A NOTE ON GENERAL AND SPECIFIC COMBINING ABILITY

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I. Introduction

The expectation of the mean square for specific combining ability has appeared in several forms in the literature [1-7]. The question concerning the "correctness" of the various forms has arisen from time to time. All forms are correct, but different assumptions and definitions are involved in obtaining the various expectations. The purpose of this paper is

(i) to set forth the specific and general combining ability problem in detail,
(ii) to set forth the assumptions and definitions required to obtain the various expectations,
(iii) to discuss the validity of the assumptions and definitions, and
(iv) to redefine specific combining ability.

II. The Experimental Situation

The linear model usually assumed for the possible crosses among p lines observed in a completely randomized design is

\[ Y_{ijh} = \mu + \alpha_i + \alpha_j + \delta_{ij} + \epsilon_{ijh}, \]

where \( Y_{ijh} \) represents an observation on the hth offspring (or group of off- springs) from the cross of line i with line j, \( \mu \) = an effect common to all observations, \( \alpha_i \) = the effect of the ith line, \( \alpha_j \) = the effect of the jth line, \( \delta_{ij} \) = an effect common to ith line crossed with jth line = nicking effect = effect due to dominance and epistasis, \( \epsilon_{ijh} \) = random component.
associated with measurement of the effects. It is assumed that there are
n offspring from each cross, that there are p(p-1)/2 crosses (i.e., there
are no reciprocals), and that the $\epsilon_{ijh}$ are random independent variates with
mean zero and common variance $\sigma^2_e$.

The addition of other additive effects for stratification, such as
blocks, does not affect the expectation of the mean squares for general and
for specific combining ability. Hence, no effects for stratification are
included in the model.

The breakdown of the degrees of freedom in the analysis of variance
for the experiment described above is:

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degrees of freedom</th>
<th>Sum of squares</th>
<th>Mean square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Among lines</td>
<td>$p-1$</td>
<td>$G$</td>
<td>$G'$</td>
</tr>
<tr>
<td>Among crosses within lines</td>
<td>$p(p-3)/2$</td>
<td>$S$</td>
<td>$S'$</td>
</tr>
<tr>
<td>Among yields of the same cross</td>
<td>$p(p-1)(n-1)/2$</td>
<td>$E$</td>
<td>$E'$</td>
</tr>
<tr>
<td>Total</td>
<td>$np(p-1)/2 - 1$</td>
<td>$T$</td>
<td></td>
</tr>
</tbody>
</table>

The normal equations for the completely randomized design with
$p(p-1)/2$ crosses, each with $n$ offspring (or with $n$ groups (plots) of
offspring) per cross, are obtained from the following table of totals,
each from $n$ observations:
The residual sum of squares is:

$$R = \sum_{i \leq j=2}^{p} \sum_{h=1}^{n} (Y_{ijh} - \mu - \alpha_i - \alpha_j - \delta_{ij})^2 ;$$ (II-2)

the partial derivatives with respect to $\mu, \alpha_i', \alpha_j', \text{ and } \delta_{ij}$ are:

$$\frac{dR}{d\mu} = -2 \sum_{i \leq j} \sum_{h=1}^{n} (Y_{ijh} - \mu - \alpha_i - \alpha_j - \delta_{ij}) ;$$ (II-3)

$$\frac{dR}{d\alpha_i} = -2 \sum_{j \neq i} \sum_{h=1}^{n} (Y_{ijh} - \mu - \alpha_i - \alpha_j - \delta_{ij}) ;$$ (II-4)

$$\frac{dR}{d\alpha_j} = -2 \sum_{i \neq j} \sum_{h=1}^{n} (Y_{ijh} - \mu - \alpha_i - \alpha_j - \delta_{ij}) ;$$ (II-5)

$$\frac{dR}{d\delta_{ij}} = -2 \sum_{h=1}^{n} (Y_{ijh} - \mu - \alpha_i - \alpha_j - \delta_{ij}) .$$ (II-6)

The various normal equations are:

For the mean $\hat{\mu}$

$$\frac{np(p-1)}{2} \hat{\mu} + n(p-1) \sum_{i=1}^{p} \hat{\alpha}_i + n \sum_{i<j} \hat{\delta}_{ij} = \sum_{i<j} \sum_{h=1}^{n} Y_{ijh} = Y_{...} .$$ (II-7)
For the $\hat{\alpha}_i$

$$n(p-1)(\mu + \hat{\alpha}_i) + n \sum_{j \neq i} (\hat{\alpha}_j + \hat{\delta}_{ij}) = \sum_{j \neq i} \sum_{h} Y_{ijh} = Y_{..}.$$  

(II-3)

For the $\hat{\alpha}_j$

$$n(p-1)(\mu + \hat{\alpha}_j) + n \sum_{i \neq j} (\hat{\alpha}_i + \hat{\delta}_{ij}) = \sum_{i \neq j} \sum_{h} Y_{ijh} = Y_{..}.$$  

(II-9)

For the $\hat{\delta}_{ij}$

$$n(\mu + \hat{\alpha}_i + \hat{\alpha}_j + \hat{\delta}_{ij}) = Y_{ij}.$$  

(II-10)

Using the following relations:

$$\sum_{i=1}^{p} \hat{\alpha}_i = \sum_{j=1}^{p} \hat{\alpha}_j = 0 \quad \text{and}$$

$$\sum_{i \neq j} \hat{\delta}_{ij} = \sum_{j \neq i} \hat{\delta}_{ij} = 0,$$

the following solutions for the estimates $\hat{\mu}, \hat{\alpha}_i, \hat{\alpha}_j,$ and $\hat{\delta}_{ij}$ are obtained:

$$\hat{\mu} = 2 \bar{Y}_{..} / np(p-1) = \frac{1}{p} \sum_{i} \bar{Y}_{..} = \bar{y},$$  

(II-13)

$$\hat{\alpha}_i = \frac{n(p-2)}{n(p-1)} \left\{ Y_{i..} - n(p-1)\hat{\mu} \right\};$$

(II-14)

$$\hat{\alpha}_j = \frac{n(p-2)}{n(p-1)} \left\{ Y_{..j} - n(p-1)\hat{\mu} \right\};$$

(II-15)

$$\hat{\delta}_{ij} = \frac{1}{r(p-2)} \left\{ (p-2)y_{ij} - Y_{i..} - Y_{..j} + 2y_{..} \right\}.$$  

(II-16)

The various sums of squares are:

Total with np(p-1)/2 - 1 d.f.

$$T = \sum_{i \leq j=2}^{p} \sum_{h=1}^{n} Y_{ijh}^2 - 2Y_{..}^2 / np(p-1).$$  

(II-17)
Among lines with p-1 d.f.

\[ G = \sum_{i=1}^{p} \frac{y_{1..}^2}{n(p-2)} - \frac{4}{np(p-2)} \]

\[ = \frac{4}{np(p-2)} \sum \left( \frac{p}{2} y_{1..} - y_{..} \right)^2 \]  \hspace{1cm} (II-18)

Among crosses within lines with p(p-1)/2 d.f.

\[ S = \sum_{i \neq j} \frac{y_{1j..}^2}{n} - \frac{\sum y_{1..}^2}{n(p-2)} + \frac{2y_{..}^2}{n(p-1)(p-2)} \]

\[ = \sum_{i \neq j} \left\{ (p-2)y_{1ij} - y_{1..} - y_{..} + \frac{2y_{..}^2}{p-1} \right\} / n(p-2) \]

\[ = \sum_{i \neq j} \left\{ (p-2)y_{1ij} - y_{1..} - y_{..} + \frac{2y_{..}^2}{p-1} \right\} / n(p-2)^2 \]  \hspace{1cm} (II-19)

Among yields within crosses with p(p-1)(p-1)/2 d.f.

\[ E = \sum_{i < j} \left\{ \sum_{h} y_{1jh}^2 - y_{1j..}^2 / n \right\} \]  \hspace{1cm} (II-20)

III. Expectation of Mean Squares

III - 1. Random effects case

If it is assumed that the p lines represent a random sample of lines from a large population, then the expectation of the mean squares \( G \) and \( S \) are, respectively:
\[ E \left[ \mathbf{c}_i \right] = \frac{1}{p-1} E \left[ \sum_{i=1}^{p} \frac{Y_{i,2}}{n(p-2)} - \frac{4Y_{2}}{np(p-2)} \right] \]

\[ = \frac{1}{np(p-1)(p-2)} \left\{ p \sum_{1}^{p} \frac{E \left[ n(p-1)(\mu+\alpha_i) + n \sum_{j \neq i} (\alpha_j+\delta_{ij}) + \sum_{j \neq i} \epsilon_{ijh} \right]^2}{2} \right. \]

\[ - 4E \left[ \frac{np(p-1)}{2} \mu + n(p-1) \sum_{i=1}^{p} \alpha_i + n \sum_{i \neq j} \delta_{ij} \right. \]

\[ + \sum_{1 < j \neq h} \epsilon_{ijh} \right] \left. \right\} = \frac{1}{np(p-1)(p-2)} \left\{ \frac{p^2}{2} \left[ n^2(p-1)^2 (\mu^2 + \sigma^2) \right. \right. \]

\[ + n^2(p-1) (\sigma^2 + \sigma^2) + n(p-1) \sigma^2 \] - \left. \left[ n^2p^2(p-1)^2 \mu + 4n^2p(p-1)^2 \sigma^2 + 2n^2p(p-1) \sigma^2 + 2np(p-1) \sigma^2 \right] \right\} \]

\[ = \sigma^2 + n\sigma^2 + n(p-2)\sigma^2 \quad \text{(III-1)} \]

\[ E \left[ S^2 \right] = \frac{2}{p(p-3)} E \left\{ \sum_{1 < j} \frac{Y_{ij}^2}{n} - \frac{p}{n(p-2)} + \frac{2Y_{2}}{n(p-1)(p-2)} \right\} \]

\[ = \frac{2}{np(p-2)(p-3)} \left\{ (p-2) \sum_{1 < j} \frac{E \left[ n(\mu+\alpha_i+\alpha_j+\delta_{ij}) + \sum_{h} \epsilon_{ijh} \right]^2}{2} \right. \]

\[ - \sum_{i} \left[ n(p-1)(\mu+\alpha_i) + n \sum_{j \neq i} (\alpha_j+\delta_{ij}) + \sum_{j \neq i} \epsilon_{ijh} \right]^2 \]

\[ + \frac{2}{p-1} E \left[ \frac{np(p-1)}{2} \mu + n(p-1) \sum_{i \neq j} \delta_{ij} + \sum_{i \neq j} \epsilon_{ijh} \right]^2 \]

\[ = \frac{2}{np(p-2)(p-3)} \left\{ \frac{(p-2)p(p-1)}{2} \left[ n^2\mu^2 + 2n^2\sigma^2 + n^2\sigma^2 + n\sigma^2 \right] \right. \]

\[ - p \left[ n^2(p-1)^2 \mu^2 + [n^2(p-1)^2 + n^2(p-1)] \sigma^2 + n^2(p-1)\sigma^2 + n(p-1)\sigma^2 \right] \]

\[ + \frac{2}{p-1} \left[ \frac{n^2p^2(p-1)^2}{2} \mu^2 + n^2(p-1)^2 p\sigma^2 + n^2p(p-1)\sigma^2 + np(p-1)\sigma^2 \right] \]

\[ = \sigma^2 + n\sigma^2 \quad \text{(III-2)} \]
In the above expectations it is assumed that the effects $\alpha_i$, $\delta_{ij}$, and $\epsilon_{ijh}$, are independent random effects with mean zero, and that

$$E[\alpha_i^2] = \sigma_\alpha^2;$$  \hspace{1cm} (III-3)

$$E[\delta_{ij}^2] = \sigma_\delta^2;$$  \hspace{1cm} (III-4)

$$E[\epsilon_{ijh}^2] = \sigma_\epsilon^2;$$  \hspace{1cm} (III-5)

$$E[\mu^2] = \mu^2;$$  \hspace{1cm} (III-6)

$$E[\epsilon_{ijh} \epsilon_{efg}] = 0 \text{ except where } ijh = efg.$$  \hspace{1cm} (III-7)

III - 2. Fixed effects case

In the fixed effects case it is assumed that the $\epsilon_{ijh}$ is a random variate independent of the other effects and that the following relations hold:

$$\sum_{i=1}^{p} \alpha_i = 0;$$  \hspace{1cm} (III-8)

$$\sum_{i=1}^{p} \delta_{ij} = 0 = \sum_{j=1}^{p} \delta_{ij};$$  \hspace{1cm} (III-9)

$$E[\alpha_i^2] = \alpha_i^2;$$  \hspace{1cm} (III-10)

$$E[\mu^2] = \mu^2;$$  \hspace{1cm} (III-11)

$$E[\delta_{ij}^2] = \delta_{ij}^2;$$  \hspace{1cm} (III-12)

$$E[\epsilon_{ijh}^2] = \sigma_\epsilon^2;$$  \hspace{1cm} (III-13)

$$E[\epsilon_{ijh} \epsilon_{efg}] = 0 \text{ except when } ijh = efg;$$  \hspace{1cm} (III-14)

$$E[\alpha_i] = \alpha_i;$$  \hspace{1cm} (III-15)

$$E[\alpha_i \mu] = \alpha_i \mu.$$  \hspace{1cm} (III-16)
\[ E[\alpha_i \delta_{ij}] = \alpha_i \delta_{ij} \quad ; \quad (III-17) \]

\[ E[\delta_{ij}\mu] = \delta_{ij}\mu \quad . \quad (III-18) \]

With the above assumptions the expected values of \( G \) and \( S \) are, respectively:

\[
E[G] = \frac{1}{n p(p-1)(p-2)} \left\{ p \Sigma E[X_i^2] - 4E[X_{i\ldots}^2] \right\}
\]

\[
= \frac{1}{n p(p-1)(p-2)} \left\{ p \Sigma \left( n(p-1) \mu + n(p-2) \alpha_i \right)^2 + n(p-1) \sigma^2 \right\}
- 4 \left\{ \frac{n^2 p^2 (p-1)^2}{4} \mu^2 + \frac{np(p-1)}{2} \sigma^2 \right\}
\]

\[
= \frac{1}{n p(p-1)(p-2)} \left\{ np(p-1)(p-2) \sigma^2 + np^2 (p-2)^2 \Sigma \alpha_i^2 \right\}
\]

\[
= \sigma^2 + \frac{n(p-2)}{p-1} \Sigma \alpha_i^2 \quad ; \quad (III-19)
\]

\[
E[S] = \frac{2}{np(p-3)} \left\{ \Sigma E[Y_{ij}^2] - \Sigma E[Y_{i\ldots}^2] \right\}
+ 2E[Y_{i\ldots}^2]/(p-1)(p-2)
\]

\[
= \frac{2}{np(p-2)(p-3)} \left\{ (p-2) \Sigma \left[ n^2 (\mu + \alpha_i + \alpha_j + \delta_{ij})^2 + \frac{np(p-1)}{2} \sigma^2 \right] \right. \\
- \left. \Sigma (n(p-1)\mu + n(p-2)\alpha_i)^2 + np(p-1)\sigma^2 \right\}
+ \frac{2}{(p-1)} \left\{ \frac{n^2 p^2 (p-1)^2}{4} \mu^2 + \frac{np(p-1)}{2} \sigma^2 \right\}
\]

\[
= \frac{2}{np(p-2)(p-3)} \left\{ n^2 (p-2) \Sigma (\alpha_i + \alpha_j)^2 - n^2 (p-2)^2 \Sigma \alpha_i^2 \right. \\
+ \left. \sigma^2 \left( \frac{np(p-1)(p-2)}{2} - np(p-1) + np \right) \right\}
\]
+ (p-2)n^2 \sum_{i<j} \delta_{ij}^2 \right) \\
= \sigma^2 + \frac{2n}{p(p-3)} \sum_{i<j} \delta_{ij}^2 \quad (III-20)

IV. Expectation of Mean Square for General Combining Ability of ith Line

IV. - 1. Random effects case

Since \( E[\alpha_i^2] = \sigma^2 \) and since it is considered that \( \alpha_i \) is a randomly selected member from an infinite population, it is inappropriate to talk about the expectation of the mean square for general combining ability for the ith line. Instead, one should talk about estimates of \( \sigma^2 \) obtained from a randomly selected line. The expectation of \( \hat{\alpha}_i^2 \) is:

\[
E[\hat{\alpha}_i^2] = E\left[ \left\{ \frac{(y_i - 2\bar{y} \cdots}{p} \right\}^2 \right]/n(p-2) \left\{ \sum_{j=1}^{p} (\alpha_j + \delta_{ij}) + \sum \Sigma \sum \epsilon_{ijh} \right. \\
- \frac{2}{p} \left( np(p-1) \alpha_i + np \sum_{i<j=2}^{p} \sum_{i<j=2}^{p} \sum_{h=1}^{n} \epsilon_{ijh} \right) \left\}^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ n(p-2)\alpha_i + \Sigma \alpha_i(n-2n(p-1)) + n \sum_{j \neq i} \delta_{ij} \\
+ \sum \sum \epsilon_{ijh} - \frac{2n}{p} \sum_{i \neq j} \delta_{ij} - \frac{2}{p} \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right] \\
- \frac{2}{p} \sum \sum \sum \sum \epsilon_{ijh} \right] ^2 \right) \\
= \frac{1}{n^2(p-2)^2} \left[ \frac{n^2(p-1)(p-2)^2}{p} \sigma^2 + \frac{n^2(p-1)(p-2)}{p} \sigma^2 \\
+ \frac{n(p-1)(p-2)}{p} \sigma^2 \right]
Therefore, the estimated component of variance from the ith for general combining ability is:

\[
\hat{\sigma}_i^2 = p \left( \frac{Y_i - 2Y}{p} \right)^2 / n^2(p-1)(p-2)^2 - \frac{S'}{n(p-2)}.
\]  

(IV-2)

IV - 2. Fixed effects case

For this case it is assumed that \( E[\alpha_i^2] = \alpha_i^2 \), \( E[\alpha_i] = \alpha_i \), \( \Sigma \alpha_i = \text{zero} \) (i.e., the p \( \alpha_i \) constitute the entire (finite) population), \( \Sigma \delta_{ij} = \text{zero} \), and the \( \epsilon_{ijh} \) are random independent variates with mean zero and variance \( \sigma_\epsilon^2 \). Thus,

\[
E[\hat{\alpha}_i^2] = \frac{1}{n^2(p-2)^2} E \left[ \left( \frac{Y_i - 2Y}{p} \right)^2 \right]
\]

\[
= \frac{1}{n^2(p-2)^2} \left[ \frac{n(p-1)^2 + n(p-2)\alpha_i + \Sigma \Sigma \epsilon_{ijh}}{2} \right]
\]

\[
- \frac{2}{p} \left( \frac{n(p-1)^2 + \Sigma \Sigma \epsilon_{ijh}}{2} \right)^2
\]

\[
= \frac{1}{n^2(p-2)^2} \left[ n^2(p-2)^2\alpha_i^2 + \frac{n(p-1)(p-2)}{p} \sigma_\epsilon^2 \right]
\]

\[
= \alpha_i^2 + \frac{p-1}{np(p-2)} \sigma_\epsilon^2
\]  

(IV-3)

which is what was obtained by Sprague and Tatum [7] and Federer [2]. Therefore, \( \alpha_i^2 \) is estimated as follows:

\[
\hat{\alpha}_i^2 = \frac{1}{n^2(p-2)^2} \left( \frac{Y_i - 2Y}{p} \right)^2 - \frac{np(p-2)}{p-1} E
\]

\[
= \frac{p-1}{np(p-2)} \left\{ \frac{p}{n(p-1)(p-2)} \left( \frac{Y_i - 2Y}{p} \right)^2 - E \right\},
\]  

(IV-4)
which is a form of the formulae obtained by Sprague and Tatum [7] and Federer [2].

It should be pointed out that \( \hat{a}_i^2 \) is not a variance component in the usual sense, but is merely the square of an estimate of a parameter. The usefulness of a quantity like \( \hat{a}_i^2 \) is not known since the squaring obliterates the algebraic sign. The quantity \( \hat{a}_i \) is, however, of use in estimating the general combining ability for a given line in relation to the other lines in the experiment.

With regard to either the random effects or the fixed effects case, there appears to be no difficulties in estimating and in using the general combining ability for the \( i \)th line. The nature of the experimental material and conditions will determine which model is appropriate.

**IV - 3. Average over all lines**

From formulae (III-1) and (III-2) we note that the estimate of \( \sigma_\alpha^2 \) for the random effects case may be obtained as

\[
\hat{\sigma}_\alpha^2 = \frac{1}{n(p-2)} \left\{ G' - S' \right\}. \tag{IV-5}
\]

It is of interest to see the relationship between the \( p \) \( \hat{\sigma}_\alpha^2 \)'s obtained from formula (IV-2) for all of the \( p \) lines and \( \hat{\sigma}_\alpha^2 \) obtained in (IV-5). Summing over \( i \) and dividing by \( p \) in (IV-2) we obtain

\[
\hat{\sigma}_\alpha^2 = \frac{1}{n^2(p-1)(p-2)} \left\{ \sum_{i=1}^{p} \left( Y_{1i} - 2Y \ldots / p \right)^2 - \frac{1}{n(p-2)} S' \right\}
= \frac{1}{n(p-2)} \left\{ G' - S' \right\} \tag{IV-6}
\]

which is identical to (IV-5).

For the fixed effects case, the estimate of \( \Sigma a_i^2 \) from formula (III-19) is

\[
\hat{\Sigma a_i^2} = \frac{p-1}{n(p-2)} \left\{ G' - E' \right\}. \tag{IV-7}
\]
Now if we sum over the $\hat{\alpha}_i^2$ obtained from formula (IV-4) we obtain

$$\sum \hat{\alpha}_i^2 = \frac{1}{n^2(p-2)^2} \sum (Y_{i..} - 2Y_{i.}/p)^2 - \frac{(p-1)}{n(p-2)} E'$$

$$= \frac{p-1}{n(p-2)} \left\{ G' - E' \right\}$$  \hspace{1cm} (IV-8)

which is formula (IV-7).

Thus, for both cases the average of the estimates from each line is identical with that obtained from all lines taken collectively.

V. Expectations for Specific Combining Ability

for the ith Line

V - 1. Case I

For the first situation the following conditions are assumed:

(i) $E_i [\delta_{ij}^2] = \sigma_{\delta_i}^2$;

(ii) $E_i [\epsilon_{ij}^2] = \sigma_{\epsilon}^2$;

(iii) $E_i [\delta_{ij} \delta_{if} \text{ for } j \neq f] = 0$;

(iv) $\epsilon_{ijh}$ are random independent variates with mean zero;

(v) the $p-1$ lines crossed with line $i$ represent a random sample of lines from an infinite population.

In (i) to (iii) the conditional expectations are given for the $i$th line (see [8] on conditional distributions and on conditional probabilities).

The expectation of the mean square for specific combining ability for the $i$th line would not depend upon which line is being considered if we make assumption that $\sigma_{\delta_i}^2 = \sigma_{\delta}^2$ a constant. Since this assumption was not made it is necessary to use conditional expectations.