines with guide-curves. The angle $\alpha$ is in these $90^{\circ}$, and the

* [It will be observed that, in working this example, we have retained the co-efficients applicable to the Prussian weights and measures, viz.: $\frac{1}{2g} = 0,016$, and the weight of the cubic foot of water 66 lbs., for which the student can at pleasure substitute 0,0155 and 62,5 respectively, also 8,02 for 7,906.—AM. ED.]

angle $\beta = 150^\circ$ to $160^\circ$, in order to have $x$ as low as possible. The ratio $\nu = \frac{r}{r_1}$ is only 1,15 to 1,30, as, otherwise, from $\beta$ being so large, the *length* of bucket would be inconvenient. In order to have the loss of mechanical effect for the entrance on the wheel as low as possible, the water is laid on to the wheel with a velocity of only 2 feet, and hence the internal radius $r_1$ is only $0,4 \sqrt{Q}$, and the external radius $r = \nu\, r_1 = 0,4\, \nu \sqrt{Q}$.

The best velocity of rotation is also to be calculated by a different rule, as the formula:

$$v_1 = \sqrt{\frac{2gh}{\frac{2 \sin. \beta \cos. \alpha}{\sin. (\beta - \alpha)} + \zeta \left(\frac{\sin. \beta}{\sin. (\beta - \alpha)}\right)^2 + \varkappa \left(\frac{r}{r_1}\right)^2}},$$

which in this case has the form:

$$v_1 = \sqrt{\frac{2gh}{\zeta \,.\, tang. \beta^2 + \varkappa \left(\frac{r}{r_1}\right)^2}},$$

gives too great values. The reason of this is, that the condition of making the velocity of discharge $= 0$, does not, on account of the prejudicial resistances, lead to the maximum effect being produced; and it is only for turbines with guide-curves, that the fulfilment of this condition gives satisfactory approximations to this maximum. On the other hand, for turbines without guide-curves, and in all cases in which $\alpha$ is nearly $90^\circ$, the influence of the prejudicial resistances on the working of the wheel becomes too great for its being possible to assume that $w = 0$, or $v = c_2$. In order to find the least velocity for these wheels, we adopt the following method: We have already (Vol. II. § 147) found that

$$(1 + \varkappa)\, c_2^2 = 2gh + v^2 - 2\, c\, v_1 \cos. \alpha - \zeta\, c^2, \text{ and as}$$
$$\cos. \alpha = \cos. 90^\circ = 0, \text{ and}$$

$$c = \frac{v_1 \sin. \beta}{\sin. (\beta - 90)} = -v_1\, tang. \beta = -\frac{r_1}{r}\, v\, tang. \beta, \text{ we may put:}$$

$$(1 + \varkappa)\, c_2^2 = 2gh + v^2 \left[1 - \zeta \left(\frac{r_1}{r}\right)^2 tang. \beta^2\right];$$

and, therefore, the velocity of discharge:

$$c_2 = \sqrt{\frac{2gh + v^2 \left[1 - \zeta \left(\frac{r_1}{r}\right)^2 tang. \beta^2\right]}{1 + \varkappa}}.$$

Hence, the loss of fall:

$$y = \frac{c_2^2 + v^2 - 2\, c_2\, v \cos. \delta + \varkappa\, c_2^2 + \zeta\, c^2}{2g}$$

$$= \frac{(1 + \varkappa)\, c_2^2 + v^2 \left[1 + \zeta \left(\frac{r_1}{r}\right)^2 tang. \beta^2\right] - 2\, v\, c_2 \cos. \delta}{2g}$$

$$= \left(2gh + 2v^2 - 2v\cos.\delta \sqrt{\frac{2gh + v^2\left[1 - \zeta\left(\frac{r_1}{r}\right)^2 tang.\,\beta^2\right]}{1+\varkappa}}\right)\frac{1}{2g}$$

$$= h - \left(v\cos.\delta \sqrt{\frac{2gh + v^2\left[1 - \zeta\left(\frac{r_1}{r}\right)^2 tang.\,\beta^2\right]}{1+\varkappa}} - v^2\right)\cdot\frac{1}{g},$$

and, therefore, the effect to be expected from the wheel:

$$L = \left(v\cos.\delta \sqrt{\frac{2gh + v^2\left[1 - \zeta\left(\frac{r_1}{r}\right)^2 tang.\,\beta^2\right]}{1+\varkappa}} - v^2\right)\frac{Q\gamma}{g}.$$

If we put $\psi$ for $1 - \zeta\left(\frac{r_1}{r}\right)^2 tang.\,\beta^2$, and $\phi$ for $\frac{\sqrt{1+\varkappa}}{cos.\,\delta}$, then we have, more simply, $L = (v\sqrt{2gh + \psi v^2} - \phi v^2)\frac{Q\gamma}{\phi g}$.

In order that this value may give a maximum, we can deduce by the higher calculus that $\phi v = \frac{gh + \psi v^2}{\sqrt{2gh + \psi v^2}}$, or, if we represent the ratio of the height due to velocity $\frac{v^2}{2g}$ to the pressure height $h$, that is, $\frac{v^2}{2gh}$, by $x$, then $\frac{\frac{1}{2} + \psi x}{\sqrt{+\psi x^2}} = \phi$, and hence, $x = \frac{\phi - \sqrt{\phi^2 - \psi}}{2\psi\sqrt{\phi^2 - \psi}}$. If from this we have got $x$, we have for the velocity $v = \sqrt{x \cdot 2gh}$, $v_1 = \frac{r_1}{r}v$, $c = -v_1\,tang.\,\beta$, and $c_2 = \sqrt{\frac{2gh + \psi v^2}{1+\varkappa}}$. Hence the sections $F = \frac{Q}{c}$, and $F_3 = \frac{Q}{c_2}$, and, lastly, the height of the wheel, or of the orifice $e = \frac{F}{2\pi r_1}$.

The other proportions, as the construction of the buckets, &c. &c., do not differ from those of turbines having guide-curves.

*Remark.* Strictly speaking, turbines with guide-curves should also be treated in this manner, but as the expressions are very complicated, and lead to a value of $\frac{c_2}{v}$, which differs very little from unity, we have deemed the investigation unnecessary.

*Example.* It is required to make the necessary calculations for the design of a turbine on Cadiat's plan, for a fall of 5 feet, with 30 cubic feet of water per second. Assuming $\beta = 150^\circ$, $\nu = 1{,}2$, and $r_1 = 0{,}4\sqrt{Q} = 0{,}4\sqrt{30} = 2{,}19$, which we shall make 2,25, and, hence, $r = 1{,}2 \cdot 2{,}25 = 2{,}70$ feet. If, further, $\zeta = 0{,}15$, and $\varkappa = 0{,}10$, and $\delta = 16^\circ$, then $\psi = 1 - \zeta\left(\frac{r_1}{r}\right)^{2n} \cdot tang.\,\beta^2 = 1 - 0{,}15 \cdot \frac{(tang.\,30^\circ)^2}{1{,}44} = 1 - 0{,}035 = 0{,}965$, and

$\phi = \frac{\sqrt{1+\varkappa}}{cos.\,\delta} = \frac{\sqrt{1{,}1}}{cos.\,16^\circ} = 1{,}091$; and, therefore,

$$x = \frac{\phi - \sqrt{\phi^2 - \psi^2}}{2\psi\sqrt{\phi^2 - \psi^2}} = \frac{1{,}091 - 0{,}475}{1{,}93n\,0{,}475} = 0{,}672, \text{ and } \sqrt{x} = 0{,}820.$$

From this we have the most advantageous velocity of rotation:

$v = \sqrt{\chi \cdot 2gh} = 0{,}82 \cdot 7{,}906 \sqrt{5} = 14{,}50$ feet. Again, $v_1 = \frac{v}{\nu} = \frac{14{,}50}{1{,}2} = 12{,}08$ feet,

$c = -v_1 \, tang. \, \beta = 12{,}08 \, tang. \, 30^\circ = 6{,}97$ feet, and

$$c_2 = \sqrt{\frac{2gh + \psi v^2}{1 + \varkappa}} = \sqrt{\frac{312{,}5 + 202{,}9}{1{,}1}} = 21{,}65 \text{ feet};$$

and now we have the section $F = \frac{Q}{c} = \frac{30}{6{,}97} = 4{,}304$ square feet, and the section $F_2 = \frac{Q}{c_2} = \frac{30}{21{,}65} = 1{,}386$ square feet. Hence, again, we have the height of the wheel:

$e = \frac{F}{2 \pi r_1} = \frac{4{,}304}{2 \cdot 2{,}25 \cdot \pi} = 0{,}304$ feet, and if we take for the orifices of discharge of the wheel, the proportional dimensions $\frac{e}{d} = \frac{4}{3}$, we have for the number of buckets:

$n = \frac{2 F_2}{e^2} = \frac{2 \cdot 1{,}386}{0{,}304^2} = \frac{2{,}772}{0{,}0924} = 30$. If the thickness of the bucket plates $s = 0{,}017$ feet, we have as the angle of dischargee

$$sin. \, \delta = \frac{F_2 - n e s}{2 \pi r e} = \frac{1{,}386 - 30 \cdot 0{,}304 \cdot 0{,}017}{2 \cdot 2{,}7e \, 0{,}304 \, \pi} = \frac{1{,}226}{1{,}6416 e \pi} = 0{,}238,$$

and, hence, $\delta = 13\frac{3}{4}^\circ$. As we assumed above that for $\psi = \frac{\sqrt{1 + \varkappa}}{\cos. \delta}$, $\delta = 16^\circ$, the velocities, sectional areas, &c., just found, will be slightly varied by the introduction of $\delta = 13\frac{3}{4}^\circ$. The efficiency of this wheel is:

$$\eta = \left(1 - (\phi^2 - \psi) \frac{v^2}{gh}\right) \frac{1}{\phi \sqrt{1 + \varkappa}} = \left(\frac{1 - 0{,}281 \cdot 178{,}5}{156{,}25}\right) \frac{1}{1{,}116 \cdot 1{,}049}$$

$= \frac{1 - 0{,}321}{1{,}17} = \frac{0{,}679}{1{,}17} = 0{,}578$, the co-efficient 7,906 being taken for Prussian measures, as in last example.

For the same fall, a Fourneyron's turbine gave an efficiency $\eta = 0{,}705$. (*See* example to last paragraph.)

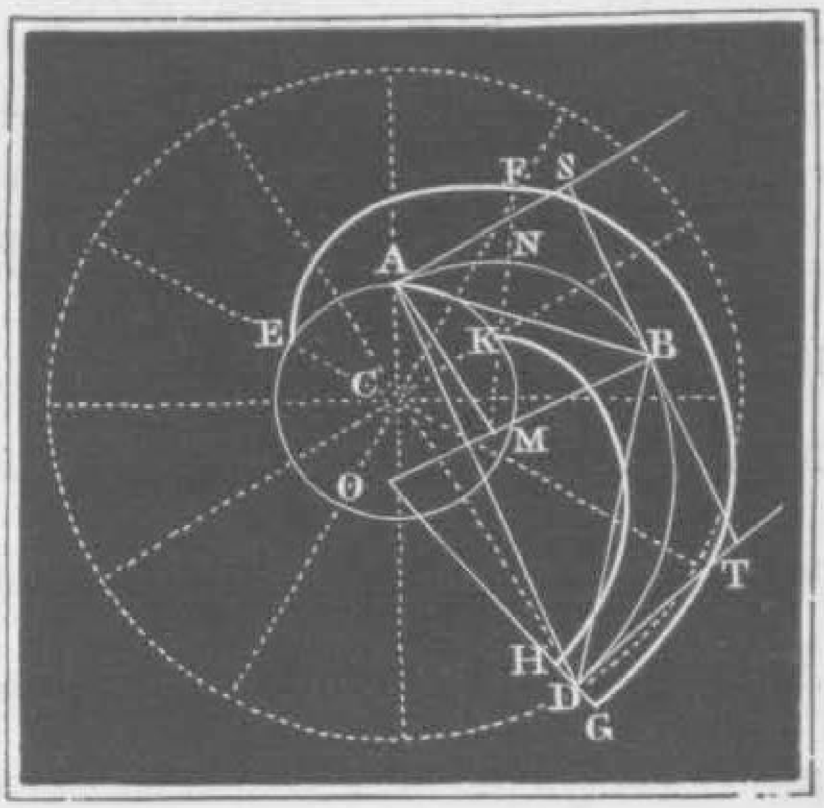
§ 160. *Whitelaw's Turbines.*—The Scottish turbine has to be treated differently from that of Cadiat, inasmuch as the water enters the wheel, in great measure, in a manner involving *shock*, and because in these turbines the dimensions and form of the wheel-channels are much more arbitrary than for the other. The angle $\delta$ may be made much less in these than in the other forms of turbine. They are peculiarly adapted for falls of great height, with small supply of water.

The width of the pressure pipe may be determined by the condition that there shall be a maximum velocity of 6 feet per second through it. So that the internal radius of the wheel, or the radius of the pressure pipe $r_1 = \sqrt{\frac{Q}{6 \pi}} = 0{,}23 \sqrt{Q}$. The external radius is made 2, 3, or 4 times this, according as the number of *arms* or discharge channels is 4, 3, or 2. The velocities $v$, $v_1$, and $c$, and, therefore, the sections $F_1$ and $F_2$, may be determined as in the case of turbines without guide-curves (last paragraph). The depth or height of the wheel $e = \frac{F}{2 \pi r_1}$, and the width of the orifices of discharge $= d = \frac{F_2}{ne}$.

In determining $v$ or $x = \frac{v^2}{2gh}$, it will be necessary to take $\zeta$ higher than 0,15, as shock cannot be avoided where the stream of water divides itself to run in so many different directions. It may be assumed, without much risk of error, that $\zeta = 0{,}20$. As the arms or wheel channels are considerably longer than in any other turbine, we must take a higher value for $\varkappa$, and assume it at least 0,15.

The arms are generally curved according to the Archimedian spiral; they may be curved to the form $ABD$, Fig. 272, composed of two circular arcs, $AB$ and $BD$. For this, the orifice of the inlet pipe, or internal periphery of the wheel, is divided into as many equal parts as there are to be arms—three in the case, Fig. 273, and from each of these draw the line $AS$, making the angle $\beta$ with the tangent at that point; or, for example, make $SAC = 270^\circ - \beta^\circ = 90^\circ + \beta^\circ$, then with the external radius $r$, describe a circle, and divide it into as many equal parts as there are to be arms, but so that between the two points $A$ and $D$ of the two peripheries, a central angle of about 135°, 150°, or 180° is included, according as the number of arms is 4, 3, or 2.

Fig. 273.




The direction of the axis $DT$ being laid off in such manner that the angle $CDT$ equals about 80°, we find the centres $M$ and $O$ of the arcs $AB$ and $BD$ forming the axis, by bisecting the angles $SAD$, $TDA$ by the straight lines $AB$ and $BD$; then draw $ST$ parallel to $AD$, and $AM$ at right angles to $AS$, $BO$ at right angles to $ST$, and $DO$ at right angles to $DT$. We see the reason of this at once, if we consider that, by division of the angles $SAD$ and $TDA$, and by drawing the parallel $ST$, the angles $MBA$ and $MAB$, and also the straight lines $MA$ and $MB$, are made equal to each other, that in like manner the angles $ODB$ and $OBD$, as also the lines $OB$ and $OD$, are made equal to each other.

To find the outsides of the pipes, $DG$ is made $= DH =$ the half width of orifice $\frac{d}{2}$, and $FN$ is made $= KN$, and the arcs $HK$ and $GF$ are drawn, so that the width $GH$ gradually passes into $FK$, &c.

*Example.* Required to design a Scottish turbine for a fall of 150 feet, with a supply of water of $1\frac{1}{2}$ cubic feet per second. In the first place, the internal radius $r_1 = 0{,}23\sqrt{Q} = 0{,}23\sqrt{1{,}5} = 0{,}282$ feet; but we shall put it $= 0{,}3$ feet, and the diameter of the pressure pipe 9 inches, or 0,75 feet; we shall have only two arms, and make the external radius $r = 4 \cdot r_1 = 1{,}2$ feet. We shall put $\beta = 150^\circ$, and $\delta = 10^\circ$, and assume $\varkappa = 0{,}15$, and $\zeta = 0{,}25$, hence:

$$\psi = 1 - 0{,}25 \cdot \left(\frac{r_1}{r}\right)^2 tang.\, \beta^2 = 1 - 0{,}25 \cdot \tfrac{1}{16}\, (tang.\, 30^\circ)^2 = 1 - 0{,}0052 = 0{,}9948, \text{ and}$$

$$\phi = \frac{\sqrt{1+\varkappa}}{\cos.\, \delta} = \frac{\sqrt{1{,}15}}{\cos.\, 10^\circ} = 1{,}0890.$$

Of the fall $h = 150$, the friction of the water in the 9 inch pipe, which may be presumed to be 200 feet long, consumes, according to Vol. I. §§ 331 and 332, an amount:

$$z = 0{,}0213 \,.\, 0{,}016 \,.\, \left(\frac{4}{\pi}\right)^2 \,.\, \frac{lQ^2}{d^5} = 0{,}0003408 \,.\, \left(\frac{4}{\pi}\right)^2 \,.\, \frac{200 \,.\, 1{,}5^2}{0{,}75^5}$$

$= 0{,}0003408 \,.\, 1{,}621 \,.\, \frac{200 \,.\, 256}{27} = 0{,}03408 \,.\, 1{,}621 \,.\, \frac{512}{27} = 1{,}05$ feet; therefore, we must only introduce $h = 148{,}95$ feet in our calculations. For the most advantageous velocity:

$$\chi = \frac{\phi - \sqrt{\phi^2 - \psi}}{2\psi\sqrt{\phi^2 - \psi}} = \frac{1{,}089 - \sqrt{1{,}1858 - 0{,}9948}}{1{,}9896\sqrt{0{,}191}} = \frac{1{,}089 - 0{,}437}{0{,}8695} = 0{,}750.$$

And hence:

$v = \sqrt{\chi \,.\, 2gh} = \sqrt{0{,}75 \,.\, 62{,}5 \,.\, 148{,}95} = 83{,}56$ feet, $v_1 = \frac{v}{4} = 20{,}89$ feet;

$c = - v_1 \text{ tang. } \beta = 20{,}89 \text{ tang. } 30^\circ = 12{,}06$ feet; and

$$c_2 = \sqrt{\frac{2gh + \psi v^2}{1 + \kappa}} = \sqrt{\frac{9309{,}4 + 6945{,}8}{1{,}15}} = 118{,}89 \text{ feet. (Prussian.)}$$

From this we have the sections: $F = \frac{Q}{c} = \frac{1{,}5}{12{,}06} = 0{,}1244$ square feet, and

$F_2 = \frac{Q}{c_2} = \frac{1{,}5}{118{,}82} = 0{,}01262$ square feet. From this again we have the height of wheel $e = \frac{F}{2\pi r_1} = \frac{0{,}1244}{0{,}6\,\pi} = 0{,}066$, and, lastly, the width of orifice:

$d = \frac{F_2}{n\,e} = \frac{0{,}01262}{2 \,.\, 0{,}066} = \frac{0{,}01262}{0{,}132} = 0{,}0956$ feet $= 1{,}15$ inches. In order to have a greater ratio $\frac{e}{d}$ between the sides of the orifices, we should have to introduce more arms or wheel channels; but as the channels are very long, even the above proportion would be found to insure a *full flow* through them. The efficiency of the wheel, neglecting the friction at the joint and losses in the pressure pipe, is:

$$\eta = \left[1 - (\phi^2 - \psi)\, 2\chi\right] \frac{1}{\phi\sqrt{1 + \kappa}}$$

$$= (1 - 0{,}191 \,.\, 1{,}5) \frac{1}{1{,}089 \,.\, 1{,}07} = \frac{0{,}7135}{1{,}167} = 0{,}611.$$

§ 161. *Comparison of Turbines.*—Let us now draw a comparison between the three turbines of Fourneyron, Cadiat, and Whitelaw. The turbine with guide-curves is unquestionably the more perfect construction, mechanically considered, as by this arrangement (when $c_2 = v$) the entire *vis viva* of the water may be taken from it, which cannot be done without this apparatus. All things considered, the velocity of rotation for all the wheels is nearly the same, viz.: $r_3\, v = 0{,}7\sqrt{2gh}$ to $\sqrt{2gh}$ for the maximum effect. This maximum effect is nearly the same for each of them, the advantage being on the side of Fourneyron's wheels, when working in its normal state, and on the side of Whitelaw's, when the supply of water is very variable. The Scottish turbine may be constructed at less cost than Fourneyron's turbines with guide-curves.

In general terms, we believe that the turbines of Fourneyron and Cadiat are better adapted for very low falls and those of moderate height (up to 30 feet) with large supplies of water, whilst for high falls and small supplies of water, Whitelaw's wheels are to be preferred.

*Remark.* In the case of turbines without guide-curves, especially when the fall is high, the water leaving the wheel retains a considerable absolute velocity $w = c_2 - v$, and hence a notable amount of *vis viva* is lost. This loss may be avoided, or at least much diminished, if the *vis viva* of the water leaving the turbine be applied to a second wheel. M. Althans, of the Sain Iron Works, has put this into practice at a mill near Ehrenbreitstein. The essential part of the construction of this wheel is represented by Fig. 274. *AEA* is a reaction wheel with four curved discharge pipes, the fall being 120 feet (compare § 147), *BB* is a larger wheel with curved buckets, set in motion by the water discharged at *A*, *A*. As the wheels revolve in opposite directions, they have to be connected with each other by reversing gearing. The outer wheel has this further advantage, that it adds to the fly, or regulating power of the machine.* (*See* "Inner-österreichisches Gewerbeblatt," Jahrgang 5, 1843.)

Fig. 274.


§ 162. *Experiments on Turbines.*—Numberless experiments on turbines of the different forms we have now been discussing are extant, but the reported results are not all trustworthy. These recipients of water power are in many respects admirable machines, but to suppose that an efficiency $\eta = 0{,}85$ to $0{,}90$ has been obtained from them, arises from some mistake. As the discharge of water through the most perfectly formed orifice has a velocity co-efficient $\phi = 0{,}97$ (Vol. I. § 312), there must be a loss of mechanical effect at entering the wheel, represented by

$\left(\frac{1}{\phi^2} - 1\right) \frac{c^2}{2g} Q\gamma = 0{,}06 \frac{c^2}{2g} Q\gamma$. As, again, the friction of water in a pipe six times as long as it is wide, consumes (Vol. I. § 331) $0{,}019 \cdot 6 \cdot \frac{v^2}{2g} Q\gamma = 0{,}114 \frac{v^2}{2g} Q\gamma$, or 11,4 per cent. of the available fall $\left(\text{as } \frac{v^2}{2g} = \frac{c_2}{2g} \text{ nearly} = h_2\right)$, we see that, deducting these resistances, there remain only 83 per cent. of effect over. If we allow only 2 per cent. for the resistance in the curved conduits, 2 per cent. for shock on the ends of the buckets, and 3 per cent. for the mechanical effect retained by the water discharged, and neglecting all other sources of loss, such as is involved in the guide-curves, &c., there remain only 76 per cent. of useful effect, and, therefore, a turbine that gives us an efficiency $\eta = 0{,}75$, may be considered as a very excellent one. The experiments of Morin and other impartial persons give results as to efficiency as high as 0,75, but never above this.

Morin's experiments were published about ten years ago, under the title "Expériences sur les roues hydrauliques à axe vertical,

* [A second wheel to receive the water from a common Barker's mill was used in model by the Editor, to illustrate his lectures before the Franklin Institute, about the year 1830–31. The model is probably now at Carlisle, Pa.—Am. Ed.]

appclées Turbines, Metz et Paris, 1838." The first experiments were made on one of Fourneyron's turbines at Moussay, external diameter of wheel = 2,8 feet, depth = 0,36 feet, fall = 24,6 feet, and the quantity of water laid on = 26 cubic feet per second. Thus there was a fall of upwards of 70 horse power at disposition. The result of these experiments, stated in general terms, was that, whether the wheel worked more or less in back-water, it gave for 180 to 190 revolutions per minute, a maximum effect of 69 per cent. of the whole power. When the number of revolutions was greater or less by from 40 to 50 per cent. of the above, the efficiency was from 7 to 8 per cent. less. These were the results when the cylindrical sluice was quite drawn up; but when the sluice was lowered to half the height of the wheel, the efficiency was reduced about 8 per cent. Had the wheel been entirely free of back-water, this falling off in efficiency must have been greater.

Experiments on a turbine at Mühlbach for a fall of 120 horse power, gave the following results: diameter of wheel 2 metres, height $\frac{1}{2}$ metre, fall 12 feet, with 86 cubic feet of water per second. With the sluice quite drawn, the wheel made 50 to 60 revolutions, and the efficiency was 0,78 according to Morin; but he has adopted too low a co-efficient of discharge in calculating the quantity of water, and, therefore, 0,75 is probably the true efficiency. For variations of from 30 to 80 revolutions, the efficiency did not vary more than 4 per cent. from the above. The efficiency was the same whether the wheel was only a few inches, or 3 feet under water. The efficiency was nearly constant for great variations in the quantity of water laid on. As the sluice was depressed, the efficiency fell off rapidly. Morin directed experiments to ascertaining the ratio $\frac{v}{\sqrt{2gh}}$, and found, as theory indicates, that this ratio increases as $v$ increases (owing to the influence of centrifugal force), and decreases as the sluice is raised.

§ 163. Redtenbacher gives the result of some experiments on turbines in Switzerland, in his work "Ueber die Theorie und den Bau der Turbinen und Ventilatoren." These were ill constructed, and gave low results.

Among other interesting results which Redtenbacher deduces from the recorded experiments on Fourneyron's turbines, we may particularly mention that these wheels, when working with their maximum effect, and with sluice fully drawn, make half the number of revolutions that they do when working free of all load but their own inherent resistances.

Combes' experiments, with models of his wheels, give less efficiency than those above mentioned, viz: 0,51 to 0,56.

Mr. Ellwood Morris, of Philadelphia, has recorded a very complete set of experiments on two turbines of Fourneyron, (*see* "Journal of the Franklin Institute," Dec. 1843.)

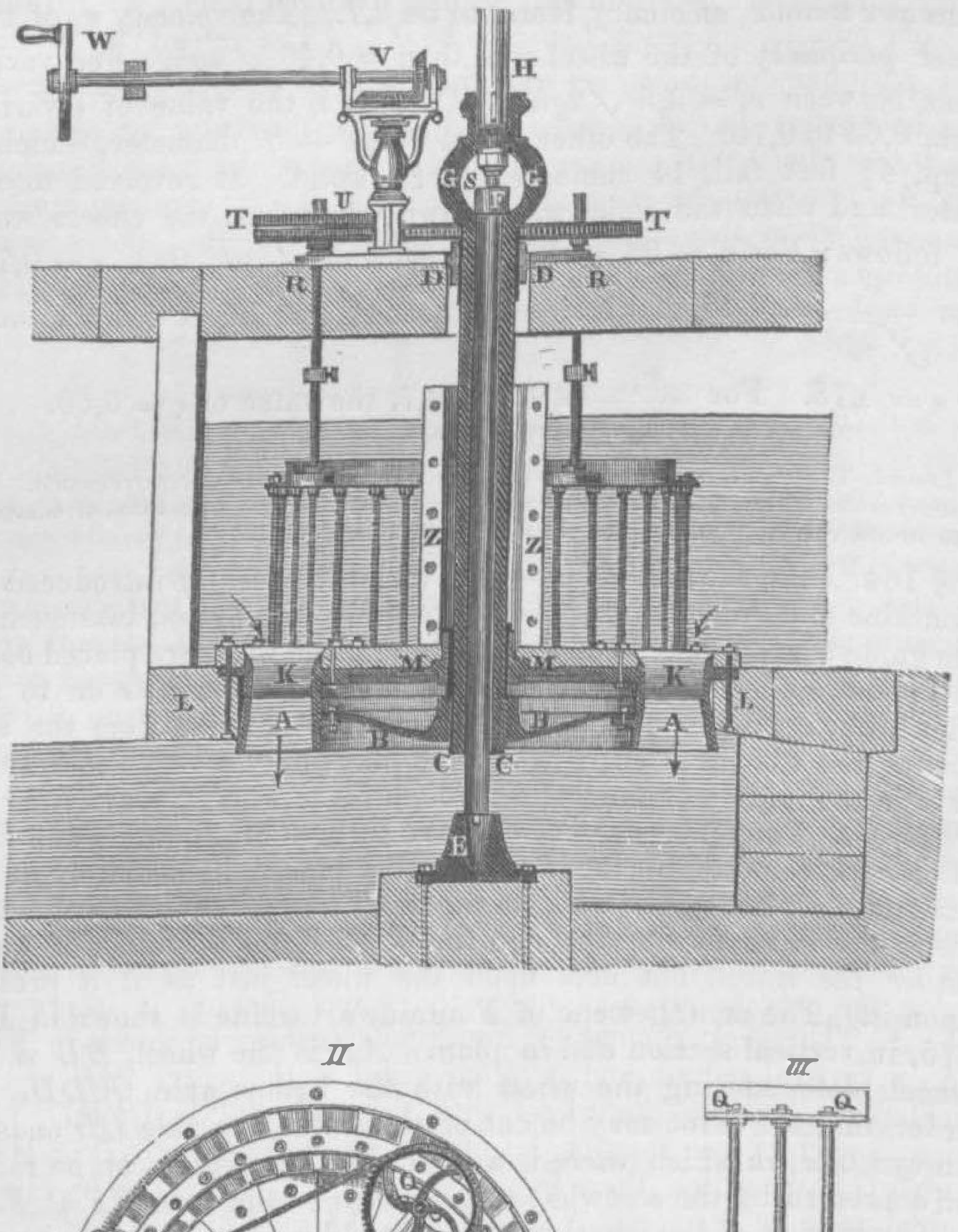
One wheel was $4\frac{2}{3}$ feet diameter, 8 inches high, 6 feet fall, 1700 cubic feet of water per minute. Sluice drawn 6 inches, 52 revolutions per minute, efficiency found to be 0,7. The velocity $v_1$ of the inner periphery of the wheel was then $= 0{,}46 \sqrt{2gh}$. For variations between $v_1 = 0{,}5 \sqrt{2gh}$ to $0{,}9 \sqrt{2gh}$, the value of $\eta$ varied from 0,64 to 0,70. The other wheel was 4′ — 5″ diameter, 6 inches deep, $4\frac{1}{2}$ feet fall, 14 cubic feet per second. It revolved *under* water, and when the sluice was drawn $4\frac{1}{2}$ inches, the effects were as follows: For $v_1 =$ 25 to 30 per cent. of $\sqrt{2gh}$, then $\eta = 0{,}71$. For $\frac{v_1}{\sqrt{2gh}} = 0{,}45$, that is $u = 49$, the maximum effect was obtained, or $\eta = 0{,}75$. For $\frac{v_1}{\sqrt{2gh}} = 0{,}5$ to $0{,}7$, the value of $\eta = 0{,}60$.

*Remark.* The results of experiments on Cadiat's wheels are probably overstated. Experiments on Whitelaw's wheels, as made by Messrs. Randolph and Co., of Glasgow, give results varying from 0,60 to 0,75 for the efficiency.

§ 164. *Fontaine's Turbine.*—The turbines recently introduced by Fontaine and Jonval, differ from those of Fourneyron, inasmuch as the guide-curves, instead of being in one place with, are placed *above* the wheel, and thus the water does not flow *outwards* on to the wheel, but from above downwards, and is discharged from the bottom of the wheel. Centrifugal force plays only a very subordinate part in the motion of the water through these wheels, gravity taking its place. The difference between the turbine of Fontaine and that of Jonval consists in the former being placed immediately on or under the surface of the water in the race; while, in Jonval's arrangement, the water flowing through the wheel forms a column of water *undere the wheel*, but acts upon the wheel just as if it pressed upon it. The arrangement of Fontaine's turbine is shown in Fig. 275, in vertical section and in plan. *AA* is the wheel, *BB* is the wheel plate, uniting the wheel with the hollow axle *CCDD*. In order that the pivot may be out of the water, the axle *CD* ends in an eye *GG*, in which there is a steel plug *FS* (which can be raised or depressed by the screw *S*) resting on the solid axle *EF* at *F*.

The motion of the wheel is transmitted by an axle *H*, firmly connected with the head of the hollow shaft. To keep the upright shaft from the water, it is surrounded by a casing, as in Fourneyron's turbines. The guide-curve apparatus *KK* is screwed on to the beams *LL*, and to it there is a plate *KMMK* united, having a cylindrical metallic bed *MM*, in which there is a collar similar to that at *DD*, for maintaining the perpendicularity and steadiness of the shaft. The form of the guide-curves *N*, and of the wheel-buckets *O*, is represented at III. For regulating the quantity of water laid on, there is a compound sluice, having as many separate valves as there are guide-curves. These valves are covered by round pieces of wood, and are fastened by screws and nuts to the cylindrical casing of the guide-curve apparatus. The sluice-rods *PQ*, *PQ* . . . are firmly united to each other by an iron ring *QQ*, which can be

Fig. 275.

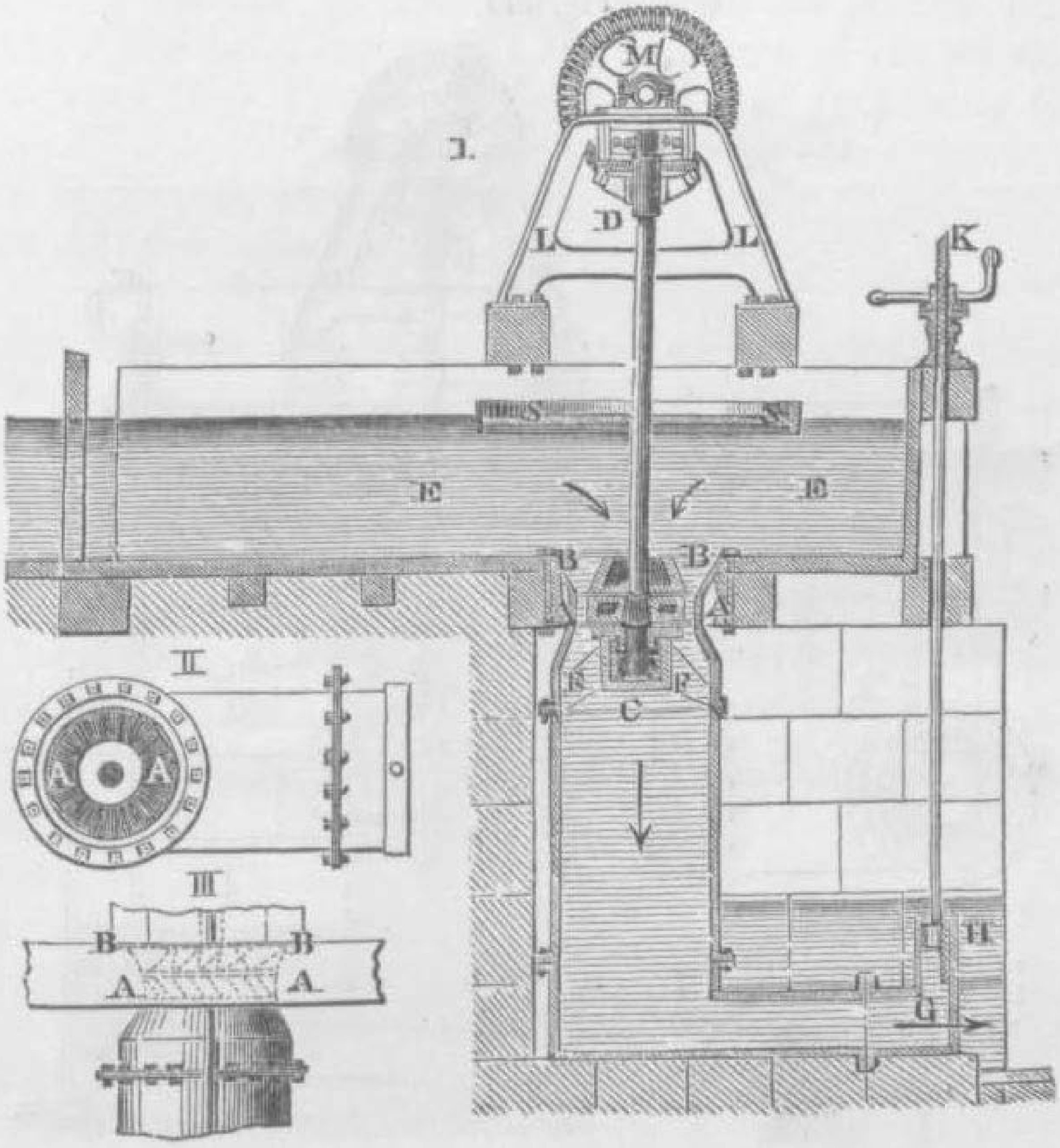



raised and depressed by three lifting-rods *QR*, *QR* . . . For this purpose the ends *R*, *R* . . . of these rods are cut as screws, and toothed wheels *T*, *T* .e. . put on them, the nave or box of these having female screws, and the peripheries of the whole being encompassed by an endless chain.

When one of these wheels is moved by means of a winch, or otherwise, it is evident that the rest must be so too, so that the three rods are moved simultaneously.

§ 165. *Jonval's Turbine.*—Figs. 276 and 277, I, II, and III, represent Jonval's turbine. Here, again, *AA* is the wheel, united to

Fig. 276.



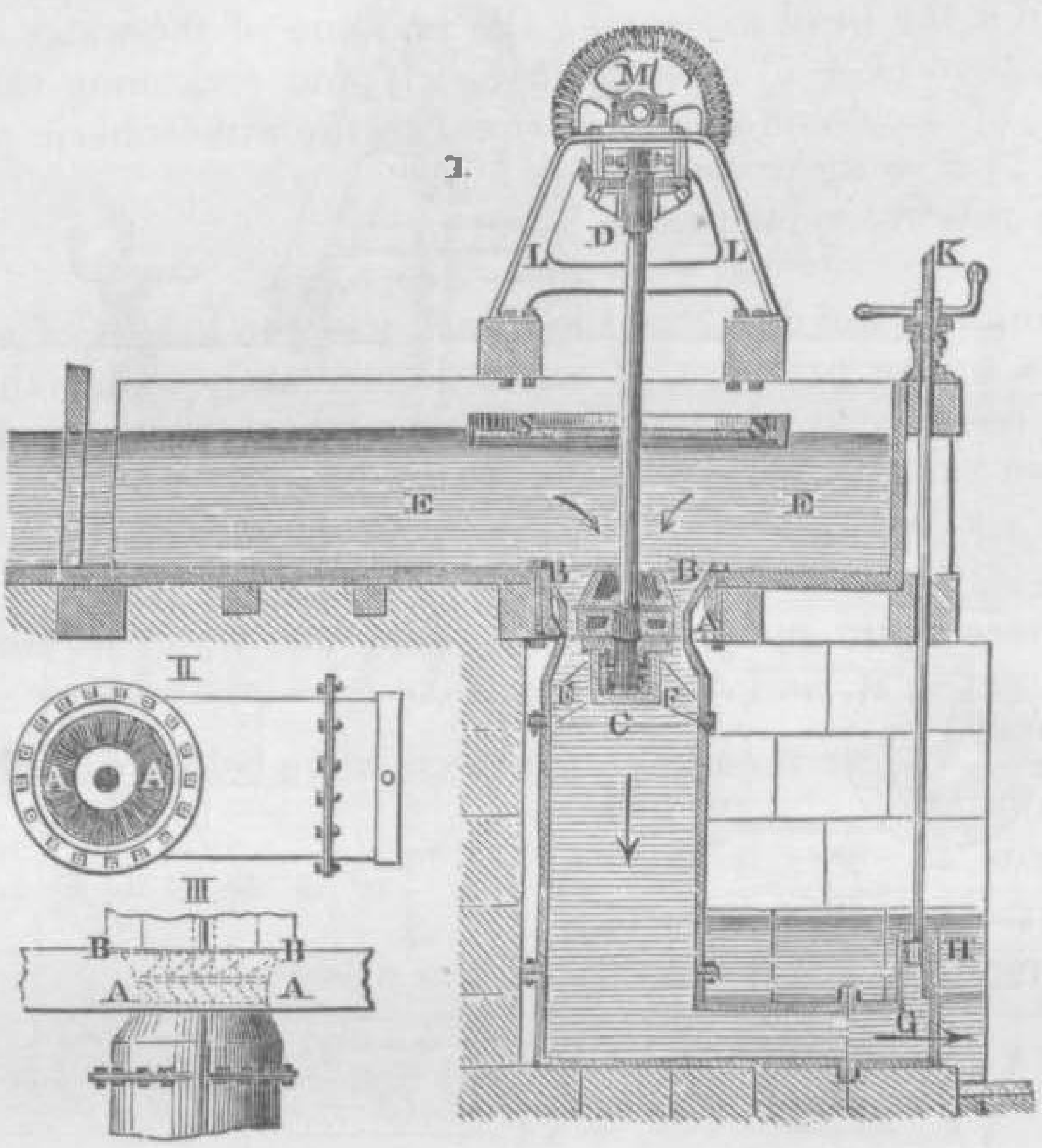

the upright shaft *CD* by a disc or plate; *BB* is the guide-curve apparatus opening as a diverging cone into the lead. The pivot rests on a footstep *C*, supported by *EE*. The relative position of the wheel and guide-curves, as also a part of the outside of the pipe in which the wheel is enclosed, is represented at II and III.

To keep the surface of the water in the lead free from agitation, a float *SS* is placed on it, and for regulating the wheel's motion, a sluice *G* is introduced, worked by a handle at *K*. According as this sluice is raised or depressed, more or less water flows away, and thus the power is regulated.

The framing *LL* supports the plumber-blocks for the upper end of the shaft, and for a horizontal shaft, through which the motion is first transmitted by a pair of mitre wheels. When the wheels are small, the reservoir or well in which the wheel is enclosed may be of cast iron; for large wheels it should be built of solid masonry.

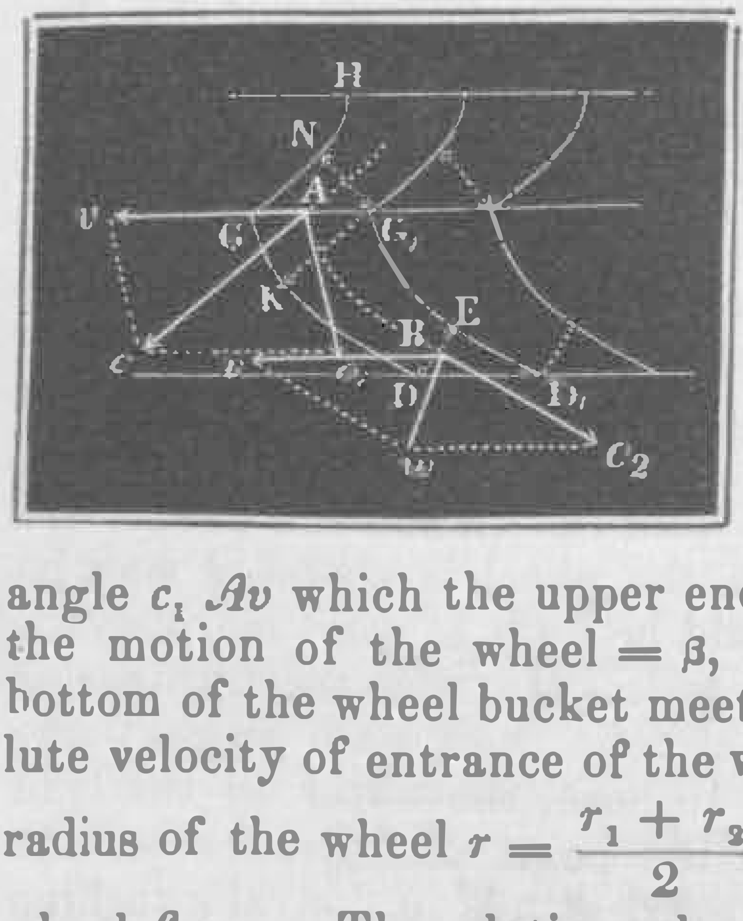
It is evident, from what we have now detailed, that the turbines of Fontaine and Jonval are essentially alike in their main proportions, and that their *theory* is the same. In both, the water in the lead stands at a certain height $h_1$ above the point of entrance on the wheel. The water in the race, however, stands in Jonval's turbine at a certain depth $h_2$ *below* the wheel, while, in Fontaine's arrangement, the race-water is *in immediate contact with* the wheel. The regulation of the wheel's motion is managed in Fontaine's by an

Fig. 277.



internal, and in Jonval's by an external sluice — the one being analogous in this respect to Fourneyron's turbine, the other to that of Cadiat.

Fig. 278.



§ 166. *Theory of Fontaine's and Jonval's Turbines.*—In developing the theory of the turbines of Fontaine and Jonval, we shall adopt the following symbols.

Let the angle of inclination of a guide-curve $NG$ to the horizon, or the angle of entrance of the water $NGG_1 = c\,Av$, Fig 278, $= \alpha$. The angle $c_1\,Av$ which the upper end of the wheel bucket $A$ makes with the motion of the wheel $= \beta$, and the angle $DD_1E$, at which the bottom of the wheel bucket meets the horizontal $= \delta$. Let the absolute velocity of entrance of the water on the wheel $Ac = c$, the mean radius of the wheel $r = \frac{r_1 + r_2}{2}$, corresponding to the velocity of the wheel $Av = v$. The relative velocity of entrance $Ac_1 = c_1$, and the velocity of discharge $Bc_2 = c_2$. Again, let $F =$ the sum of the areas of all the sections $NG_1$ of the water flowing out of the guide-curve apparatus, $F_1$ the sum of the upper sections $G_1K$, and $F_2$ the sum of the lower sections $DE$ of the wheel channels.

If, again, $\zeta$ be the co-efficient of resistance in the guide-curve canals, and $x$ the head measuring the pressure of the water entering the wheel, then $(1 + \zeta)\, c^2 = 2g\,(h_1 - x)$; and reckoning the height $a$ (34 feet) of a column of water equal to the atmospheric pressure, then $(1 + \zeta)\, c^2 = 2g\,(a + h_1 - x)$.

For the relative velocity, we have:

$$c_1^2 = c^2 + v^2 - 2\,c\,v\,\cos.\,\alpha.$$

If, again, $b =$ the depth of the wheel, $y =$ the height of a column of water $=$ to the pressure of water immediately under the wheel, and $\varkappa$ the co-efficient of resistance in the wheel channels, then, for the relative velocity of discharge, we have:

$$(1 + \varkappa)\, c_2^2 = 2g\,(b + x - y) + c_1^2 = 2g\,(a + h_1 + b - y) + v^2 - 2\,c\,v\,\cos.\,\alpha - \zeta\, c^2$$

If we here again endeavor to take from the water as much effect as is inherent in it, and, therefore, make $c_2 = v$, and also $c = \frac{v\,\sin.\,\beta}{\sin.\,(\beta - \alpha)}$, we then have for the relative velocity of discharge:

$$\left[2\,\frac{\sin.\,\beta\,\cos.\,\alpha}{\sin.\,(\beta-\alpha)} + \zeta\left(\frac{\sin.\,\beta}{\sin.\,(\beta-\alpha)}\right)^2 + \varkappa\right] v^2 = 2g\,(a + h_1 + b - y),$$

and, therefore, the best velocity of the wheel:

$$v = \sqrt{\frac{2g\,(a + h_1 + b - y)}{2\,\frac{\sin.\,\beta\,\cos.\,\alpha}{\sin.\,(\beta-\alpha)} + \zeta\left(\frac{\sin.\,\beta}{\sin.\,(\beta-\alpha)}\right)^2 + \varkappa}}.$$

The pressure-height $y$, when the turbine revolves in free air, is equal to the atmospheric pressure $a$; but when the turbine is in back water, it $= a + h_2$, where $h_2$ is the height of the surface of the water above the bottom of the wheel; and lastly, when the wheel is above the race water, as in Jonval's arrangement, $y = a - h_2 + z$, where $h_2 =$ the depth of the race surface underneath the bottom of the wheel, and $z$ is the height due to the velocity of the water flowing through the sluice from the reservoir to the tail-race. The total fall for the case of the wheel revolving free of back water is $h = h_1 + b$; when the wheel is in back water, $h = h_1 + b - h_2$; and when the wheel is above the tail water, $h = h_1 + b + h_2$. Hence, for the two first cases:

$$v = \sqrt{\frac{2gh}{2\,\frac{\sin.\,\beta\,\cos.\,\alpha}{\sin.\,(\beta-\alpha)} + \zeta\left(\frac{\sin.\,\beta}{\sin.\,(\beta-\alpha)}\right)^2 + \varkappa}}.$$

and, for the latter:

$$v = \sqrt{\frac{2g\,(h - z)}{2\,\frac{\sin.\,\beta\,\cos.\,\alpha}{\sin.\,(\beta-\alpha)} + \zeta\left(\frac{\sin.\,\beta}{\sin.\,(\beta-\alpha)}\right)^2 + \varkappa}},$$

and, when the orifice $G$ by which the vessel communicates with the tail-race is large, or when the water flows away very slowly:

$$z = \frac{1}{2g}\left(\frac{Q}{G}\right)^2 = 0.$$

§ 167. From the velocity $v = c_2$, the absolute velocity of entrance $c = \frac{v \sin. \beta}{\sin. (\beta - \alpha)}$, and the pressure height:

$$x = a + h_1 - (1 + \zeta)\frac{c^2}{2g} = a + h_1 - (1 + \zeta)\frac{v^2 \sin. \beta^2}{2g \sin. (\beta - \alpha)^2}$$

may be calculated. Neglecting prejudicial resistances:

$$x = a + h_1 - \frac{h \sin. \beta}{2 \cos. \alpha \sin. (\beta - \alpha)},$$

and neglecting the atmospheric pressure:

$$x = h_1 - \frac{h \sin. \beta}{2 \cos. \alpha \sin. (\beta - \alpha)}.$$

$x = 0$, or more correctly $x =$ the external pressure of the atmosphere when $h_1 = \frac{h \sin. \beta}{2 \cos. \alpha \sin. (\beta - \alpha)}$. The loss of water involved in the free space necessarily left, depends on the difference between the internal pressure ($x$), and the external pressure at this point, and is different in the two turbines now under consideration. That the water may flow on in a connected stream, $x$ must never descend to 0, that is, we must have $a + h_1 > \frac{h \sin. \beta}{2 \cos. \alpha \sin. (\beta - \alpha)}$. Again, that the water may not recede from the bottom of the wheel, we must never have $y = 0$, that is, we must have:t

$$a - h_2 + z > 0, \text{ or } h_2 < a + z, \text{ or } h_2 < a + \frac{1}{2g}\left(\frac{Q}{G}\right)^2.$$

Hence, when the area of the orifice $G$ is large, we must have $h_2 < a$. From this we see that the height of the wheel above the surface of the tail-race must never reach to the water-barometric height of 34 feet.

If, in Jonval's turbine, the reservoir be high and narrow, so that the velocity of the water in it is considerable, there arise losses of effect at this point from friction, resistance in curves, impact, &c., &c. On this account, it is advisable to make the reservoir wide in proportion to the wheel's diameter.

§ 168. *The Mechanical Effect of Fontaine's and Jonval's Turbines.*—The effect of these turbines may be deduced exactly as that of Fourneyron's has been, by subtracting from the total mechanical effect of the fall $Q h \gamma$, the effect consumed by different prejudicial resistances, &c. The loss in the guide-curve apparatus $L_1 = \zeta . \frac{c^2}{2g} Q\gamma$, and that in the wheel-channel $L_2 = \varkappa \frac{c^2}{2g} Q\gamma$. Again, the loss of the *vis viva* retained by the water at its exit from the wheel, is

$$= \frac{w^2}{2g} Q\gamma = \frac{\left(2\, v\, sin. \frac{\delta}{2}\right)^2}{2g} Q\gamma.$$

In Jonval's turbines, there has to be added to these the loss of effect involved in the velocity of discharge ($w_1$) through the sluice

$$= \frac{w_1^{\,2}}{2g} Q\gamma = \frac{1}{2g} \cdot \frac{Q^3}{G^2}\gamma.$$

Hence the total effect of the wheel:

$$L = \left(h - \left[\zeta\, c^2 + \varkappa\, c_2^{\,2} + \left(2\, v\, sin. \frac{\delta}{2}\right)^2 + w_1^{\,2}\right] \cdot \frac{1}{2g}\right) Q\gamma.$$

We see from this that the loss of effect increases as the angle $\delta$ increases, and as the velocity $w_1$ is greater, or as the velocity of discharge and sluice-opening $G$ are less.

When the sluice is fully drawn, and the reservoir is wide, $w_1$ may be assumed $= 0$. Hence, in Jonval's turbine, the efficiency decreases as the quantity of water diminishes, or as the sluice is lowered. In Fontaine's turbines, the same relative effects are produced for different positions of the sluice as in Fourneyron's turbines. It appears, therefore, that the efficiency of the turbines now under discussion, cannot be much more or less than that of Fourneyron's in the same circumstances. Experiments, hereafter cited, confirm this.

§ 169. *Construction of Fontaine's and Jonval's Turbines.*—We have now to determine the general rules for the proportions and construction of these wheels.

The angles $\beta$ and $\delta$ of the wheel-buckets are taken arbitrarily—the latter, however, as small as possible, *i. e.*, 15° to 20°, and $\beta = 100°$ to 110°. From these we have the guide-curve angle $\alpha$, if, for the sake of preventing all impact at entrance of the water, we put

$$c_1\, sin.\, \beta = c_2\, sin.\, \delta = v\, sin.\, \delta, \text{ and } \frac{c_1}{v} = \frac{sin.\, \alpha}{sin.\, (\beta - \alpha)}.$$

Hence, by combination: $\dfrac{sin.\, \alpha}{sin.\, (\beta - \alpha)} = \dfrac{sin.\, \delta}{sin.\, \beta}$; and we have

$$\frac{sin.\, (\beta - \alpha)}{sin.\, \alpha\, sin.\, \beta} = \frac{1}{sin.\, \delta}, \text{ or } cotg.\, \alpha = cotg.\, \beta + \frac{1}{sin.\, \delta}.$$

From the angles $\alpha$ and $\beta$, we have the velocity of the wheel

$$v = \sqrt{\frac{2gh}{2\dfrac{sin.\, \beta\, cos.\, \alpha}{sin.\, (\beta - \alpha)} + \zeta\left(\dfrac{sin.\, \beta}{sin.\, (\beta - \alpha)}\right)^2 + \varkappa}},$$

and the velocity of entrance of the water $c = \dfrac{v\, sin.\, \beta}{sin.\, (\beta - \alpha)}$; and from this we have the sectional areas $F = \dfrac{Q}{c}$, and $F_2 = \dfrac{Q}{c}$.

The width of the wheel, or length of the buckets measured radially, must be made in suitable proportion (as small as possible),

$\nu = \frac{d}{r}$ to the mean radius of the wheel. In the turbines hitherto made $\nu = 0{,}3$ to $0{,}4$. This being done, we havec

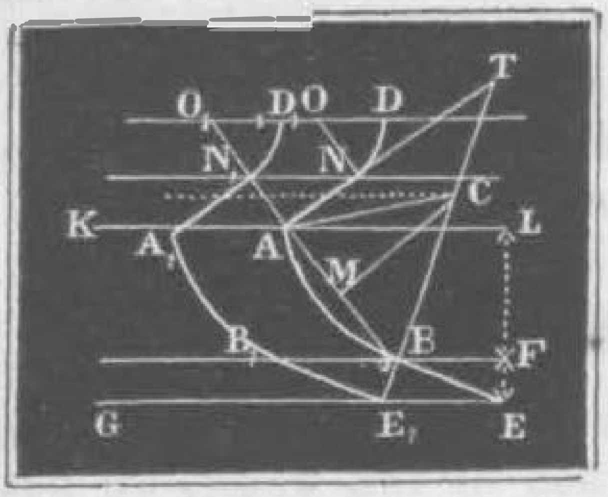
$$F = 2\,\pi\,r\,d\,sin.\,\alpha = 2\,\pi\,.\,\nu\,r^2\,sin.\,\alpha,$$

and, therefore, $r = \sqrt{\frac{F}{2\,\pi\,\nu\,sin.\,\alpha}}$, and $d = \nu\,r$.

If we further assume a proportion $\psi = \frac{e}{d}$ of the width of the orifice measured at the mean circumference of the wheel to the length of the buckets (in existing turbines this is $\frac{1}{2}$), we have $e = \psi\,d$, and, hence, the number of buckets $n = \frac{2\,\pi\,r\,sin.\,\alpha}{e} = \frac{F}{d\,e}$. The height of the wheel $b$ is made about the same as the width $d$.

§ 170. *Construction of the Buckets.*—The buckets are surfaces of double curvature, the generatrix of which passes, on the one hand, through the axis at right angles, and on the other through a leading line which we may suppose drawn on a cylinder of the mean radius $r$. As by developing a cylinder as a plane, a rectangular surface is produced, lines may be drawn on this surface, which, when the right angle is round the cylinder, will serve as the leading line for the bucket surfaces. These developed leading lines may, however, be constructed of arcs and their tangents. If $KL$, Fig. 279, be the developed circle, in which the wheel and guide-curve apparatus meet, the line $AND$ of the guide-curve, may be found by making

Fig. 279.




$AA_1 = \frac{2\,\pi\,r}{n}$, and by drawing $AN$, $A_1N_1$ . . . , so that the angle of inclination, $NAL = N_1A_1L \ldots = \alpha$. Again, let fall $AO$ perpendicular to $A_1N_1$. Draw a line parallel to $KL$ at a distance equal to the height of the guide-curves. From the point $O$ where the perpendicular $AO$ intersects this line, describe the arc $N_1D_1$, and similarly, from another point $O$, the arc $ND$, &c. &c., then $AND$, $A_1N_1D_1$ are the *developed* guide lines of the guide-curves. To find the guide lines of the wheel buckets, draw at the distance $EL = b$ the height of the wheel, the line $EG$ parallel to $KL$, make $EE_1 = \frac{2\,\pi\,r}{n}$, and draw the straight lines $EB$, $E_1B_1$, &c., so that the angle $BEG = B_1E_1G$ becomes equal to the angle of discharge $\delta$. Again, let fall $E_1B$ perpendicular to $BE$, and lay off $AB$, so that the angle $ABC = \frac{\beta + \delta}{2}$. If, lastly, from the centre $M$ of the line $AB$, there be raised the perpendicular $MC$, this cuts $BT$ at the centre $C$ of the arc $AB$, forming the upper part of the developed guide line of a wheel bucket, whilst the straight lines $BE$, $B_1E_1$, &c., form the lower part.

It is evident that this construction of the guide and wheel curves, insures that water leaves them with the sections $AN_1$ and $BE_1$ respectively.

*Example.* It is required to give the leading dimensions and proportions of a Jonval's turbine for a fall of 12 feet in height, with 8 cubic feet of water per second. Assuming $\delta = 20°$, and $\beta = 105°$, we have for the angle of the guide curves:

$$cotg.\ \alpha = cotg.\ \beta + \frac{1}{sin.\ \delta} = cotg.\ 105° + \frac{1}{sin.\ 20°} = -0,26795 + 2,92380 = 2,65585,$$

and, therefore, $\alpha = 20°, 38'$. Assuming $\zeta = 0,15$, and $\varkappa = 0,10$, the best velocity for the wheel:

$$v = \sqrt{\frac{2gh}{2\frac{sin.\ \beta\ cos.\ \alpha}{sin.\ (\beta - \alpha)} + \zeta\left(\frac{sin.\ \beta}{sin.\ (\beta - \alpha)}\right)^2 + \varkappa}} = \frac{8.02\sqrt{12}}{\sqrt{1,813 + 0,1407 + 0,1000}}$$

$= \frac{27,78}{\sqrt{2,0537}} = 19,38$ feet, and from this we have the velocity of entrance:

$$c = \frac{v\ sin.\ \beta}{sin.\ (\beta - \alpha)} = 18,77 \text{ feet.}$$

The sections $F = \frac{Q}{c} = \frac{8}{18,77} = 0,4262$ square feet, and $F_2 = \frac{Q}{v} = \frac{8}{19,38} = 0,4127$ square feet, and if we take the ratio $\nu = \frac{d}{r} = \frac{1}{3}$, the mean radius:

$r = \sqrt{\frac{F}{2\pi\nu\ sin.\ \alpha}} = \sqrt{\frac{0,4262}{\frac{2}{3}\pi\ sin.\ 20°, 38'}} = 0,7598$ feet, and the width of wheel

$d = \nu \cdot r = \frac{0,7598}{3} = 0,2532$ feet. From the space occupied by the buckets, each of these calculated dimensions should be somewhat increased.

The width of the channels $e = \frac{1}{2}d = 0,1266$ feet, and $n$ the number of buckets $= \frac{F}{de} = \frac{0,4262}{0,2532 \cdot 0,1266} = \frac{0,4262}{0,032} = 13,29$, for which we may, however, adopt 16.

The height of the wheel $b$ is made $= d = 0,2532$. The radius of the reservoir may be made somewhat greater than $r + \frac{d}{2} = 0,7598 + 0,1266 = 0,8864$, or about 1 foot, and hence the area of it will be $\pi = 3,1416$ square feet. The velocity:

$w_1 = \frac{Q}{\pi} = \frac{8}{3,1416} = 2,546$ feet, and the height due to this velocity:

$z = 0,0155 \cdot 2,546^2 = 0,1005$ feet. The effect of this wheel, when the sluice is completely drawn, would be:

$$L = \left(h - \left[\zeta c^2 + \varkappa c_2^2 + \left(2\ v\, sin.\frac{\delta}{2}\right)^2 + w_1^2\right] \cdot \frac{1}{2g}\right)Q\gamma$$

$= (12 - [0,15 \cdot 18,77^2 + 0,10 \cdot 19,38^2 + (2 \cdot 19,38\ sin.\ 10°)^2 + 2,546^2] \cdot 0,0155)\ 8 \cdot 62,5$
$= (12 - 103,66 \cdot 0,0155) \cdot 500 = 10,39 \times 500 = 5195$ ft. lbs., $= 9,4$ horse power. The losses of effect in the reservoir would reduce this to 4800 ft. lbs., so that the efficiency would be something near 0,80; for as the power expended is $62,5 \times 8 \times 12 = 6000$ ft. lbs., $4800 \div 6000 = ,80$. These calculations are for English measures.

§ 171. *Experiments on Fontaine's and Jonval's Turbines.*—Very trustworthy experiments on these wheels are detailed in the "Comptes Rendues de l'Académie des Sciences à Paris, 1846." There are also some earlier experiments by MM. Alcan and Grouvelle. (*See* Bulletin de la Société d'Encouragement, tome xliv.")

These experiments show that in Fontaine's turbines, as in Fourneyron's, the efficiency is greatest when the sluice is quite drawn up, and that the efficiency is less affected by variations of *head*, than by variations in the quantity of water supplied. The turbine at Vadeney, near Chalons-sur-Marne, the efficiency of which was determined by Alcan and Grouvelle, has 1,6 metres (5,24 feet) external diameter, 0,12 metres (nearly 5 inches) in height, the fall was $5\frac{1}{2}$ feet, the quantity of water about 93 gallons per second. The principal result of this experiment was that for $u = 30$ to 50 per minute, the efficiency was 0,67. One of Fourneyron's wheels of an early date, made for the same fall, gave $\eta = 0,60$. Morin's experiments were made on a turbine for a powder mill, at Bouchet. The diameter was 1,2 metres, the width 0,25 metres. There were 24 guide-curves and 58 wheel buckets. It had a fallcof about 12 metres, and 6 cubic feet per second supply. Experiments were made with 2ç 3, and 4 inches of the sluice drawn, and the following results obtained. Sluice quite open $u = 45$, the efficiency a maximum, and $= 0,69$ to 0,70.

When the sluice was shut so as to reduce the expenditure by $\frac{1}{4}$, $\eta$ was reduced to 0,57. The efficiency varied little with the velocity of the wheel, for when making 35 revolutions per minute, $\eta$ was still $= 0,64$, and for 55 revolutions, $\eta = 66$. It appears, too, that the greatest power exerted, and at which the wheel moved irregularly, was about $1\frac{1}{2}$ times that with which the wheel produced its maximum effect. The wheel was a few inches in back water during the experiments. We see from these experiments, that Fontaine's turbine may be considered among the first-class of hydraulic wheels. The circumstance of the pivot being out of water is an advantage (though obtained at considerable expense, and by a method inapplicable to large machines). The "graissage atmosphérique" of Decker and Laurent accomplishes the same end, the lower end of the upright shaft being surrounded by a bell, analogous to a diving-bell, which revolves with it. The air in the bell is kept of the necessary density by a small air-pump.

§ 172. *Jonval's Turbines.* — The experiments on Jonval's turbines gave equally favorable results as those on Fontaine's. Messrs. Köchlin and Co. have detailed experiments on one constructed by them at Mühlhausen, in the "Bulletin de la Société Industr. de Mülhause, 1844." This turbine was 3,1 feet in diameter, 8 inches high. It was placed $2' - 8''$ under the surface of the water in the lead, the fall being, however, $5\frac{1}{2}$ feet, and the supply being 125 gallons per second. The efficiency for $u = 73$ to 95 per minute was 0,75 to 0,90. Morin considers, however, that the quantity of supply was reckoned too low, and that, therefore, this high efficiency must be reduced from 0,63 to 0,71.

Colonel Morin made experiments with a turbine of 0,81 metres external diameter, 0,12 metres internal width, 18 buckets, fall $5\frac{1}{2}$ feet, supply 45 to 65 gallons per second. Morin comes to the following conclusions from all his experiments. In the normal state, the water having impeded entrance and exit, the number of revolutions was 90 per minute and $\eta = 0,72$. By putting contracting pieces on the wheel, the efficiency did not become much less (0,63) until the section was very considerably diminished.

The efficiency did not vary for variations of velocity 25 per cent. above and below that for the maximum effect. By depressing the sluice, the efficiency was diminished, so that it is evidently a very imperfect *regulator* for the wheel. When the section of the aperture for the discharge of the water was reduced to 0,4 of that for the normal condition, $\eta$ was reduced to 0,625.

Redtenbacher gives some experiments on a turbine of Jonval's, the maximum efficiency for the sluice fully drawn having been = 0,62. As in the case of Fourneyron's turbines, these experiments indicate that the wheel working without load makes about twice as many revolutions as when furnishing its maximum effect in its normal state.

§ 173. *Comparison of different Turbines with each other.*—If we compare the turbines of Fontaine and Jonval with those of Fourneyron, we find that in Fontaine's turbines the water is less deviated from its original direction of motion than in Fourneyron's, so that for the same velocity of entrance the resistance is less in the one than in the other. Thus the velocity of entrance in Fontaine's wheel may be made greater, and, therefore, the wheel may be made less in diameter than Fourneyron's. The guide-curves of Fontaine's wheels take on the water in more nearly parallel layers than they do in Fourneyron's wheels, where a divergence of the stream entering the wheel cannot possibly be avoided.

On the other hand, Fourneyron's wheels have certain advantages. The pressure on the pivot is reduced to the *weight* of the machine in motion, whilst in Fontaine's, the *whole weight of water* is borne by the pivot, thus involving greater friction, *cæteris paribus*. Again, in Fourneyron's turbines, the particles of water move with the same velocity of rotation, which is not the case in the newer turbine, in which the velocity of the outer particles is much greater than that of the inner. This gives rise to eddying motions, consuming mechanical effect, and causing irregularities in the motion of the water through the wheel. The turbine of Fourneyron is also more easily constructed than that of Fontaine, particularly the buckets.

*Remark* 1. The Fontaine turbines are well adapted for *tide-mills.*

*Remark* 2. Jonval's turbines are considered to present advantages in respect of their being placed so that they can be easily got at. The limit at which they may be placed above the tail-race has been already pointed out to be 34 feet; but from experiments of M. Marazeau, and from certain theoretical considerations of Morin, it appears that the height of the turbine above the water in the race must not exceed even lower limits than the above, because otherwise, the water is very apt to *lose* its continuity immediately under the wheel, and thus *effect* is lost.

§ 174. *Comparison between Turbines and other Water Wheels.*—Turbines, from their nature, are applicable to fallse of any height, from 1 to 500 feet. Vertical water wheels are limited in their applications to falls under 60 feet as the highest. The efficiency of turbines for very high falls is less than for smaller falls, on account of the hydraulic resistances involved, and which increase as the square of the velocity. Vertical water wheels having from 20 to 40 feet fall, give a greater efficiency than any turbine. For falls of from 10 to 20 feet, they may be considered as being very nearly on a *par* in point of efficiency; and, for very low falls, turbines give a higher efficiency than any vertical wheel that could be substituted for them. Poncelet's wheels, for falls of from 3 *to* 6 *feet*, are on a *par* with turbines, but only within these limits. Turbines are unaffected by back-water, whilst vertical wheels lose effect in this condition. Variations in supply of water affect the efficiency of vertical water wheels less than they do that of turbines. This gives the vertical water wheel an hydraulic economical advantage, which is in some cases of great importance. When water becomes scarce, the best effect from what is available may *always* be depended upon from a good vertical wheel, whilst the turbine falls off in efficiency as its sluice is lowered, from causes which in our discussion of the theory of turbines we have fully explained.

§ 175. Variations of velocity on either side of the normal conditions, have the same result in the two kinds of wheels, but the turbines have a decided advantage, in that they make a greater number of revolutions per minute than any vertical wheels. The velocity of rotation is limited to from 4 to 8 feet per second, whilst in turbines this velocity, having a certain ratio to the height of fall, is generally much greater. The application of water to operations requiring great velocity, is, therefore, most advantageously made by turbines; whilst for operations requiring slow motions, the vertical wheel is to be preferred. It is a question of practical discretion, to decide as to whether it is better to reduce the velocity of turbines, or to raise the velocity of vertical wheels by means of the gearing that is to transmit their water-power to the work to be done.

For variable resistances, such as rolling mills, forge hammers, &c., the vertical wheel is certainly to be preferred, because its great mass serves better for regulating the motion than the smaller turbine, which for such work requires the addition of a fly-wheel.

In respect to economy of construction, turbines are at least as cheap as vertical wheels. When the fall is considerable and the quantity of water great, the turbine is the cheaper machine of the two. The turbine almost necessarily involves the use of iron in its construction, and hence cannot always be adopted. The durability, or the maintenance of a turbine, is probably less than that of a vertical wheel, *cæteris paribus.*

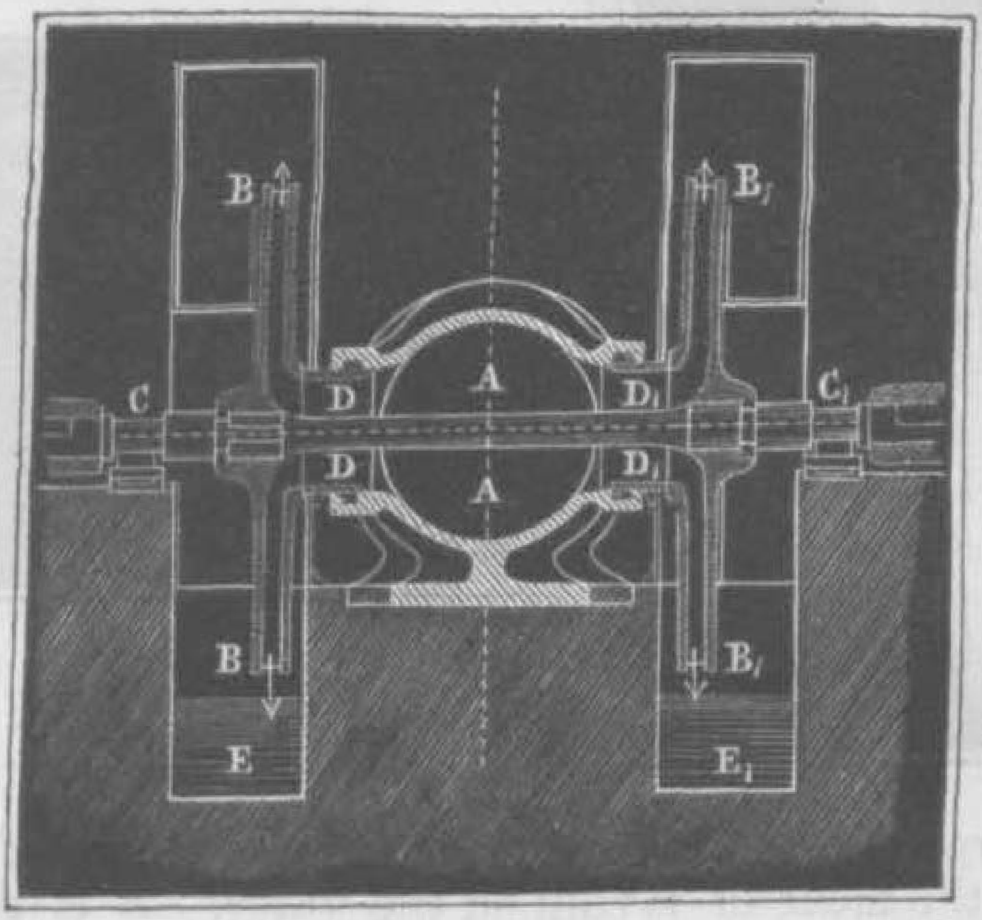
In respect to workmanship, it is manifest that the guide-curve turbines require greater *skill* than vertical water wheels do for their construction, with the same relative degree of perfection. Also, deviation from the scientific rules for their construction is of much more prejudicial consequence for turbines, than in the case of vertical wheels. This latter circumstance is the cause of the failure of many of the turbines that have been erected, and operates against their more general introduction.

Turbines, it must be borne in mind, require *clean* water to be laid on, for they would be greatly damaged bysand, mud,deaves, branches, ice, &c., passing through them, and their efficiency lessened. This is not the case with vertical wheels.

§ 176. *Turbines with Horizontal Axis.*—Examples of distorted ingenuity have been displayed in putting turbines, particularly Jonval's and Whitelaws', on horizontal axes. This mode of construction can never be advantageous, though it may have some local convenience suggesting its adoption.

Jonval and Redtenbacher have proposed the arrangement shown in Fig. 280. Where *AA* is the lead pipe, *BB* the one, and $B_1B_1$

Fig. 280.

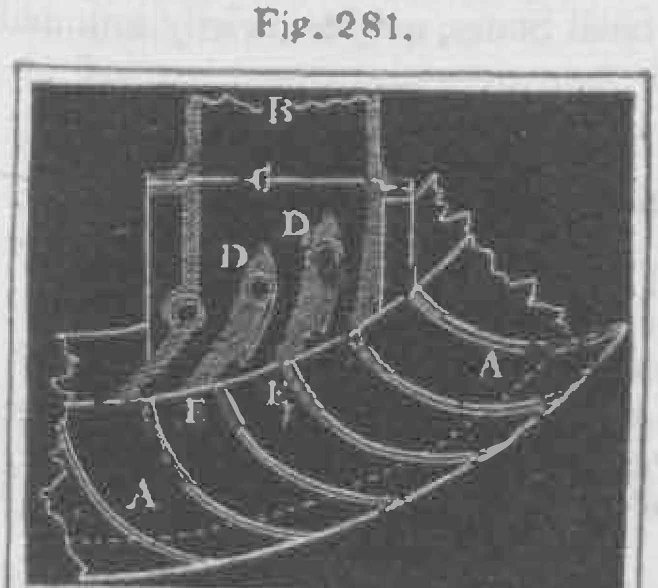



the other wheel, $CC_1$ the horizontal axis, and *DD* and $DD_1$ the jointing-rings (Vol. II. § 151), *E* and $E_1$ being the tail-race.

A throttle valve in the main or lead pipe is the means of regulation.

Herr Schwamkrug, of Freyberg, has recently erected a vertical wheel, working on the principle of the pressure turbine. The wheel is like one of Poncelet's, but the water is introduced on the *inside* by a pipe, so that it flows through the wheel near the bottom of it. Fig. 281 shows the arrangement adopted.

Fig. 281.




The guide-curves *DE*, $D_1E_1$ are movable on centres, and serve to regulate the discharge of water. This construction has advantages in respect of the wheel being little exposed to the action of the water, and as the water acts on a very small arc, the wheel must have a greater diameter than a turbine, and hence in cases where slow motion is required, may do away with the necessity of intermediate gear for reducing speed. But such a wheel would necessarily be more costly than a turbine, and its efficiency would certainly be less.

The same principle might be applied, as shown in elevation in Fig. 282, to a Fontaine's turbine. Such a machine is applicable for all falls, but never advantageously.

Before concluding this subject, we may add that Poncelet's turbines have been quite recently applied in Switzerland, under the name of tangential wheels.

Fig. 282.

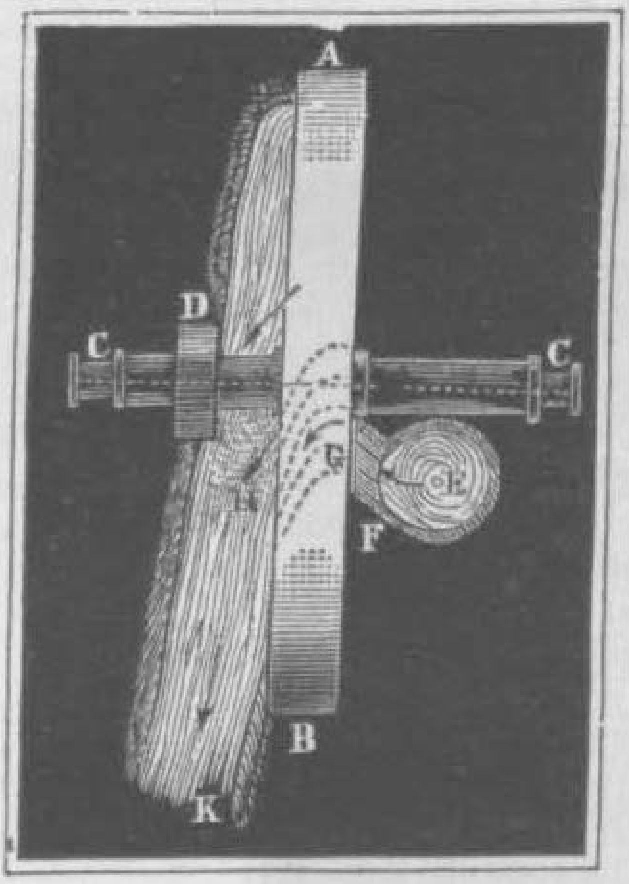



Fig. 283.

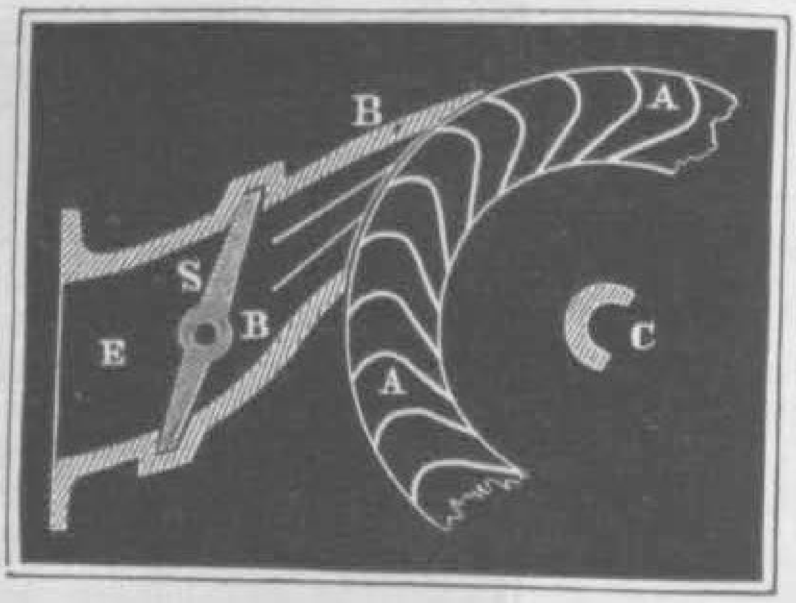



Fig. 283 represents a horizontal section of a part of one of these wheels, and the mode of laying on the water. $S$ is the regulating sluice, in advance of which the lead is divided into three channels by guide plates. The water is discharged in the interior of the wheel in such manner that the pivot is protected from the water.

[Mr. Ellwood Morris, in the "Journal of the Franklin Institute," for November, 1842 (third series, vol. iv., p. 303), in discussing the advantages of Fourneyron's turbines, makes the following remarks: "In conclusion, the chief points of advantage promised by the use of turbines upon the mill seats of the United States, may be briefly summed up as follows:—

1. They act with perfect success in back-water.
2. They are not liable to obstruction from ice.
3. They require but little gearing to get up a high velocity at the working point.
4. They use to advantage every inch of fall.
5. They are equally applicable to very high and very low falls.
6. They are equal in power to the best overshot wheels.
7. They may vary greatly in velocity without losing power.
8. They are very compact and occupy but little room.
9. They may be very accurately regulated to an uniform speed.
10. They are perfectly simple, and not likely to get out of order.
11. They are not very expensive.
12. They are very durable.

"Upon one account or another," he adds, "the turbine is superior to all other water wheels, and consequently must be regarded as the very best hydraulic motors now known to mechanics."—Am. Ed.]

*Literature.* The literature on turbines has of late years become very extensive. We have already mentioned several treatises and papers on the subject. The following are some of the more important works:—

Fourneyron's original paper appeared in the "Bulletin de la Société d'Encouragement, 1834." Morin's "Experimental Inquiry," already quoted, followed in 1838. In 1838, Poncelet published his "Théorie des Effets mécaniques de la Turbine Fourneyron," in the "Comptes Rendues," and as a separate treatise. In D'Aubuisson's "Hydraulique," the turbine is treated of, but only superficially. In 1843, Combes published, "Recherches théoretiques et experimentales sur les Roues à réaction ou à tuyaux," a tract of considerable importance, as it for the first time recognizes the necessity of taking into consideration the hydraulic resistances, which Poncelet and Redtenbacher have neglected to do. Redtenbacher's work, "Théorie und Bau der Turbinen und Ventilatoren, Manheim, 1844," is founded on Poncelet's theory, and is the best and most complete work on the subject. On the newer turbines, there appears in the "Comptes Rendues," tome xxii., 1846, "Rapport sur un Mémoire de M. M. A. Koechlin, concernant une nouvelle, Turbine (Jonval) construite dans leurs ateliers, par Poncelet, Piobert, et Morin." Also, "Note sur la Théorie de la Turbine de Koechlin, par Morin," et "Note sur l'Application de la Théorie du Mouvement des Fluides aux expériences de M. Maroseau, par Morin." In the "Comptes Rendues," &c., t. xxiii., 1846, there appears a paper "Expériences et Notes sur la Turbine de M. Fontaine-Baron, par Morin." The "Bulletin de la Société d'Encouragement, 1844-45," contains notices of the turbines of Jonval and Fontaine.

Armengand's publication "Industrielle," contains good drawings and descriptions of the turbines of Cadiat, Callon. Fourneyron, and Gentilhomme. In the "Polytech. Centralblatt, bd. vii., 1846," Parro's turbine is described. Nagel's turbine is described in Dingler's "Journal, bd. xcv.," and Passot's turbine, in the same Journal, bd. xciv. Bourgeois' screw, is a *turbine-helice*, or with screw-formed channels. See "Polytechnisches Centralblatt, bd. i., 1847." In the "Proceedings of the Institution of Civil Engineers for 1842," there is a notice of turbines by Prof. Gordon. In the "Transactions of the Society of Arts of Scotland, 1805," there is a notice of a turbine erected at Mr. J. G. Stuart's flax-mill, at Balgonie, in Fifeshire. This is the first turbine erected in Britain, and is one of the largest ever made. Its efficiency is reckoned to be = 0,70.