The Method of Virtual Reality for Creation a Disk–Ridging Tool

Mati Heinloo and Jüri Olt
Estonian University of Life Sciences
Kreutzwaldi 56, 51014 Tartu, Estonia
Mati.Heinloo@eau.ee, olt@eau.ee

ABSTRACT

The present paper concentrates the main attention to the description of the method of virtual reality for creation a disk–ridging tool and to study its complex working process. The influence of the angular velocity of rotation of the disk–ridging tool and its adjustment angle to the quality of virtual tillage and to the force system, applied to the blades of the virtual disk ridging tool, is studied. The method of virtual reality is illustrated by the virtual projections of a single blade, by the virtual projections of assembly of the blades of the disk–ridging tool and by the processed virtual regions. The force systems, applied to a single blade and to the assembly of the blades of the disk–ridging tool, are characterized by graphs of the dependencies of the forces and moments on the angle of rotation of the disk–ridging tool. Substantial dependence of the quality of the virtual processing a lateral surface of a virtual ridge on the value of the adjustment angle and on the angular velocity of the disk–ridging tool is detected. The results of this paper can be used by designers of potato field tillage machines with the forced driven disk–ridging tools, by the agricultural engineers, using the features of computer graphics, and by the users of modern computer package Mathcad.

1. INTRODUCTION

Low reproduction coefficient and high cost of seed potatoes have prompted the need for new technologies. Sinijärv (1982) has made a proposal for a new technology of growing seed potatoes, including the increasing the growing space of potatoes step by step. This technology, as well as the idea of creating a new combined wide row potato field tillage machine, had been described by Olt (2001) and Olt, Heinloo (2001c) in details. Thorough study of the working process of the rotating free–active tools, driven only by soil, in the structure of this machine had been made by Olt (2001) in the thesis for PhD in Agricultural Engineering. The results of this thesis were published by Olt (1999), Heinloo, Olt (2000), Heinloo, Olt. (2001a–c, 2002a) and reviewed by Heinloo, Olt (2002b–c).

Figure 1. The principal scheme (a) of the combined wide row potato field processing machine and its working process (b), where
1– supporting wheel,
2 – S-tine,
3 – free-active circular links,
4 – duckfoot shear,
5 – free-active disk–ridging tool,
6 – ridge bottom former,
α – angle of attack,
Δc – shift of the S-tine,
v_m – speed of the machine

The principal scheme of a new combined wide row potato field tillage machine, described by Olt (2001) and Olt, Heinloo (2001c) in details, is seen in Fig. 1.

Fig. 2 shows the first combined wide row potato field tillage machine created, according to the scheme in Fig. 1, for the experiments. The results of the experimental study on the ability of disk–ridging tools in moulding ridges of a potato field are discussed by Olt (1999) in detail.

Heinloo, Olt (2004a, 2004b, 2004c), differently from their previous papers, have presented the results of study a forced driven disk–ridging tool, driven by transmission from tractor in the structure combined wide row potato field tillage machine.

This paper concentrates the main attention to the description of the method of virtual reality on creation a disk–ridging tool for the combined wide row potato field processing machine, supporting the new technology of growing seed potatoes of Sinijärv (1982).

2. DESCRIPTION OF A DISK–RIDGING TOOL

In the present paper a disk–ridging tool is considered as a disk of radius \( r_m \) with trapezoidal blades, clamped to it (Figs. 3, 4) under adjustment angle \( \xi \) to the radial direction of the disk.

The trapezoidal blades of the disk–ridging tool have the width \( d \) and the angle of inclination \( \zeta \) to the plane of the disk of the disk–ridging tool. The disk–ridging tool has an angle of attack \( \alpha \) (Fig. 5) to the direction of its forward motion together with whole machine and the angle of
inclusion $\beta$ (Figs. 4, 5) to the lateral surface of the ridge. The disk–ridging tool has the complex motion in the structure of potato field tillage machine: it traversing together with the whole machine and rotates relatively to the frame of the machine.

Fig. 4 shows the disk–ridging tool in a working position on the lateral surface of a ridge of a potato field. According to the agricultural engineering requirements the tolerance between the upper edges of the lateral surface of the potato ridge is $\delta$ and the distance between the centre of disk and the lateral surface is $H$.

### 3. DETERMINATION OF POSITIONS OF A DISK–RIDGING TOOL

The positions of the disk–ridging tool can be determined by the co-ordinate systems, shown in Fig. 5. The axes of the motionless observation system $Oxyz$ are directed as follows. Axis $Oz$ is perpendicular to the lateral surface of the ridge. Axis $Oy$ is perpendicular to the axis $Oz$ and shows the direction of traversing of the machine, carrying the disk–ridging tool. Axis $Ox$ is perpendicular to the axes $Oz$ and $Oy$ and forms together with these axes the rectangular right hand co-ordinate system. The auxiliary co-ordinate system $O_1x_3y_3z_2$ was supposed to be in forward motion together with the whole machine. Its position at the initial position is $Ox_2y_2z_2$. The position of rigidly connected to the tool co-ordinate system $O_1x_3y_3z_2$ determines the position of the disk–ridging tool relative to the system $Oxyz$ by the angle of attack $\alpha$, by the angle of inclination $\beta$, by the angle of rotation $\varphi$ and by the co-ordinates of the origin $O_1$.

![Figure 5. Used co-ordinate systems](image)

The co-ordinates $x, y, z$ of a point $P$ (Fig. 5) in the co-ordinate system $Oxyz$ are determined by the transformation

$$
\begin{bmatrix}
  x(\alpha, \beta, \varphi, r, \psi, z_2) \\
  y(\alpha, \beta, \varphi, r, \psi, z_2, t) \\
  z(\alpha, \beta, \varphi, r, \psi, z_2, H)
\end{bmatrix}
= A(\alpha)B(\beta)C(\varphi)
\begin{bmatrix}
  x_3(r, \psi) \\
  y_3(r, \psi) \\
  z_2
\end{bmatrix}
+ \begin{bmatrix}
  0 \\
  v_m t \\
  H
\end{bmatrix},
$$

(1)

where $x_3(r, \psi) = r \cos(\psi)$, $y_3(r, \psi) = r \sin(\psi)$, $v_m$ – the velocity of forward motion of the machine (origin $O_1$); $H$ – the distance between the lateral surface of the ridge and origin $O_1$; $r$ and $\psi$ – polar radius and polar angle of a point $P$ (Fig. 5) of the disk–ridging tool in the co-ordinate system $Ox_3y_3z_2$ and

$$
A(\alpha) = \begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos(\alpha) & -\sin(\alpha) \\
  0 & \sin(\alpha) & \cos(\alpha)
\end{bmatrix},
B(\beta) = \begin{bmatrix}
  \cos(\beta) & 0 & \sin(\beta) \\
  0 & 1 & 0 \\
  -\sin(\beta) & 0 & \cos(\beta)
\end{bmatrix},
C(\varphi) = \begin{bmatrix}
  \cos(\varphi) & \sin(\varphi) & 0 \\
  -\sin(\varphi) & \cos(\varphi) & 0 \\
  0 & 0 & 1
\end{bmatrix}.
$$

During computations the values of the initial values of parameters of the disk–ridging tool

were specified and have in the present paper the following final values (Figs. 1–3): 
\[ \delta = 0.015 \text{ m}; \quad h = 0.05 \text{ m}; \quad d = 0.2 \text{ m}; \quad \xi = 20^\circ (-20^\circ); \quad \alpha = 8^\circ; \quad \beta = 4^\circ; \quad H = 0.043 \text{ m}; \quad \zeta = 7.8^\circ; \quad T = 0.5 \text{ s}; \quad N = 1000; \quad M = 8; \quad \omega = 16 (8, 12) 1/\text{s}; \quad \pi = 3.1416; \quad r_m = 0.325 \text{ m}. \]

4. DELVING PROCESS OF A BLADE INTO A SOIL

For study hereafter the forces, applied to the underground part of the blades of the disk-ridging tool, the detail know-how of the delving process of the blades into the soil of the ridge is needed.

Let us suppose that the process of delving a blade into a soil of a ridge is the following. Firstly delves into a soil of a ridge the corner point 3 of a blade. Then, during some time, the underground part of a blade has the triangular form (hatched region 2 in Fig. 6a). After delving the corner point 1 of the blade into a soil of a ridge the underground part of the blade changes to the tetragonal form (hatched regions 1 and 2 in Fig. 6b). Finally, after delving the point 6 of the blade into a soil the underground part of a blade changes to the pentagon form (hatched regions 1, 2 and 3 in Fig. 6c). The blade leaves the soil of the ridge on the opposite order.

5. VIRTUAL PROJECTIONS OF A BLADE

Let \( \psi_j = \frac{\pi}{2} + \frac{2\pi j}{M} \) is the polar angle of the end of a \( j \)-th blade (Fig. 3), \( \varphi_i = \omega t_i \) – the angle of rotation (Fig. 5) at the \( i \)-th \( (i = 0, 1, 2, \ldots, N) \) position of the disk-ridging tool, \( M \) – the number of the blades, \( \omega \) – the angular velocity of the disk-ridging tool and \( t_i \) – the moment of the time at the \( i \)-th position. The co-ordinates of the corner points 1, 2, 3, 6 (Fig. 6) of the \( j \)-th blade at the \( i \)-th position of the disk-ridging tool are

\[
\begin{align*}
x_{1_{i,j}} &= x(\alpha, \beta, \varphi_i, r_m, \psi_j, -h),
&= y(\alpha, \beta, \varphi_i, r_m, \psi_j, -h, t_i),
&= z(\alpha, \beta, \varphi_i, r_m, \psi_j, -h, H),
\end{align*}
\]

\[
\begin{align*}
x_{2_{i,j}} &= x(\alpha, \beta, \varphi_i, \rho, \psi_j + \psi_{\rho}, 0),
&= y(\alpha, \beta, \varphi_i, \rho, \psi_j + \psi_{\rho}, 0, t_i),
&= z(\alpha, \beta, \varphi_i, \rho, \psi_j + \psi_{\rho}, 0, H),
\end{align*}
\]

\[
\begin{align*}
x_{3_{i,j}} &= x(\alpha, \beta, \varphi_i, \rho, \psi_j + \psi_{\rho}, -h_1),
&= y(\alpha, \beta, \varphi_i, \rho, \psi_j + \psi_{\rho}, -h_1 t_i),
&= z(\alpha, \beta, \varphi_i, \rho, \psi_j + \psi_{\rho}, -h_1 H),
\end{align*}
\]

\[
\begin{align*}
x_{6_{i,j}} &= x(\alpha, \beta, \varphi_i, r_m, \psi_j, 0),
&= y(\alpha, \beta, \varphi_i, r_m, \psi_j, 0, t_i),
\end{align*}
\]

\[ z_{0,j} = z(\alpha, \beta, \varphi_j, r_m, \psi_j, 0, H) . \]

Let us compose the following vectors of corner’s co-ordinates of the \( j \)-th blade at the \( i \)-th position

\[
\begin{align*}
\mathbf{s}_x(i, j) &= \begin{pmatrix}
  x(\alpha, \beta, \varphi_j, r_m, \psi_j, -h) \\
  x(\alpha, \beta, \varphi_j, r_m, \psi_j, 0) \\
  x(\alpha, \beta, \varphi_j, r_m, \psi_j, +\psi_\rho, 0) \\
  x(\alpha, \beta, \varphi_j, r_m, \psi_j, +\psi_\rho, -h)
\end{pmatrix}, \\
\mathbf{s}_y(i, j) &= \begin{pmatrix}
  y(\alpha, \beta, \varphi_j, r_m, \psi_j, -h, t) \\
  y(\alpha, \beta, \varphi_j, r_m, \psi_j, 0, t) \\
  y(\alpha, \beta, \varphi_j, r_m, \psi_j, +\psi_\rho, 0, t) \\
  y(\alpha, \beta, \varphi_j, r_m, \psi_j, +\psi_\rho, -h, t)
\end{pmatrix},
\end{align*}
\]

\[
\mathbf{s}_z(i, j) = \begin{pmatrix}
  z(\alpha, \beta, \varphi_j, r_m, \psi_j, -h) \\
  z(\alpha, \beta, \varphi_j, r_m, \psi_j, 0) \\
  z(\alpha, \beta, \varphi_j, r_m, \psi_j, +\psi_\rho, 0) \\
  z(\alpha, \beta, \varphi_j, r_m, \psi_j, +\psi_\rho, -h)
\end{pmatrix}, \quad (2)
\]

where \( \varphi_i = \omega T_i / N \), \( \psi_\rho = \arcsin \left( \frac{d \sin(\zeta)}{\rho} \right) \), \( \rho = \sqrt{d^2 + r_m^2 - 2r_m d \cos(\zeta)} \), \( \psi_j = \frac{\pi}{2} + \frac{2\pi}{M} j \), \( h_1 = h + d \tan(\zeta) \), \( \varphi_i = \frac{\omega T_i}{N} i \), \( t_i = \frac{T}{N} i \). Here \( i = 0, 1, 2, \ldots, N \); \( j = 0, 1, 2, \ldots, M \); \( \omega \) – angular velocity, \( T \) – given interval of time, \( N \) – number of positions in the computations, \( M \) – number of blades of the disk–ridging tool. In the environment of computer package Mathcad the vectors (2) can be used for drawing virtual projections of \( j \)-th blade at \( i \)-th position. Fig. 7 shows the virtual projections of the 0-th blade on the co-ordinate planes Oxy (Fig. 7a), Oyz (Fig. 7b), Oxz (Fig. 7c) of the motionless observation system Oxyz (Fig. 5) at the 0-th position, created in the environment of computer package Mathcad. In Fig. 7 the hatching denotes the region to be processed by the blades of the virtual disk-ridging tool.
Figure 7. Virtual projections of the 0–th blade on the co-ordinate planes Oxy (a), Oyz (b), Ozx (c) of the motionless observation system Oxyz (Fig. 5) at the 0-th position

By animation, the motion of the blade in Fig. 7 one can confirm the assumption made in Fig. 6.

6. VIRTUAL PROJECTION OF ASSEMBLY OF BLADES

Let us compose new vectors $S_x(j)$, $S_y(j)$, $S_z(j)$ by putting the vectors $s_x(i,j)$, $s_y(i,j)$, $s_z(i,j)$.
above the vectors $s_x(i, j+1), s_y(i, j+1), s_z(i, j+1)$ ($i = 0, 1, 2, \ldots, N; j = 0, 1, 2, \ldots, M$), accordingly. The Mathcad programs

$$S_X(i) = \begin{cases} S_X \leftarrow s_X(i, 0) \\ for \ j \in 1..M \\ S_X \leftarrow stack(S_X, s_X(i, j)) \end{cases}, \quad S_Y(i) = \begin{cases} S_Y \leftarrow s_Y(i, 0) \\ for \ j \in 1..M \\ S_Y \leftarrow stack(S_Y, s_Y(i, j)) \end{cases},$$

$$S_X(i) = \begin{cases} S_X \leftarrow s_X(i, 0) \\ for \ j \in 1..M \\ S_X \leftarrow stack(S_X, s_X(i, j)) \end{cases}.$$

do this procedure. In the environment of computer package Mathcad vectors $S_x(i), S_y(i), S_z(i)$ imagine the virtual projections of assembly of $M$ blades at the $i$-th position on the co-ordinate planes of the motionless observation system $Oxyz$ (Fig. 5). This assembly of $M$ blades is named below as virtual disk-ridging tool.

Fig. 8 shows the virtual projections at the 0-th position of the virtual disk-ridging tool with 8 blades on the co-ordinate planes $Oxy$ (Fig. 8a), $Oyz$ (Fig. 8b), $Ozx$ (Fig. 8c) of the system $Oxyz$. These projections were obtained in the environment of computer package Mathcad by using the vectors $S_x(0), S_y(0), S_z(0)$. In Fig. 2.5 the blades of the virtual disk-ridging tool are connected together with an auxiliary line.
Figure 8. Virtual projections of the 0–th disk–riding tool on the co-ordinate planes
Oxy (a), Oyz (b), Ozx (c) of the system Oxyz at the 0-th position
Fig. 8 was used for making a video clip with the motion of the virtual disk-ridging tool.

7. DETERMINATION OF THE UNDERGROUND PART OF A BLADE OF THE DISK–RIDGING TOOL

The co-ordinates $x_{4_{i,j}}$ and $y_{4_{i,j}}$ of the point 4 (Fig. 6) were found out form conditions $z_{4_{i,j}} = 0$, $z_{5_{i,j}} = 0$ and are

$$x_{4_{i,j}} = x_{3_{i,j}} + \frac{x_{2_{i,j}} - x_{3_{i,j}}}{z_{3_{i,j}} - z_{2_{i,j}}} z_{3_{i,j}}, \quad y_{4_{i,j}} = y_{3_{i,j}} + \frac{y_{2_{i,j}} - y_{3_{i,j}}}{z_{3_{i,j}} - z_{2_{i,j}}} z_{3_{i,j}}.$$

The point 5 may be located on three sides of a blade, thus its co-ordinates $x_{5_{i,j}}$ and $y_{5_{i,j}}$ are

$$x_{5_{i,j}} = x_{3_{i,j}} + \frac{x_{1_{i,j}} - x_{3_{i,j}}}{z_{3_{i,j}} - z_{1_{i,j}}} z_{3_{i,j}}, \quad y_{5_{i,j}} = y_{3_{i,j}} + \frac{y_{1_{i,j}} - y_{3_{i,j}}}{z_{3_{i,j}} - z_{1_{i,j}}} z_{3_{i,j}},$$

if $z_{1_{i,j}} \geq 0$ (Fig. 6a) and

$$x_{5_{i,j}} = x_{1_{i,j}} + \frac{x_{6_{i,j}} - x_{1_{i,j}}}{z_{1_{i,j}} - z_{6_{i,j}}} z_{1_{i,j}}, \quad y_{5_{i,j}} = y_{1_{i,j}} + \frac{y_{6_{i,j}} - y_{1_{i,j}}}{z_{1_{i,j}} - z_{6_{i,j}}} z_{1_{i,j}},$$

if $z_{1_{i,j}} \leq 0$ and $z_{6_{i,j}} \geq 0$ (Fig. 6b),

$$x_{5_{i,j}} = x_{2_{i,j}} + \frac{x_{6_{i,j}} - x_{2_{i,j}}}{z_{2_{i,j}} - z_{6_{i,j}}} z_{2_{i,j}}, \quad y_{5_{i,j}} = y_{2_{i,j}} + \frac{y_{6_{i,j}} - y_{2_{i,j}}}{z_{2_{i,j}} - z_{6_{i,j}}} z_{2_{i,j}},$$

if $z_{6_{i,j}} \leq 0$ (Fig. 6c).

Let us compose now the following vectors

$$s_{1x}(i,j) = \begin{pmatrix} x_{3_{i,j}} \\ x_{4_{i,j}} \\ x_{5_{i,j}} \\ x_{3_{i,j}} \end{pmatrix}, \quad s_{1y}(i,j) = \begin{pmatrix} y_{3_{i,j}} \\ y_{4_{i,j}} \\ y_{5_{i,j}} \\ y_{3_{i,j}} \end{pmatrix}, \quad s_{1z}(i,j) = \begin{pmatrix} z_{3_{i,j}} \\ z_{4_{i,j}} \\ z_{5_{i,j}} \\ z_{3_{i,j}} \end{pmatrix},$$

$$s_{2x}(i,j) = \begin{pmatrix} x_{3_{i,j}} \\ x_{4_{i,j}} \\ x_{5_{i,j}} \\ x_{3_{i,j}} \end{pmatrix}, \quad s_{2y}(i,j) = \begin{pmatrix} y_{3_{i,j}} \\ y_{4_{i,j}} \\ y_{5_{i,j}} \\ y_{3_{i,j}} \end{pmatrix}, \quad s_{2z}(i,j) = \begin{pmatrix} z_{3_{i,j}} \\ z_{4_{i,j}} \\ z_{5_{i,j}} \\ z_{3_{i,j}} \end{pmatrix}.$$
In the environment of the computer package Mathcad vectors $s'_{x}(i,j)$, $s'_{y}(i,j)$, $s'_{z}(i,j)$ can be used for imagination of virtual projections of the underground part of $j$-th blade at $i$-th position on the co-ordinate planes of the observation system Oxyz. Fig. 9 shows the virtual projections (hatching denote the region to be processed) of the underground part of the 0-th blade in 0-th position blade on the co-ordinate planes Oxy (Fig. 7a), Oyz (Fig. 7b), Ozx (Fig. 7c) of the system Oxyz (Fig. 5).
Figure 9. Virtual projections of the underground part of the 0-th blade on the co-ordinate planes Oxy (a), Oyz (b), Ozx (c) of the motionless observation system Oxyz (Fig. 5) at the 0-th position.
8. IMAGINATION OF PROCESSED AREA

Let us compose now vectors $S'_x(j)$, $S'_y(j)$, $S'_z(j)$ by putting the vectors $s'''_x(i,j)$, $s'''_y(i,j)$, $s'''_z(i,j)$ above the vectors $s'''_x(i+1,j)$, $s'''_y(i+1,j)$, $s'''_z(i+1,j)$ ($i = 0, 1, 2, \ldots, N; j = 0, 1, 2, \ldots, M$), accordingly. The Mathcad programs

$$S'_x(j) = \begin{cases} S'_x \leftarrow s'''_x(0,j) \\ for \ i \in 1..N \\ S'_x \leftarrow \text{stack}(S'_x, s'''_x(i,j)) \end{cases}$$

$$S'_y(j) = \begin{cases} S'_y \leftarrow s'''_y(0,j) \\ for \ i \in 1..N \\ S'_y \leftarrow \text{stack}(S'_y, s'''_y(i,j)) \end{cases}$$

$$S'_z(j) = \begin{cases} S'_z \leftarrow s'''_z(0,j) \\ for \ i \in 1..N \\ S'_z \leftarrow \text{stack}(S'_z, s'''_z(i,j)) \end{cases}$$

do this procedure. In the environment of computer package Mathcad vectors $S'_x(j)$, $S'_y(j)$, $S'_z(j)$ can be used for presentation the virtual projections at all positions of the underground part of $j$-th blade on the co-ordinate planes of the system Oxyz (Fig. 5). The set of these projections determine the lateral space region on of the virtual ridge, processed by $j$-th blade. All $N$ virtual projections of all $M$ blades form the space layer, processed by assembly of $M$ blades. Figs. 10 – 12 show the virtual projections of this layer on the co-ordinate plane Oxy, when $\xi = 20^o$ (Fig. 3) and the combined wide row potato field tillage machine together with disk–ridging tool moves in the direction of y-axis of the system Oxyz (Fig. 3) with the traversing velocity $v_m = 2.6$ m/s and the disk–ridging tool rotates with the angular velocities $\omega = 8$ 1/s (Fig. 10), $\omega = 12$ 1/s (Fig. 11), $\omega = 16$ 1/s (Fig. 12).
From Figs. 10 – 12 one can turn out that the quality of processing depends substantially from the angular velocity of rotation of the disk–ridging tool. Fig. 12 shows that the disk–ridging tool with $M = 8$ blades, working with forced angular velocity $\omega = 16\;1/s$ and traversing with velocity $v_m = 2.6\;m/s$ gives sufficient quality of processing of the space lateral layer of a ridge in direction of co-ordinate plane Oxy, when $\xi = 20^\circ$.
Figs. 13 and 14 show the virtual projections of the processed space layer to the co-ordinate planes Oyz (Fig. 13) and Oxz (Fig. 14). It follows from Figs. 13 and 14 that the disk–ridging tool with given in this paper parameters is able process a soil up to depth 5.6 cm, but no uniformly.
Figure 13. Virtual projection of the processed part of the lateral layer on the co-ordinate plane Oyz ($\omega = 16 \text{ 1/s, } \xi = 20^\circ$).

Figure 14. Virtual projection of the processed part of the lateral layer on the co-ordinate plane Oxz ($\omega = 16 \text{ 1/s, } \xi = 20^\circ$).

It turned out that the value of the adjustment angle $\xi$ (Fig. 2.1) of the disk–ridging tool considerably affects the quality of processing. Fig. 15 shows that in the case, when $\omega = 16 \text{ 1/s, but } \xi = -20^\circ$ the layer has several regions, not processed.
9. AREA OF THE UNDERGROUND PART OF A BLADE

The area of the underground part of a blade is

\[ S_{i,j} = \frac{1}{2} \sqrt{n_{x,i,j}^2 + n_{y,i,j}^2 + n_{z,i,j}^2}, \]

where

\[ n_{x,i,j} = (y_{5,i,j} - y_{3,i,j})(z_{4,i,j} - z_{5,i,j}) - (z_{5,i,j} - z_{3,i,j})(y_{4,i,j} - y_{5,i,j}), \]
\[ n_{y,i,j} = (z_{5,i,j} - z_{3,i,j})(x_{4,i,j} - x_{5,i,j}) - (x_{5,i,j} - x_{3,i,j})(z_{4,i,j} - z_{5,i,j}), \]
\[ n_{z,i,j} = (x_{5,i,j} - x_{3,i,j})(y_{4,i,j} - y_{5,i,j}) - (y_{5,i,j} - y_{3,i,j})(x_{4,i,j} - x_{5,i,j}). \]

If \( z_{1,i,j} \geq 0 \) (Fig. 4a), \( S_{i,j} = n^{(1)}_{i,j} + n^{(2)}_{i,j} \), where

\[ n^{(1)}_{x,i,j} = \frac{n^{(1)}_{x,i,j} y_{1,i,j}^2 + n^{(1)}_{y,i,j} z_{1,i,j}^2 + n^{(1)}_{z,i,j} x_{1,i,j}^2}{2}, \]
\[ n^{(2)}_{x,i,j} = \frac{n^{(2)}_{x,i,j} y_{1,i,j}^2 + n^{(2)}_{y,i,j} z_{1,i,j}^2 + n^{(2)}_{z,i,j} x_{1,i,j}^2}{2}, \]
\[ n^{(1)}_{y,i,j} = (y_{1,i,j} - y_{3,i,j})(z_{4,i,j} - z_{5,i,j}) - (z_{5,i,j} - z_{3,i,j})(y_{4,i,j} - y_{5,i,j}), \]
\[ n^{(2)}_{y,i,j} = (y_{1,i,j} - y_{3,i,j})(z_{4,i,j} - z_{5,i,j}) - (z_{5,i,j} - z_{3,i,j})(y_{4,i,j} - y_{5,i,j}), \]
\[ n^{(1)}_{z,i,j} = (x_{1,i,j} - x_{3,i,j})(y_{4,i,j} - y_{5,i,j}) - (y_{5,i,j} - y_{3,i,j})(x_{4,i,j} - x_{5,i,j}), \]
\[ n^{(2)}_{z,i,j} = (x_{1,i,j} - x_{3,i,j})(y_{4,i,j} - y_{5,i,j}) - (y_{5,i,j} - y_{3,i,j})(x_{4,i,j} - x_{5,i,j}). \]
\begin{align*}
n^{(2)}_{x_{i,j}} &= \frac{1}{3} \left( y_{4_{i,j}} - y_{3_{i,j}} \right) \left( z_{4_{i,j}} - z_{3_{i,j}} \right) - \left( z_{4_{i,j}} - z_{3_{i,j}} \right) \left( y_{4_{i,j}} - y_{3_{i,j}} \right), \\
n^{(2)}_{y_{i,j}} &= \frac{1}{3} \left( x_{4_{i,j}} - x_{3_{i,j}} \right) \left( z_{4_{i,j}} - z_{3_{i,j}} \right) - \left( x_{4_{i,j}} - x_{3_{i,j}} \right) \left( z_{4_{i,j}} - z_{3_{i,j}} \right), \\
n^{(2)}_{z_{i,j}} &= \frac{1}{3} \left( x_{4_{i,j}} - x_{3_{i,j}} \right) \left( y_{4_{i,j}} - y_{3_{i,j}} \right) - \left( y_{4_{i,j}} - y_{3_{i,j}} \right) \left( x_{4_{i,j}} - x_{3_{i,j}} \right),
\end{align*}

if \( z_{i,j} \leq 0 \) and \( z_{6_{i,j}} \geq 0 \) (Fig. 4b) and

\[ S_{i,j} = n^{(2)}_{i,j} + n^{(3)}_{i,j} + n^{(4)}_{i,j}, \]

where

\[ n^{(3)}_{x_{i,j}} = \frac{\sqrt{n^{(1)}_{x_{i,j}} + n^{(3)}_{x_{i,j}} + n^{(3)}_{z_{i,j}}}}{2}, \quad n^{(3)}_{y_{i,j}} = \frac{\sqrt{n^{(4)}_{x_{i,j}} + n^{(4)}_{y_{i,j}} + n^{(4)}_{z_{i,j}}}}{2}, \quad n^{(4)}_{z_{i,j}} = \frac{\sqrt{n^{(1)}_{x_{i,j}} + n^{(3)}_{x_{i,j}} + n^{(3)}_{z_{i,j}}}}{2}, \quad n^{(4)}_{y_{i,j}} = \frac{\sqrt{n^{(4)}_{x_{i,j}} + n^{(4)}_{y_{i,j}} + n^{(4)}_{z_{i,j}}}}{2}, \quad n^{(4)}_{z_{i,j}} = \frac{\sqrt{n^{(1)}_{x_{i,j}} + n^{(3)}_{x_{i,j}} + n^{(3)}_{z_{i,j}}}}{2}, \]

if \( z_{6_{i,j}} \leq 0 \) (Fig. 4c).

Fig. 16 shows the dependence of the area of the underground part of the \( j \)-th blade \((j = 0)\) on angle \( \varphi \) of rotation of the disk–ridging tool.

\[ S_{i,0} \]

Figure 16. The dependence of the underground area of the 0-th blade, on angle of rotation \( \varphi \) of the disk–ridging tool, when \( \xi = 20^\circ \).

10. CENTRE OF MASS OF THE UNDERGROUND PART OF A BLADE

The co-ordinates of the centre of mass of the underground part of a blade are

\[ x_{M_{i,j}} = \frac{x_{4_{i,j}} + x_{3_{i,j}} + x_{5_{i,j}}}{3}, \quad y_{M_{i,j}} = \frac{y_{3_{i,j}} + y_{4_{i,j}} + y_{5_{i,j}}}{3}, \quad z_{M_{i,j}} = \frac{z_{3_{i,j}} + z_{4_{i,j}} + z_{5_{i,j}}}{3}, \]

if \( z_{i,j} \geq 0 \) (Fig. 6a),

\[ x_{M_{i,j}} = \frac{x_{i,j} n^{(1)}_{i,j} + x_{2_{i,j}} n^{(2)}_{i,j}}{n^{(1)}_{i,j} + n^{(2)}_{i,j}}, \quad y_{M_{i,j}} = \frac{y_{i,j} n^{(1)}_{i,j} + y_{2_{i,j}} n^{(2)}_{i,j}}{n^{(1)}_{i,j} + n^{(2)}_{i,j}}, \quad z_{M_{i,j}} = \frac{z_{i,j} n^{(1)}_{i,j} + z_{2_{i,j}} n^{(2)}_{i,j}}{n^{(1)}_{i,j} + n^{(2)}_{i,j}}, \]

where

\[ x_{1_{i,j}} = \frac{x_{1_{i,j}} + x_{4_{i,j}} + x_{5_{i,j}}}{3}, \quad y_{1_{i,j}} = \frac{y_{1_{i,j}} + y_{4_{i,j}} + y_{5_{i,j}}}{3}, \quad z_{1_{i,j}} = \frac{z_{1_{i,j}} + z_{4_{i,j}} + z_{5_{i,j}}}{3}, \]
\[ x_{2_{i,j}} = \frac{x_{3_{i,j}} + x_{4_{i,j}} + x_{5_{i,j}}}{3}, \quad y_{2_{i,j}} = \frac{y_{3_{i,j}} + y_{4_{i,j}} + y_{5_{i,j}}}{3}, \quad z_{2_{i,j}} = \frac{z_{3_{i,j}} + z_{4_{i,j}} + z_{5_{i,j}}}{3}, \]

if \( z_{i_{i,j}} \leq 0 \) and \( z_{6_{i,j}} \geq 0 \) (Fig. 6b) and
\[ x_{M_{i,j}} = \frac{x^{(1)}_{1_{i,j}} n^{(3)}_{i,j} + x^{(2)}_{2_{i,j}} n^{(4)}_{i,j} + x^{(3)}_{3_{i,j}} n^{(1)}_{i,j}}{n^{(3)}_{i,j} + n^{(2)}_{i,j} + n^{(1)}_{i,j}}, \quad y_{M_{i,j}} = \frac{y^{(1)}_{1_{i,j}} n^{(3)}_{i,j} + y^{(2)}_{2_{i,j}} n^{(4)}_{i,j} + y^{(3)}_{3_{i,j}} n^{(1)}_{i,j}}{n^{(3)}_{i,j} + n^{(2)}_{i,j} + n^{(1)}_{i,j}}, \quad z_{M_{i,j}} = \frac{z^{(1)}_{1_{i,j}} n^{(3)}_{i,j} + z^{(2)}_{2_{i,j}} n^{(4)}_{i,j} + z^{(3)}_{3_{i,j}} n^{(1)}_{i,j}}{n^{(3)}_{i,j} + n^{(2)}_{i,j} + n^{(1)}_{i,j}}, \]

where
\[ x^{(1)}_{1_{i,j}} = \frac{x_{1_{i,j}} + x_{4_{i,j}} + x_{6_{i,j}}}{3}, \quad y^{(1)}_{1_{i,j}} = \frac{y_{1_{i,j}} + y_{4_{i,j}} + y_{6_{i,j}}}{3}, \quad z^{(1)}_{1_{i,j}} = \frac{z_{1_{i,j}} + z_{4_{i,j}} + z_{6_{i,j}}}{3}, \]
\[ x^{(3)}_{3_{i,j}} = \frac{x_{3_{i,j}} + x_{4_{i,j}} + x_{5_{i,j}}}{3}, \quad y^{(3)}_{3_{i,j}} = \frac{y_{3_{i,j}} + y_{4_{i,j}} + y_{5_{i,j}}}{3}, \quad z^{(3)}_{3_{i,j}} = \frac{z_{3_{i,j}} + z_{4_{i,j}} + z_{5_{i,j}}}{3}. \]

if \( z_{6_{i,j}} \leq 0 \) (Fig. 6c).

11. MODELLING THE FORCE SYSTEM

The projections of the vector \( \vec{N} \), normal to the \( j \)-th blade on the \( i \)-th position, on axes Ox, Oy, Oz of co-ordinate system Oxyz are
\[ N_{x_{i,j}} = (y_{2_{i,j}} - y_{3_{i,j}})(z_{2_{i,j}} - z_{3_{i,j}}) - (z_{2_{i,j}} - z_{3_{i,j}})(y_{2_{i,j}} - y_{3_{i,j}}), \]
\[ N_{y_{i,j}} = (z_{2_{i,j}} - z_{3_{i,j}})(x_{2_{i,j}} - x_{3_{i,j}}) - (x_{2_{i,j}} - x_{3_{i,j}})(z_{2_{i,j}} - z_{3_{i,j}}), \]
\[ N_{z_{i,j}} = (x_{2_{i,j}} - x_{3_{i,j}})(y_{2_{i,j}} - y_{3_{i,j}}) - (y_{2_{i,j}} - y_{3_{i,j}})(x_{2_{i,j}} - x_{3_{i,j}}). \]

The transformation
\[ \begin{pmatrix} x_{M_{3_{i,j}}} \\ y_{M_{3_{i,j}}} \\ z_{M_{2_{i,j}}} \end{pmatrix} = C(\phi)^{-1} B(\beta)^{-1} A(\alpha)^{-1} \begin{pmatrix} x_{M_{i,j}} \\ y_{M_{i,j}} - v_m I \\ z_{M_{i,j}} \end{pmatrix} \]
transforms the co-ordinates of the centres of mass of the underground parts of the blades, in the system Oxyz into the correspondent co-ordinates in the system O1x3y3z2 (Fig. 5) rigidly connected to the disk–ridging tool. The velocity of the centre of mass of \( j \)-th blade at the \( i \)-th position in the system Oxyz is determined by the following formulas
\[ v_{x_{i,j}} = \omega \cos(\beta) \left[ y_{M_{3_{i,j}}} \cos(\phi) - x_{M_{3_{i,j}}} \sin(\phi) \right], \]

\[ v_{y_{i,j}} = \omega \left\{ \sin(\alpha) \sin(\beta) \left[ x_{M_{i,j}} \cos(\varphi_i) - x_{M_{i,j}} \sin(\varphi_i) \right] \right\} + v_m, \]
\[ v_{z_{i,j}} = \omega \left\{ \cos(\alpha) \sin(\beta) \left[ x_{M_{i,j}} \sin(\varphi_i) - y_{M_{i,j}} \cos(\varphi_i) \right] \right\}, \]

which are derived from transformation (1) after its expanding, substitution \( x_i(r, \psi) = x_{M_{i,j}}, \)
\( y_i(r, \psi) = y_{M_{i,j}} \) and differentiation by time \( t. \)

The projection of the area \( S_{y_{i,j}} \) of the underground part of a \( j \)-th blade at \( i \)-th position on the
plane, perpendicular to the direction of the velocity \( v_{i,j} \) of the blade in the system \( Oxyz, \)
is
\[ S_{y_{i,j}} = S_{z_{i,j}} \left| \frac{N_{y_{i,j}} v_{y_{i,j}} + N_{z_{i,j}} v_{z_{i,j}}}{\sqrt{N_{x_{i,j}}^2 + N_{y_{i,j}}^2 + N_{z_{i,j}}^2}} \right| v_{x_{i,j}} + v_{y_{i,j}}^2 + v_{z_{i,j}}^2. \]

Force, applied to the centre of mass of underground part of \( j \)-th blade at the \( i \)-th position, can
be determined by the formula Goryachkin (1968)
\[ \vec{F}_{v_{i,j}} = -\left( k_a + \rho c v_{i,j}^2 \right) S_{v_{i,j}} \frac{\vec{v}_{i,j}}{v_{i,j}}, \]
where \( k_a \) is the initial resistivity of soil, \( \rho \) – density of a soil, \( c \) – shape coefficient of a blade,
\( \vec{v}_{i,j} \) – velocity of the centre of mass of the underground part of the \( j \)-th blade at the \( i \)-th posi-
tion in the system \( Oxyz \) (Fig. 5), \( v_{i,j} \) – modulus of the velocity \( \vec{v}_{i,j}. \) The projections on axes
Ox, Oy, Oz of the forces \( \vec{F}_{v_{i,j}} \) are
\[ F_{x_{i,j}} = -k_a S_{v_{i,j}} \frac{v_{x_{i,j}}}{v_{i,j}} - \rho c S_{v_{i,j}} v_{y_{i,j}} v_{x_{i,j}}, F_{y_{i,j}} = -k_a S_{v_{i,j}} \frac{v_{y_{i,j}}}{v_{i,j}} - \rho c S_{v_{i,j}} v_{y_{i,j}} v_{y_{i,j}}, \]
\[ F_{z_{i,j}} = -k_a S_{v_{i,j}} \frac{v_{z_{i,j}}}{v_{i,j}} - \rho c S_{v_{i,j}} v_{z_{i,j}} v_{z_{i,j}}. \]

The computations show that for \( \xi = 20^\circ \) and \( \xi = -20^\circ, \) the disk–riding tool has the same
working depth. Let us compare below the parameters of the force systems, applied to the
disk–riding tool for \( \xi = 20^\circ \) and \( \xi = -20^\circ, \) when \( k_a = 50000 \text{ Pa}; \rho = 1300 \text{ kg/m}^3; c = 0.6, \)
\( \omega = 16 \text{ 1/s}. \)

Fig. 17 shows the dependence of \( F'_{x_{i,0}} (N), F'_{y_{i,0}} (N), F'_{z_{i,0}} (N) \) on angle of rotation \( \varphi \) (deg),
when \( \xi = 20^\circ \) (Fig. 17a) and \( \xi = -20^\circ \) (Fig. 17b).
Figure 17. The dependence the projections of the force $\vec{F}_{i,j}$ on the angle of rotation $\varphi$, when $\xi = 20^\circ$ (a) and $\xi = -20^\circ$ (b).

The projections of the resultant force $\vec{F}_i$ of the force system, applied to the disk–ridging tool at the $i$-th position are

$$F_{x_i} = \sum_{j=0}^{7} F'_{x_i,j}, \quad F_{y_i} = \sum_{j=0}^{7} F'_{y_i,j}, \quad F_{z_i} = \sum_{j=0}^{7} F'_{z_i,j}$$

Fig. 18 shows the dependence of $F_x(N), F_y(N), F_z(N)$ on the angle of rotation $\varphi$ (deg), when $\xi = 20^\circ$ (Fig. 18a) and $\xi = -20^\circ$ (Fig. 18b).
The moments $M'_{x_{i,j}}$, $M'_{y_{i,j}}$, $M'_{z_{i,j}}$ of the force $F_{i,j}$, applied to the centre of mass of the underground part of the $j$-th blade in the $i$-th position, relative to the axes Ox, Oy, Oz (Fig. 5) accordingly, are determined by formulas

$$M'_{x_{i,j}} = F'_{y_{i,j}} \left( y_{M_{i,j}} - v_{m_i} t_i \right) - F'_{z_{i,j}} \left( z_{M_{i,j}} - H \right)$$
$$M'_{y_{i,j}} = -F'_{z_{i,j}} \left( x_{M_{i,j}} + F'_{x_{i,j}} \left( z_{M_{i,j}} - H \right) \right.$$  
$$M'_{z_{i,j}} = F'_{y_{i,j}} \left( x_{M_{i,j}} - F'_{x_{i,j}} \left( y_{M_{i,j}} - v_{m_i} t_i \right) \right).$$

Fig. 19 shows the dependence the moments $M'_{x_{i,0}}$ (Nm), $M'_{y_{i,0}}$ (Nm), $M'_{z_{i,0}}$ (Nm) on the angle of rotation $\varphi$, when $\xi = 20^\circ$ (Fig. 19a) and $\xi = -20^\circ$ (Fig. 19b).
Figure 19. The dependence of the moments $M_{x,i,0}$, $M_{y,i,0}$, $M_{z,i,0}$ on the angle of rotation $\phi$, when $\xi = 20^\circ$ (a) and $\xi = -20^\circ$ (b).

The resultant moments relative to the axes $Ox$, $Oy$, $Oz$ (Fig. 5), applied to the disk-riding tool in the $i$-th position, are determined by formulas

$$M_{x,i} = \sum_{j=0}^{7} M_{x,i,j}^\prime, \quad M_{x,j} = \sum_{j=0}^{7} M_{x,j}^\prime, \quad M_{x,i,j} = \sum_{j=0}^{7} M_{x,i,j}^\prime,$$

accordingly.

Fig. 20 shows the dependence the moments $M_x$ (Nm), $M_y$ (Nm), $M_z$ (Nm) on the angle of rotation $\phi$ (deg), when $\xi = 20^\circ$ (Fig. 20a) and $\xi = -20^\circ$ (Fig. 20b).
Figure 20. The dependence of the moments $M_x, M_y, M_z$ on the angle of rotation $\phi$, when $\xi = 20^\circ$ (a) and $\xi = -20^\circ$ (b).

The following transformation

$$
\begin{pmatrix}
M_{x_j} \\
M_{y_j} \\
M_{z_j}
\end{pmatrix}
= C(\phi)^{-1} B(\beta)^{-1} A(\alpha)^{-1}
\begin{pmatrix}
M_{x_i} \\
M_{y_i} \\
M_{z_i}
\end{pmatrix}
$$

transforms the moments $M_{x_j}, M_{y_j}, M_{z_j}$ into the corresponding moments $M_{x_1}, M_{y_1}, M_{z_1}$ relative to the axes $O_1x_3, O_1y_3, O_1z_3$ (Fig. 5).

Fig. 21 shows the dependence the moments $M_{x_3}$ (Nm), $M_{y_3}$ (Nm), $M_{z_2}$ (Nm) on angle of rotation $\phi$ (deg), when $\xi = 20^\circ$ (Fig. 21a) and when $\xi = -20^\circ$ (Fig. 21b).
Figure 21. The dependence the moments $M_{x3}$, $M_{y3}$, $M_{z2}$ on the angle of rotation $\phi$, when $\xi = 20^\circ$ (a) and $\xi = -20^\circ$ (b).

It follows from Figs. 17 – 21 that the force system, applied to a blade and to the virtual disk–ridging tool substantially depends from value of the adjustment angle $\xi$ (Fig. 3). In the case of the value $\xi = 20^\circ$ the values of forces and moments are approximately twice greater than in the case $\xi = -20^\circ$. It means that the disk–ridging tool, which blades have the adjustment angle $\xi = 20^\circ$, has better ability for moving soil from the lateral surface of the ridge to its top.

12. CONCLUSIONS

The method of virtual reality, considered in this paper, can be used to predict useful information for experimental study of the working process of machine elements. By application of this method to the analysis of a virtual working process of a virtual disk–ridging tool the suitable values of it parameters are found out. One can conclude now that for the fixed working depth, velocity of forward motion and number of blades of the disk–ridging tool the quality of processing a soil in the ridge and the force system, applied to the disk–ridging tool are substantially depend on the value of the adjustment angle $\xi$ and on the angular velocity $\omega$ of the disk–ridging tool. The ability to move soil from the bottom of the ridge to its top depends to a large extent on the value of the adjustment angle $\xi$.

13. REFERENCES

Olt, J. 2001. Combined Technology of the Wide Row Potato Growing, Thesis for PhD in

Agricultural Engineering, Agricultural Machinery, 106 p.


